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MULTIPLICITIES IN DEEP HADRON-HADRON SCATTERING*

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ABSTRACT

We derive an expression for the average multiplicity for an inclusive experiment $a + b \to c + X$, where a, b, and c are hadrons and c is detected with a large transverse momentum. If E_c is the energy of c in the center-of-mass of a and b, we find

$$\langle n(s,E_c)\rangle_{\text{deep}} \sim (C_{e^+e^-}+C_X)\ln E_c^2 + C_h \ln\left(\frac{\sqrt{s}}{2E_c} - 1\right)^2$$

as s and the transverse momentum of c get large. If $E_c > \sqrt{s}/4$, the last term is absent. $C_{e^+e^-}$ and C_h are related to the average multiplicities in annihilation and small angle hadron-hadron scattering respectively, while C_X is determined by the final hadron density in the hole plateau region. We also apply our method to the semi-inclusive reaction $a+b \to c+d+X$ where c and d both have large $p_\perp$. Finally, we speculate about the value of C_X, and about nonasymptotic relations between multiplicities in various reactions.

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The idea that hadrons are, in some sense, composed of point-like constituents has motivated a number of recent speculations about the average multiplicities, <n>, in various high energy reactions. In e^+e^- annihilation,[1] the massive photon is assumed to decay into a parton-antiparton pair, each of which then decays into hadrons. In h-h scattering[1,2,3] (where h is a hadron), the major contribution to the multiplicity is supposed to come from pionization which populates the low energy region in the center-of-mass of the colliding hadrons. In deep inelastic e-h or ν-h scattering, a number of authors[1,3,4] have used the ideas of the parton model to picture the virtual γ or W knocking a parton to a distant region of phase space where it turns into hadrons, while the original hadron, which is now missing one constituent, evolves into the final state for which it is destined. For large energies and photon (or W) masses, Cahn, Cleymans, and Colglazier[4] have derived relationships between the average multiplicities in these reactions by making two further assumptions. These are (1) partons which are widely separated in phase space do not strongly affect each other,[5] and (2) most produced hadrons are distributed in energy as dE/E, typical of a bremsstrahlung process. With these additional assumptions, it is also possible to conjecture about the dependence of <n> on the relevant kinematic variables.

In this note, we wish to apply these intuitive ideas to study the average multiplicities in hadron-hadron scattering where at least one particle is observed in the final state with a large transverse momentum. Except in certain limits, large will generally mean $|p_\perp| \gtrsim 1$-2 GeV. However, we can further divide this range of $|p_\perp|$. The two subdivisions are $N\frac{\sqrt{s}}{2} \gtrsim |p_\perp| \gtrsim 1$, and $|p_\perp| \gtrsim N\frac{\sqrt{s}}{2}$, where N is some reasonable fraction. Now, it is entirely possible that different mechanisms of production dominate in these two regions, and so the results we derive below might be valid in one, but not the other, of these domains. On the one hand, this is an especially interesting point to pursue experimentally, since at least two different mechanisms which use partons have been proposed for deep scattering.[3,6] On the other hand, although our approach seems closer in spirit to that of Ref. 3, our results may be consistent with a parton interchange theory as developed in Ref. 6 due to the possibility of hadronic bremsstrahlung. The important problem of determining exactly how model-independent our results are deserves a more thorough treatment than it receives in the present work.

With these remarks in mind, let us begin by considering the inclusive process $a+b \to c+X$ where c has a large transverse momentum, $p_{\perp c}$. In the following, we shall always work in the center-of-mass of a and b, unless we specify otherwise. The picture we have in mind for the production of particles with large $p_\perp$ involves two stages. First, a high energy parton from a scatters through a large angle off a high energy parton from b. The parton distribution in phase space immediately after the scattering is shown in Fig. 1.

Next, the two partons which are each isolated in phase space separately evolve into hadrons in a way which is more or less independent of each other, as well as independent of the remaining pieces of a and b. At sufficiently high energies and momentum transfers, we expect that each such isolated parton will contribute $\sim C_{e^+e^-} \ln E_p$ to $\langle n \rangle$ for this reaction. $C_{e^+e^-}$ is related to the average multiplicity in e^+e^- annihilation by $\langle n \rangle_{e^+e^-} \sim C_{e^+e^-} \ln Q^2$, and E_p is the parton's energy in the a+b center-of-mass. This result follows from the observation that these partons are isolated in phase space, and should produce final state particles just as the isolated partons in e^+e^- annihilation do. That is, we will have hadrons distributed, on the average, as dE/E in a cylinder in phase space pointing in the direction of the liberated parton plus, perhaps, a finite parton fragmentation region. A possible exception to this is when the parton-parton scattering angle is small in the parton-parton center of mass.[7] (See below for a discussion of this point.) These regions are shown in Fig. 2, where we display the average final hadron distribution for our deep scattering event.

In addition to the e^+e^- plateaus developed by the isolated partons, the remaining pieces of a and b develop certain plateaus and fragmentation regions. Furthest from the origin we have the fragmentation regions of a and b. Moving in along the $p_\parallel$ axis, we next encounter two plateau regions. Here we also expect a distribution of hadrons like dE/E, but the coefficient in this case is C_h. C_h is defined by the asymptotic relation $\langle n\rangle_h \sim C_h \ln s$ where $\langle n\rangle_h$ is the average multiplicity for hadron-hadron collisions if we do not require a particle with large $p_\perp$. Next, we have the hole fragmentation regions[8] which result when the remaining partons try to heal the wound left in a and b by the removal of the two partons. Finally, we have two more plateaus of density C_x, as yet unknown (we shall return to this point later), and a finite overlap region at the origin where the tails of all the final hadron distributions come together.

At sufficiently large s and $p_{\perp c}$, the major contribution to the average multiplicity of the reaction $a+b \to c+X$ comes from the six plateau regions, and is given by

$$\langle n(s,p_{\perp c},E_c)\rangle_{deep} \cong (C_{e^+e^-}+C_x)\ln E_c^2 + \theta(s-16E_c^2)C_h \ln\left(\frac{\sqrt{s}}{2E_c}-1\right)^2 \qquad (1)$$

Notice that the right-hand side does not depend on $p_{\perp c}$. E_c is the energy of c in the a+b center-of-mass and the θ function is included so that the term $\propto C_h$ will not contribute when $4E_c \geq \sqrt{s}$. This, of course, is required by energy conservation. We have neglected terms which stay finite as s, $p_{\perp c} \to \infty$. These correction terms include the contribution to the multiplicity from the finite fragmentation and overlap regions, as well as certain factors which multiply the arguments of the logs. Some of these factors come from averaging over the possible orientations of the undetected parton cylinder, and depend in detail on the parton-parton scattering amplitude and the parton distributions in a and b. The others arise when we write E_p in terms of E_c, since $E_c = f_p E_p$, where f_p is some finite fraction. However, none of these complications change the asymptotic formula (1).

This formula has several interesting features. When $E_c \sim \langle m_\perp \rangle$ (say, about 1 GeV) Eq. (1) gives $\langle n \rangle \sim C_h \ell n s$, and we recover the well-known expression for the multiplicity in ordinary hadron-hadron scattering. On the other hand, when E_c^2 is some finite fraction of s, and the scattering angle is greater than zero, we find $\langle n \rangle \sim (C_{e^+e^-} + C_x) \ell n\, E_c^2$, and the multiplicity becomes essentially independent of the second term.[9]

There are two, somewhat technical aspects of Eq. (1) and its derivation we would now like to discuss. First, as implied above, we note that if we detect a hadron with a large $p_\perp$, we can be fairly certain that it is the most energetic hadron in its cylinder, and therefore has some fixed, finite fraction of its parent parton's energy. The reason is that the probability of knocking a parton to a distant region of phase space falls rapidly with the parton's energy. So, if we observe a widely scattered hadron with energy $E_c \gg \langle m_\perp \rangle$, it is unlikely that another hadron with energy $> E_c$, was produced by the same parton, since then the parton's energy would have had to have been extremely large.[10]

Second, we note that the derivation of Eq. (1) is based on an average over configurations of final hadrons as pictured in Fig. 2. We can now ask what mechanisms for the production of hadrons by isolated partons are consistent with this picture. One possibility, which is obviously consistent with Fig. 2, is that the isolated partons form cylinders of hadrons to communicate with the wee partons in the center-of-mass a+b. However, there are at least two other very plausible possibilities: (1) In the center-of-mass of the two scattered partons, a straight cylinder joining the partons may develop, or (2) in the rest frame of the hole (i.e., the rest frame of the scattered parton before it was scattered), a straight cylinder may develop between the hole and the scattered parton. When $p_{\perp c}$ and s get large however, both of these alternatives also give the result (1). The reason is that for $|p_\perp| > \mathcal{O}(\langle m_\perp \rangle)$, both alternative distributions become, when viewed in the center-of-mass of a and b, approximately straight cylinders pointing toward the origin, as in Fig. 2. Appreciable deviations from these asymptotes occur only when $|p_\perp| \lesssim \mathcal{O}(\langle m_\perp \rangle)$. But, this is the region where the parton cylinders begin to overlap the hadron cylinders lying along the $p_\parallel$ axis. These corrections can only affect C_x or the finite pionization region near the origin, and therefore both these possibilities will result in asymptotic multiplicities given by Eq. (1). Notice that we do not mean to imply that these differences are moot or untestable; in fact, we believe they are quite important. We only mean that they all result in the same asymptotic expression for $\langle n \rangle_{\text{deep}}$.

We would now like to discuss some limitations and possible corrections to Eq. (1).[7] First, our picture is not the correct one in the limit that the scattering angle of $c \to 0$, even though $p_{\perp c}$ gets large. In this limit, the central pionization region in Fig. 2 spreads out along the $p_{\parallel}$ axis and forms an additional plateau which will signficantly contribute to $\langle n \rangle$ of such events. Second, there may be corrections to our formula coming from events in which two partons scatter through a small angle in their center of mass. It is possible that such partons contribute $C_{e^+e^-} \ln P_{\perp p}$ to $\langle n \rangle$ rather than $C_{e^+e^-} \ln E_p$. If such a mechanism exists, it will alter our expression for $\langle n \rangle_{\text{deep}}$ in certain kinematic regions. However, these corrections will become less important as the energy and scattering angle of the observed particle increase.

Another interesting reaction to observe is the semi-inclusive process $a+b \to c+d+X$ where both c and d come out with large $|p_\perp|$ in opposite hemispheres, each being produced by one of the isolated partons. For this process the multiplicity is asymptotically given by

$$\langle n(s, E_c, E_d) \rangle_{\text{deep}} \cong (C_{e^+e^-} + C_x) \ln E_c E_d + \theta\left(s - 16E_c^2\right) C_h \ln \left(\frac{\sqrt{s}}{2E_c} - 1\right)$$

$$+ \theta\left(s - 16E_d^2\right) C_h \ln \left(\frac{\sqrt{s}}{2E_d} - 1\right) \tag{2}$$

where, again, we have neglected the terms which remain finite as s, E_c and $E_d \to \infty$. Notice the similarity between this expression and expression (1) for the one particle inclusive case. However, in this experiment we can, on the average, deduce the energy of the partons which produce c and d if we know f_p (or, equivalently, $C_{e^+e^-}$). While this knowledge does not strongly affect the asymptotic relation (2), we can get a firmer handle on the possible corrections from small angle parton-parton scattering if we know E_p and $E_{p'}$. Furthermore, it is important to know the parton energies if we wish to derive nonasymptotic relationships between the multiplicities discussed here, and multiplicities in other high energy reactions. Let us turn now to a brief discussion of such relations.

We expect Eqs. (1) and (2) to be valid when clear hadronic and partonic plateaus in $\ln E$ have developed. In the absence of such plateaus, we cannot predict the s or $p_\perp$ dependence of $<n>$. However, we can relate $<n(s, E_c, E_d)>$ to the multiplicities in other reactions. There are a number of detailed forms which such relations could take, but most simple options follow from one of four general arguments:

(1) In $e(\nu)$-h scattering[4] we remove a parton with energy E_p from the struck hadron. We then have a hadron with a hole in it and a parton distant in phase space. In whatever way the hole and liberated parton evolve in e-h scattering, they do the same thing in deep h-h scattering. If the results of Cahn, Cleymans, and Colglazier[4] are correct, this argument evaluated asymptotically predicts $C_x = C_{e^+e^-}$.

(2) The hole turns into hadrons just as the parton does. This also implies $C_x = C_{e^+e^-}$.

(3) The hole does not significantly affect the final hadron multiplicity so that the final hadron distribution along the $p_\parallel$ axis looks as if no partons were removed. Then the $p_\parallel$ distribution contributes to $<n>_{deep}$ just the average multiplicity of an ordinary h-h reaction. Asymptotically this gives $C_x = C_h$.

(4) The hole and parton dispose of themselves in similar ways, but these contributions to $<n>_{deep}$ simply add to the contributions coming from the original hadrons <u>sans</u> holes.[7] This argument predicts $C_x = C_h + C_{e^+e^-}$, asymptotically.

All of these arguments are plausible. While each is based on a seemingly different notion about the way parton distributions overlap and turn into hadrons, more than one may be correct. For instance, if $C_h = C_{e^+e^-}$, possibilities 1, 2 and 3 may all be true. On the other hand, it is possible that $C_h = C_{e^+e^-}$ with, say, only one of these arguments correct, nonasymptotically. It would, therefore, be very interesting to know which of the relations that follow from these options are satisfied, and over what ranges of s and $p_\perp$. Tests of such expressions, as well as asymptotic determinations of C_x, C_h, and $C_{e^+e^-}$ can yield much valuable information about the nature of partons and about the correct way to construct the amplitudes which describe the development of partons into final state hadrons.

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7. J. D. Bjorken, private communication.
8. J. D. Bjorken, Report No. SLAC-PUB-1114, Stanford Linear Accelerator Center (1972).
9. This is true except in the limit where the scattered partons have essentially all of the incident hadrons' energy in which case the term proportional to C_x is absent.

10. We can make this argument slightly more quantitative as follows: Suppose the probability to produce a parton at large $p \propto E_p^{-N}$, where N is some fairly large number. On the average the kth hadron produced in the parton cylinder will have energy $f_p^k E_p = E_h$, where $C_{e^+e^-} \ln f_p = -1$. If we observe an energetic hadron with energy E_h, the probability that it is the kth hadron produced by a parton $\propto f_p^{kN}/E_h^N$. Therefore, if $f_p^N \ll 1$, we can be fairly certain that the hadron we observe is the most energetic in the parton's cylinder.

FIGURE CAPTIONS

1. Parton distribution immediately after hard parton-parton scattering.

2. Average final hadron distribution for a typical deep scattering event.

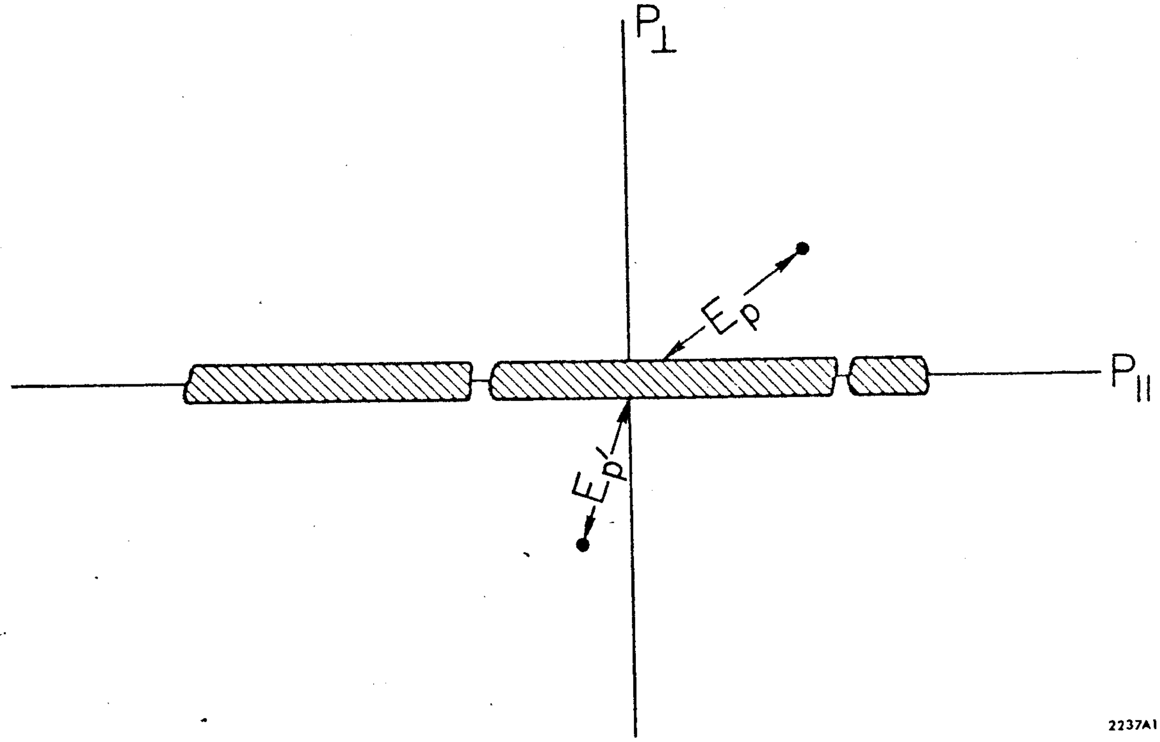
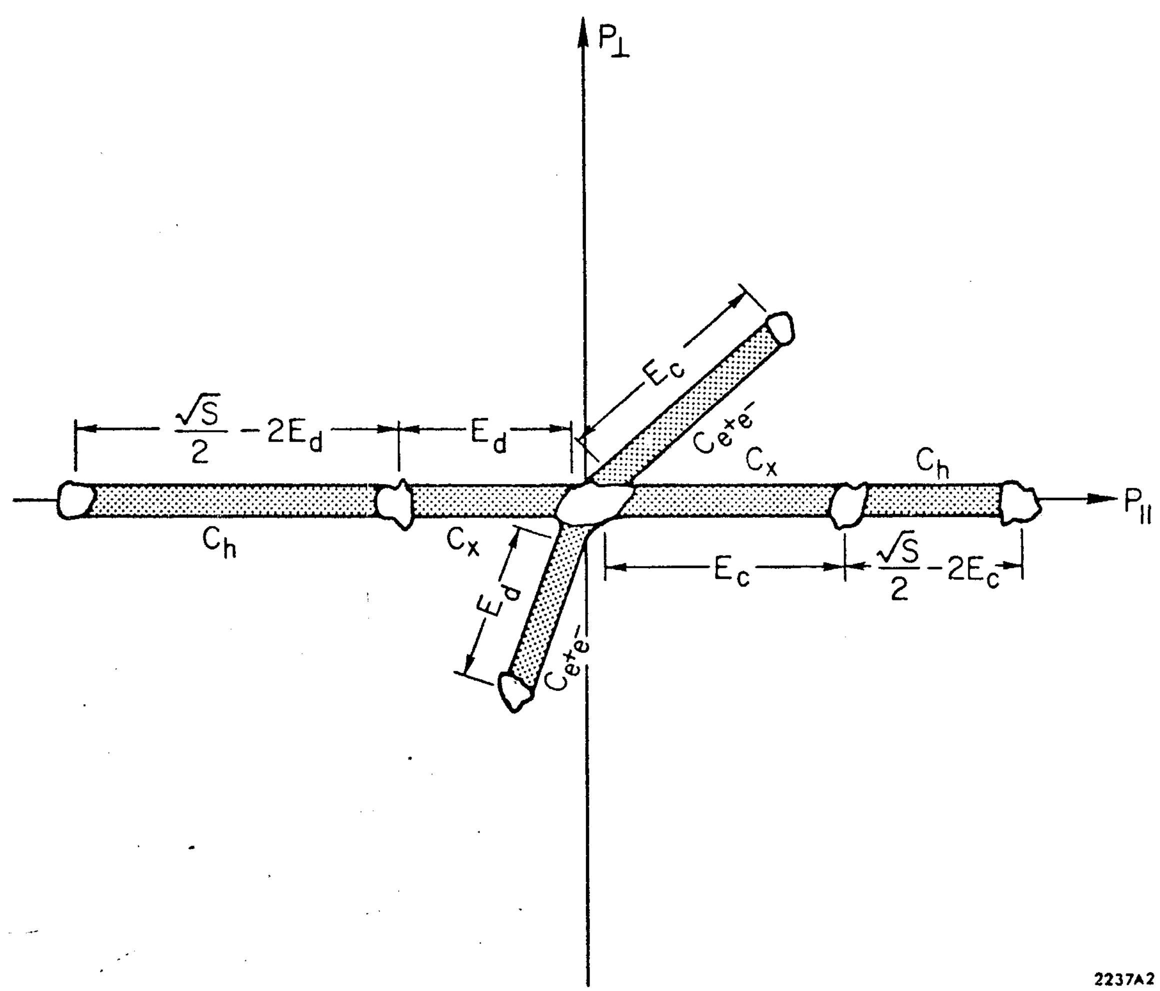


Fig. 1



$P_\perp$
$P_\parallel$
$\frac{\sqrt{S}}{2} - 2E_d$
E_d
E_c
$C_{e^+e^-}$
C_x
C_h
C_h
C_x
E_d
$C_{e^+e^-}$
E_c
$\frac{\sqrt{S}}{2} - 2E_c$
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Fig. 2