SYNCHROTRON FREQUENCY SPLITTING IN THE SLAC STORAGE RING*

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For multibunch operation of storage rings, stabilization against longitudinal oscillations has been obtained experimentally by introducing a synchrotron frequency shift between bunches.^{1,2} This frequency shift was obtained by using an RF cavity operating at a harmonic of the revolution frequency but not of the main RF accelerating frequency. Initial operation of the SLAC storage ring (SPEAR) will be with a single bunch of electrons and a single bunch of positrons. It is intended to split the synchrotron frequency of the two bunches with a non-RF harmonic cavity. The effect of this cavity on the stored beam's parameters is investigated. Although primary interest is on the interacting buckets, the effect on adjacent buckets is also examined since the ring might eventually be operated with several filled buckets.

Analysis

The ring is symmetric with two interacting regions opposite each other as shown in Fig. 1. Distances are measured from one of the interaction regions. The main accelerating cavity gap is situated an integral number of half-wavelengths from the interaction region. The frequency-splitting cavities are placed in some straight sections. For the general case, the i th cavity gap is placed at a position L_i from the interaction region. The voltage on the gap is

$$V_i \sin \omega_i \left(t + \frac{\theta_i}{\Omega} \right),$$

where the cavity frequency is $\omega_i = k_i \Omega$, k_i the harmonic number, Ω the angular frequency of the beam, and θ_i the phase angle at time $t=0$. The position of the synchronous particle in the m th positron and the m th electron bunch at $t=0$ is ℓ_m^+ and ℓ_m^- , respectively.

The case of two cavities is considered in this analysis, one the main accelerating cavity and the other the frequency-splitting cavity. The energy gain per turn from these cavities for the synchronous particle in the m th bunch is given by

$$W_m^\pm = eV_1 \sin k_1 (\pm L_1 + \ell_m^\pm + \theta_1) + eV_2 \sin k_2 (\pm L_2 + \ell_m^\pm + \theta_2), \quad (1)$$

where the subscripts one and two refer to the main accelerating and the frequency-splitting cavity respectively, and $L_1 = n\pi/k_1$ with n integral. For convenience the phase θ_1 is chosen so that when $V_2=0$ the position of the synchronous particle in the m th bunch is given by

$$\ell_m^\pm = \frac{2\pi m}{k_1}, \quad m=0, 1, 2 \dots (k_1-1) \quad (2)$$

The radiation demand per turn for these synchronous particles with $V_2=0$ is given by

$$W_{\text{rad}} = eV_1 \sin \bar{\Phi}_s, \quad (3)$$

where $\bar{\Phi}_s = k_1 \theta_1$ is the synchronous phase angle with $V_2=0$. For the case of $V_2 \neq 0$ the phase θ_2 is chosen so that the $m=0$ bunches collide in the interaction regions. This choice gives

$$k_2 \theta_2 = \pi/2 - k_2 \ell_0, \quad (4)$$

with $\ell_0 = \ell_0^+ = \ell_0^-$ which is determined by

$$V_1 \sin \bar{\Phi}_s = V_1 \sin k_1 (\theta_1 + \ell_0) + V_2 \cos k_2 L_2. \quad (5)$$

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The values of the other $\ell_m^\pm$ are determined by

$$W_m^\pm = W_{\text{rad}} \quad (6)$$

The equations of motion for a nonsynchronous particle in the m th bunch with an energy deviation about the synchronous energy $\epsilon = E - E_s$ can be derived from the following Hamiltonian:

$$H^\pm = \frac{2\pi\alpha}{E_s} \left(\frac{\epsilon^2}{2} \right) + \frac{eV_1}{k_1} \left\{ \left[\cos k_1 (X_m^\pm + \theta_1) + k_1 X_m^\pm \sin k_1 (\ell_m^\pm + \theta_1) \right] \right. \\ \left. + \frac{eV_2}{k_2} \left[\cos k_2 (X_m^\pm + L_2 + \theta_2) + k_2 X_m^\pm \sin k_2 (\ell_m^\pm + L_2 + \theta_2) \right] \right\} \quad (7)$$

where $X_m^\pm$ is the particle position and the independent variable is the turn number. α is the momentum compaction and E_s the energy of the synchronous particle. The phase trajectories are curves defined in the $\epsilon - X_m$ phase space for constant values of the above Hamiltonian, and the phase trajectory enclosing the maximum area is called the separatrix. The small amplitude synchrotron oscillation frequencies are given by

$$(\nu_m^\pm)^2 = - \left(\frac{2\pi\alpha}{E_s} \right) \left\{ eV_1 k_1 \cos k_1 (\ell_m^\pm + \theta_1) + eV_2 k_2 \cos k_2 (\ell_m^\pm + L_2 + \theta_2) \right\}. \quad (8)$$

In particular for $m=0$ we have $\ell_0^+ = \ell_0^- = \ell_0$ and with the phase θ_2 chosen by Eq. (4) the frequency splitting for the colliding bunches is obtained from

$$(\nu_0^+)^2 - (\nu_0^-)^2 = - \left(\frac{4\pi\alpha}{E_s} \right) e k_2 V_2 \sin k_2 L_2. \quad (9)$$

If the frequency splitting is small we find

$$\frac{\Delta\nu_s}{\nu_s} = - \frac{2\pi\alpha e k_2 V_2}{\nu_s^2 E_s} \sin k_2 L_2. \quad (10)$$

where $\Delta\nu_s = \nu_0^+ - \nu_0^-$.

Results

The small amplitude synchrotron oscillation frequencies, the synchronous particle positions and the separatrices for the m th bunches have been calculated for the SPEAR design parameters. To illustrate some of the results, Fig. 2 shows the frequency split $\Delta\nu_s/\nu_s$ as a function of V_2 for several values of k_2 . For all cases calculated, the main accelerating harmonic number $k_1=40$. The shift in the collision point from the interaction point between the m th electron and positron bunch is equal to $(\ell_m^+ - \ell_m^-)/2$ and is shown in Fig. 3 for various values of k_2 with the value of V_2 chosen such that $\Delta\nu_s/\nu_s=0.2$. The separatrices of the $m=0$ bunches with $\Delta\nu_s/\nu_s=0.2$ show very little change from the separatrices for $V_2=0$.

For a desired $\Delta\nu_s/\nu_s$, the higher harmonics k_2 require a lower voltage V_2 and hence give a smaller collision point displacement. For multibunch operation, it is important to use higher value of k_2 if the collision point is to be left in the narrow "low-beta" region of the interaction zone. Furthermore, from Eq. (9) we see that it is important not to locate the frequency-splitting cavity an integral number of half wavelengths (at its frequency) from the interaction point.

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References

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FIG. 1--Diagram of cavity and bunch locations in the ring.

FIG. 2--Frequency split as a function of second cavity voltage for various harmonics.

FIG. 3--Shift in collision point for all buckets with one bucket set for zero shift.