SLAC-PUB-777
September 1970
(ACC)

POSSIBLE SOURCES OF RESIDUAL POWER LOSS IN RF SUPERCONDUCTING CAVITIES*

Mario Rabinowitz

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

Abstract

The residual power loss in rf superconducting cavities has not been well understood. Various power loss sources are considered and their dependence on frequency and temperature are compared with experimental results. One power loss mechanism stands out as the most likely source of the residual power loss.

It has been observed that a residual power loss exists in rf superconducting resonant cavities at very low temperatures, contrary to theoretical expectations[1,2] which predict that for angular frequencies, ω, below the quantum region, the power loss should vanish as the temperature, T, approaches 0 K. The good agreement between these theories and experiment at higher reduced temperatures suggests that they are valid, and that the discrepancy lies in the presence of a very low, non-superconducting power loss mechanism. The nature of this loss has not been well understood. This is due in part in the past to a lack of agreement between results reported by various investigators on nominally equivalent samples. However, the more recent experimental data in which the residual power loss has been further reduced tend to be more consistent. These results indicate that for very

*This work was supported by the U. S. Atomic Energy Commission.

(To be published in Lettere al Nuovo Cimento.)

high Q superconducting cavities, the residual surface resistance, R_{res}, is independent of temperature and field level, is $\propto \omega^2$, and is present even for extremely small stored energy.[3,4,5,6]

Magnetic flux trapped due to an incomplete Meissner-Ochensenfeld effect has been suggested as the source of R_{res}.[7,8,9,10] However, there has been no theoretical analysis to show that R_{res} due to trapped fluxoids should be $\propto \omega^2$. On the contrary, other frequency dependencies have been proposed. Haden, Hartwig, and Victor[7] predicted that $R_{res} \propto \omega$ on the basis of a hysteresis loss model at stationary fluxoid sites. On the basis of the same stationary trapped flux, Victor and Hartwig[8] proposed that the losses are simply ohmic and that "the frequency dependence of the residual loss is that of normal loss." Similarly, Pierce[9] considered the trapped flux to be stationary and conjectured that these fluxoids may result in R_{res} equivalent to anomalous surface resistance with $R_{res} \propto \omega^{2/3}$. However, as we shall next see, stationary fluxoids or any small normal region within the superconducting surface will give $R_{res} \propto \omega^2$ and independent of T.

There are two sources of power loss in a stationary normal region of average radius a $\leq \delta$, the anomalous skin depth. One is an induced current loss which results from the time varying magnetic flux. The other is due to the presence of an electric field in the a.c. superconductor due to the time varying current. Interestingly, both give a power loss $\propto \omega^2$ and independent of T. Let us consider the latter loss first. The component of the electric field perpendicular to the interface will be continuous across the boundary if there is no net surface charge density and the permittivity is the same in both regions. The tangential component of the electric field, E, is continous across the superconducting to normal boundary in all cases.

$$E = \frac{m}{e}\left(\frac{1}{ne}\,\frac{dj}{dt}\right) = \left(\frac{im\omega}{ne^2}\right) j = \rho_s\, j \quad , \tag{1}$$

where n is the number density of electrons; m is the electron mass; e is the electron charge; j is the superconducting transport current density $= j_0 e^{-x/\lambda} e^{i\omega t} \equiv j_0' e^{i\omega t}$; and the superconducting electrical resistivity is

$$\rho_s = i \left(\frac{m\,\omega}{n e^2} \right) . \tag{2}$$

From Maxwell's equations,

$$\nabla^2 E_z = i\,\omega\mu\,\sigma_n' \,E_z , \tag{3}$$

where μ is the permeability of the normal region, and σ_n' is its effective conductivity. Assuming that the field E_z inside the normal cylindrical region is parallel to its axis, the solution to (3) is

$$E_z = E\,J_0\left(\sqrt{\frac{2}{i}}\;\frac{r}{\delta}\right) \Big/ J_0\left(\sqrt{\frac{2}{i}}\;\frac{a}{\delta}\right) , \tag{4}$$

where J_0 is the Bessel function of the first kind and zero order, and r is the radial distance from the cylinder axis. For $a = \delta$, E_z is reduced by only 6% at its lowest point on the axis. Therefore, to a good approximation for $a \le \delta$, E_z may be considered to be roughly constant in the normal region; and similarly for j_n, the current density in it. This conclusion would be valid at any cross section parallel to E for any orientation of the cylinder with respect to the electric field.

The average power loss per unit volume in a normal region:

$$\frac{dP'}{dV} = \frac{1}{2}\rho_n' j_{n0}^2 = \frac{1}{2\rho_n'} \left|E_0\right|^2 , \tag{5}$$

where ρ_n' is the effective resistivity of the normal region. (There is an additional power loss from the normal electrons which leave the normal region and enter the surrounding superconducting region. This loss should be relatively small.) $\rho_n' = G\rho_n$, where ρ_n is the normal-state resistivity of the stationary normal region, and is essentially the temperature-independent residual resistivity for most metals. $G \sim \ell/(2a)$; where the electron mean free path $\ell \gg 2a$.

By substituting Eq. (1) into Eq. (5), we have the power loss in the normal region in terms of the supercurrent:

$$\frac{dP'}{dV} = \frac{1}{2}\left[\frac{|\rho_s|^2}{\rho_n'}\right] j_0'^2 = \frac{1}{2}\rho j_0'^2 \quad , \tag{6}$$

where

$$\rho = \frac{|\rho_s|^2}{\rho_n'} = \left[\frac{m^2\omega^2}{n^2 e^4 G}\right]\frac{1}{\rho_n} \propto \frac{\omega^2}{\rho_n} \tag{7}$$

is the effective resistivity of the stationary normal region if the supercurrent were to flow through it.

The residual surface resistance, $R_{res} = F\rho/\lambda$, where the superconducting penetration depth, λ, is frequency independent, and where the factor F is related to the geometry of the normal region and its depth below the conducting surface.

$$\rho_n = \frac{m}{ne^2\tau} = \frac{mv_F}{ne^2\ell} \quad , \tag{8}$$

where τ is the relaxation time between electron collisions, and v_F is the Fermi velocity of the electrons. Since ρ_n and the other variables in Eq. (7) are essentially temperature independent, and $\lambda = \lambda_0 \Big/ \left[1 - (T/T_c)^4\right]^{1/2}$ is effectively temperature independent for $(T/T_c) < \frac{1}{2}$, R_{res} is also temperature independent.

Combining Eq. (7) and (8),

$$R_{res} = \frac{F}{\lambda}\left[\frac{m^2\omega^2}{n^2e^4G}\right]\frac{1}{\rho_n} = F\left[\frac{m\omega^2\tau}{ne^2G\lambda}\right] \doteq F\left[\frac{2am}{ne^2 v_F\lambda_0}\right]\omega^2. \tag{9}$$

Therefore, $R_{res} \propto \omega^2$. R_{res} is independent of ℓ, for $\ell \gg 2a$, and hence will be temperature independent even before ρ_n is constant, and will be independent of purity and lattice defects in the normal region. Equation (9) predicts the correct functional dependence for R_{res}. There can be a number, N, of such normal regions, in which case $R_{res} \propto N$.

Now let us consider the induced current power loss for the same normal cylinder, whose axis we'll assume for simplicity to be parallel to the magnetic field $H = H_p \cos \omega t$ applied at the surface of the superconductor. If the axis of the cylinder is a distance d below the surface of the superconductor, the effective magnetic field it sees is $F_1 H$, where $F_1 = e^{-(d-a)/\lambda}$ and λ is the superconducting penetration depth. The voltage induced in the cylinder is

$$V = \pi r^2 \omega \mu F_1 H_p \sin \omega t \tag{10}$$

The instantaneous power dissipation is

$$P = \int V\, dI = \int_0^a \pi r^2 \omega \mu F_1 H_p \sin \omega t \left(r \mu F_1 H_p \omega h \sin \omega t / 2 \rho'_n \right) dr$$

$$= \left[\pi h \omega^2 a^4 \mu^2 / 8 \rho'_n \right] \left(F_1 H_p \sin \omega t \right)^2 \tag{11}$$

where μ is the permeability, and h is the height of the cylinder.

Thus the average (induced current) power loss per unit area of the cavity is approximately

$$P'/A = \frac{1}{2} \left[h F_1^2 \omega^2 \mu^2 a^4 / 16 r_1 (h_1 + r_1) \rho'_n \right] H_p^2 = \frac{1}{2} R_i H_p^2 \tag{12}$$

where h_1 and r_1 are, respectively, the height and radius of the inside of the cavity. The (induced current) residual surface resistance is

$$R_i = \left[h F_1^2 \omega^2 \mu^2 a^4 / 16 r_1 (h_1 + r_1)\, G\, \rho_n \right] . \tag{13}$$

Both R_{res} and R_i are $\propto \omega^2$ and independent of T. The total residual resistance is:

$$R_{res} + R_i \leq R_{res,total} \tag{14}$$

for most typical superconducting cavities.

If the normal region is an oscillating fluxoid due to the Lorentz force interaction between the trapped flux and the oscillating current, then Rabinowitz[11] has shown that its effective resistivity (in m k s units) is:

$$\rho = \left[\frac{\omega^2 \pi^2 a^4 H^3 H_0 \mu^4}{\rho_n^2 \left(\omega^2 M - p\right)^2 + \mu^4 \omega^2 \pi^2 a^4 H^2 H_0^2} \right] \rho_n \tag{15}$$

where H is the magnetic field in the fluxoid, H_0 is $H_{c2}(0)$ for type II and is $H_c(0)$ for type I, μ is the permeability, M is the effective mass per unit length of the fluxoid, and p is the pinning constant per unit length. At low temperatures, this would give R_{res} independent of temperature. R_{res} thus obtained would be considerably higher than for a stationary normal region and an oscillating fluxoid could easily be the dominant source of residual power loss. When viscous losses dominate, the second term in the denominator of Eq. (15) is dominant, and R_{res} would have no frequency dependence. A trapped fluxoid would not oscillate if it were very strongly pinned, and/or the Lorentz force on it were very weak. This might be possible if the fluxoid were nearly parallel to the current and/or far below the conducting surface. There is no obvious reason to expect the pinning force to increase substantially as the temperature is reduced. However, if it doesn't, one is left with the perplexing problem of why an otherwise dominant power loss doesn't manifest itself until very high field levels are reached. It certainly would be satisfyingly harmonious as well as physically significant if the fluxoid went from an oscillating mode to a stationary mode as $T \rightarrow 0$. Then the fluxoid would be responsible for both critical power dissipation, as shown by Rabinowitz,[12] as well as residual power dissipation.

It is interesting to note that if $p \gg \omega^2 M$, and $\rho_n^2 p^2 \gg \omega^2 \pi^2 a^4 H^2 H_0^2 \mu^4$, then Eq. (15) reduces to

$$\rho = \left[\pi^2 a^4 H^3 H_0 \mu^4 / p^2\right] \frac{\omega^2}{\rho_n} . \tag{16}$$

This has the same dependence on ω and ρ_n as do Eq. (7) and (13), and the same dependence on a as Eq. (13). Hence Eq. (16) would also imply $R_{res} \propto \omega^2$ and independent of T, in this limit.

The simple view of flux flow power loss only deceptively appears to give a power loss $\propto \omega^2$. In this view, the power loss per unit length,

$$P'/h_1 = (\eta v) v \propto v^2 , \tag{17}$$

where η is the flow viscosity, v is the fluxoid velocity, and ηv is the viscous force/length retarding the fluxoid motion. Only for simple harmonic motion would v be $\propto \omega$, so that, in general, Eq. (17) would not give a power loss $\propto \omega^2$.

The fact that R_{res} is within ~ 10, the same for TE_{011} and TM-mode cavities,[3,5] makes it unlikely that dielectric loss is responsible for residual power loss except as a secondary contribution. This is because the TE_{011} mode ideally has a 0 electric field everywhere on its surface, and one would thus expect many orders of magnitude larger dielectric loss for TM- than for the TE_{011}-mode. The dielectric power loss is $\propto E_0^2 \omega \sin \delta \doteq E_0^2 \omega \operatorname{tn} \delta$. The electric field $E = E_0 \cos \omega t$, and $\operatorname{tn}\delta$ is the loss factor. For radiation damping, $\operatorname{tn}\delta \propto \omega^3$. For quantum absorption, $\operatorname{tn}\delta \underset{\sim}{\propto} \omega^2$. The collision broadening contribution to the dielectric loss is related to phonon generation in the dielectric with the ensuing ionic collisions. Grissom and Hartwig[13] have calculated $\operatorname{tn}\delta \propto T/E_0^2$ and independent of ω, for collision broadening. Breckenridge[14] has derived the dielectric loss due to crystal imperfections, and finds $\operatorname{tn}\delta \propto n(\omega\tau)/T\left[1 + (\omega\tau)^2\right]$, where n is the defect density, and τ is the average time between transitions. Evidently none of the dielectric loss mechanisms has the correct dependence on T and/or ω.

The field emission of electrons at microprotrusions where the electric field is locally enhanced can also be ruled out as the source of the residual power loss. In addition to power dissipation when these electrons are accelerated by the fields of the cavity and strike the cavity wall, there is a power loss at the microprotrusion when the electrons are field emitted, even at 0° K. The latter is due to the fact that the electrons are field emitted at an average energy below the Fermi energy, and are replaced in the metal by electrons at approximately the Fermi energy. Either loss would be strongly dependent on signal level, though essentially independent of T for $0 < T < T_c$.

In conclusion, the most likely source of the residual power loss is a small ($a \leq \delta$) stationary normal region(s) within the superconducting surface.

Thanks are due to P. B. Wilson for helpful discussions.

References

1. MATTIS D.C. and BARDEEN J., Phys. Rev. 111, 412 (1958).
2. ABRIKOSOV A. A., GORKOV L. P., and KHALATNIKOV I. M., Sov. Phys. JETP 8, 182 (1959).
3. FLECHER P. et al., IEEE Trans. Nucl. Sci. NS-16, 1018 (1969).
4. SZECSI L., Proc. 7th Intl. Conf. High Energy Acc., Yerevan (1969).
5. TURNEAURE J. P. and WEISSMAN I., J. Appl. Phys. 39, 4417 (1968).
6. TURNEAURE J. P. and VIET N. T., Appl. Phys. Letters 16, 333 (1970).
7. HADEN C. R., HARTWIG W. H., and VICTOR J. M., IEEE Trans. Magnets MAG-2, 331 (1966).
8. VICTOR J. M. and HARTWIG W. H., J. Appl. Phys. 39, 2539 (1968).
9. PIERCE J. M., Ph.D. dissertation, Stanford Univ., Stanford, Calif. (1967).
10. SCHWETTMAN H. A. et al., Report No. HEPL-503, Stanford Univ. (1967).
11. RABINOWITZ M., Proc. 1970 Appl. Superconductivity Conf. (to be published in J. Appl. Phys.). SLAC-PUB-714, Stanford Univ. (1970).
12. RABINOWITZ M., Appl. Phys. Letters 16, 419 (1970).
13. GRISSOM D. and HARTWIG W. H., Univ Texas, Austin Lab. Tech. Report 6 (1965).
14. BRECKENRIDGE R. G., Imperfections in Nearly Perfect Crystals, Edited by Shockley et al. (John Wiley and Sons, N. Y., 1952).