# INTEGRALLY CHARGED QUARKS AND $2 \gamma$ DECAY OF $\pi^{\circ}$ AND $\eta^{*}$ 

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[^0]It is shown that the Nambu-Han model of integrally charged quarks can account for both $\pi^{0} \rightarrow 2 \gamma$ and $\eta \rightarrow 2 \gamma$ decay rates with a suitable specification of $0^{-}$ and $1^{-}$meson states in "charm" space.

Recently Bing-Lin Young ${ }^{1}$ made a definitive formulation of $\pi^{\circ} \longrightarrow 2 \gamma$ decay by means of subtracted dispersion relation in two variables. The decay amplitude consists of three terms; (1) the subtraction term which is completely determincd by the Bjorken limit ${ }^{2}$ from the equal time commutator of spatial components of electromagnetic current (called ETC term hereafter), (2) the term given by the vector meson dominance model of Gell-Mann, Sharp and Wagner ${ }^{3}$ (namely, $\rho-\omega$ double pole term, called VMD term in the following), and (3) higher mass contributions which are neglected. Young showed that the quark model of Gcll-Mann and Zweig ${ }^{4}$ gave fair agreement with the experimental $\pi^{0}$ life time. Lately, Barry and Sakurai ${ }^{5}$ argued that, using slightly different values for $\gamma-\omega$ and $\omega \rho \pi$ coupling constants, the VMD term alone could account for the required magnitude of the amplitude. However, in the quark model, the ETC term is small compared to the VMD term in any case $(-25 \%)$. Considering the uncertainties in the values of various coupling constants which can be as large as 10 to $20 \%$, Barry-Sakurai's analysis will not rule out the quark model at this point. On the other hand, the work of Young and Barry-Sakurai does indicate that the integrally charged quark model, favored from the analysis of the radiative correction to $\beta$ decay ${ }^{6}$, would be incompatible with the $\pi^{0}-2 \gamma$ decay rate. For in this model, the ETC term is threc times as large as that of the ordinary quark model, as pointed out first by Okubo ${ }^{7}$, and hence it is not easy to obtain the agreement by adjusting parameters within reasonable limits, although not impossible. The purpose of the present paper is to point out a simple possibility to avoid the difficulty and at the same time to obtain a large amplitude for $\eta \rightarrow 2 \gamma$ decay.

Among models of integrally charged quarks ${ }^{8}$, we will study specifically the Nambu-Han model for a reason given later. In this model, three integrally charged triplets with charge configuration $(1,0,0),(1,0,0)$ and $(0,-1,-1)$ (called $S, U, B$, respectively) are assumed. For convenience, we introduce, beside the unitary spin matrices $\lambda_{a}(a=0,1, \ldots 8)\left(\lambda_{0}=\sqrt{2 / 3}\right)$, the "charm" spin matrices $\rho_{a}(a=0,1, \ldots 8)$ defined exactly as $\lambda_{a}$. Then, in terms of generalized currents

$$
\begin{equation*}
\mathrm{j}_{\mu}^{(\mathrm{a}, \mathrm{~b})}=\bar{\psi}\left(\lambda_{\mathrm{a}} / 2\right) \times\left(\rho_{\mathrm{b}} / 2\right) \gamma_{\mu} \psi \tag{1}
\end{equation*}
$$

the electromagnetic current is given by

$$
\begin{equation*}
\mathrm{j}_{\mu}^{\mathrm{em}}=\sqrt{6}\left[\mathrm{j}_{\mu}^{(3,0)}+\frac{1}{\sqrt{3}} \mathrm{j}_{\mu}^{(8,0)}+\frac{2}{\sqrt{3}} \mathrm{j}_{\mu}^{(0,8)}\right] \tag{2}
\end{equation*}
$$

The last term, which is unitary singlet, carries the charm quantum number C , which is $1,1,-2$ for $S, U$, and $B$, respectively.

In order to proceed further, we have to determine the charm space configuration of $0^{-}$and $1^{-}$mesons. There are two possibilities, the charm singlet states

$$
\begin{equation*}
(a, 0)(a=0,1, \ldots 8) \tag{3}
\end{equation*}
$$

and the "mixed" states

$$
\begin{equation*}
1 / \sqrt{3}(a, 0)-\sqrt{2 / 3}(a, 8) \tag{4}
\end{equation*}
$$

or

$$
\begin{equation*}
-\sqrt{2 / 3}(a, 0)-1 / \sqrt{3}(a, 8) \tag{4'}
\end{equation*}
$$

$(a, 0)$ corresponds to equal mixture of $S, U$ and $B$ configuration; namely, $(S, \bar{S})+$ $(U, \bar{U})+(B, \bar{B})$, while (4) and (4') corresponds to pure $(B, \bar{B})$ and $(S, \bar{S})+(U, \bar{U})$ configuration, respectively. For each of these configurations, we can calculate
$\gamma-\mathrm{V}$ coupling constant $\mathrm{G}_{\mathrm{V}}{ }^{9}$, defined by $\left\langle\left. 0\right|_{\mu} \mathrm{j}_{\mu} \mid \mathrm{V}\right\rangle=\epsilon_{\mu}(\mathrm{V}) \mathrm{G}_{\mathrm{V}}$, assuming $<0 \mathrm{j}_{\mu}^{(\mathrm{a}, \mathrm{b})} \mid(\mathrm{c}, \mathrm{d})>=\mathrm{G}_{0} \delta_{\mathrm{ac}} \delta_{\mathrm{bd}}$ in the symmetry limit. Thus, for (4) and (4') the unitary singlet current couples to the vector mesons and we have, as a result, $\mathrm{G}_{\omega}= \pm \mathrm{G}_{\rho}$, using the $\omega-\phi$ mixing angle from the nonet symmetry. Since the experiment favors the $\mathrm{SU}_{3}$ value $\mathrm{G}_{\omega}=-(1 / 3) \mathrm{G}_{\rho}$, we are forced to take the charm singlet state (3) for the vector nonet. This requirement that the unitary singlet current shall not couple to the vector nonet is precisely the reason why we take Nambu-Han model among others. This was first pointed out by Cabibbo, Maiani and Preparata ${ }^{10}$, who investigated Nambu-Han model in some detail. Besides the vector nonet $(a, 0)$, there can exist in principle other vector mesons of higher masses with a general index ( $a, b$ ). Especially, we will show later (in footnote (12)) that a vector meson $(0,8)$, designated by $\omega^{\prime}$, must exist in order to satisfy certain sum rules. $\omega^{\prime}$ need not be a real vector meson, but may represent a board background needed to saturate sum rules. Here we list the symmetry values for $G_{V}$ 's.

$$
\begin{equation*}
G_{\rho}: G_{\omega}: G_{\phi}: G_{\omega^{\prime}}=3:-1:-\sqrt{2}: 2 \sqrt{3} \tag{5}
\end{equation*}
$$

For the pseudoscalar mesons, our assignment is different from that of Ref. 10. We recall that in order to make the radiative correction for $\beta$ decay finite it is necessary to take $V+A$ interaction for positively charged $S$ and $U$ triplets, and V-A for negatively charged B triplet. Thus the weak axial vector current is defined by

$$
\begin{align*}
\tilde{\mathrm{j}}_{5 \mu}^{(a)} & =-\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{S})-\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{U})+\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{B})  \tag{6}\\
& =-\sqrt{2 / 3} \mathrm{j}_{5 \mu}^{(\mathrm{a}, 0)}-4 / \sqrt{3} \mathrm{j}_{5 \mu}^{(a, 8)},
\end{align*}
$$

which should be combined with the vector current $j_{\mu}^{(a)}=j_{\mu}^{(a)}(S)+j_{\mu}^{(a)}(U)+j_{\mu}^{(a)}(B)=$ $\sqrt{6} \mathrm{j}_{\mu}^{(a, 0)}$ in the form $\mathrm{j}_{\mu}^{(a)}-\tilde{\mathrm{j}}_{5 \mu}^{(a)}$. The decay constant $\mathrm{f} \pi^{\text {is then defined by }}$

$$
\begin{equation*}
<0\left|\widetilde{\mathrm{j}}_{5 \mu}^{(3)}\right| \pi^{\mathrm{o}}, \mathrm{p}>=\operatorname{ip}_{\mu}{ }^{\mathrm{f}} \pi . \tag{7}
\end{equation*}
$$

This means that for $\mathrm{j}_{5 \mu}^{(\mathrm{a})}=\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{S})+\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{U})+\mathrm{j}_{5 \mu}^{(\mathrm{a})}(\mathrm{B})=\sqrt{6} \mathrm{j}_{5 \mu}^{(\mathrm{a}, 0)}$, we have $<0\left|j_{5 \mu}^{(3)}\right| \pi^{0}>=-3 \operatorname{ip}_{\mu}{ }^{\mathrm{f}} \pi$, which obviously leads to a wrong Adler-Weisberger sum rule, since we can expect no particle other than $\pi$ which couples to $\partial^{\mu}{ }_{j}{ }_{5 \mu}^{(3)}$. A simple way to avoid this difficulty is to assign the state (4) to $0^{-}$octet instead of (3). Then we obtain $-<0\left|j_{5 \mu}^{(3)}\right| \pi^{0}>=\langle 0| \tilde{\mathrm{j}}_{5 \mu}^{(3)}\left|\pi^{0}\right\rangle=\operatorname{ip}_{\mu}{ }^{\mathrm{f}} \pi^{\text {. }}$

We now derive Young's formulation of $\pi^{\circ}-2 \gamma$ decay using spectral representation of the amplitude. The $\pi^{0} \rightarrow 2 \gamma^{\text {amplitude is given by }}$

$$
\begin{align*}
\mathrm{T}_{\mu \nu} & \left.=\mathrm{ie} \int^{2} \int \mathrm{e}^{\mathrm{iq} \cdot \mathrm{x}}<0\left|\left(\mathrm{j}_{\mu}^{\mathrm{em}}(\mathrm{x}), \mathrm{j}_{\nu}^{\mathrm{em}}(0)\right)_{+}\right| \pi^{\mathrm{o}}, \mathrm{p}\right\rangle  \tag{8}\\
& =\epsilon_{\mu \nu \rho \tau} \mathrm{p}^{\rho} \mathrm{q}^{\tau} \mathrm{T}_{\pi}\left(\mathrm{q}^{2}, \mathrm{k}^{2}\right)
\end{align*}
$$

where $k=p-q$. We may assume the Jost-Lehman-Dyson representation ${ }^{11}$ for $T_{\pi}$,

$$
\begin{equation*}
T_{\pi}\left(q^{2}, k^{2}\right)=\int_{0}^{\infty} d s \int d^{4} u \frac{\psi(u, s)}{s-(q-u)^{2}} \tag{9}
\end{equation*}
$$

The four-vector variable $u$ has a finite support defined by the pion momentum $p_{\mu}$, so that $(q-u)^{2}$ is of the order of $m_{\pi}^{2}$ for $q^{2}=(p-q)^{2}=0$. If we assume that the spectral function is peaked around the average $\rho-\omega$ mass $\mathrm{m}^{2}$, one may be tempted to approximate (9) by $\mathrm{m}^{-2} \int \mathrm{ds} \int \mathrm{d}^{4} \mathrm{u} \psi(\mathrm{u}, \mathrm{s})$, the integral being completely determined by the ETC of spatial components of $j_{\mu}^{\mathrm{em}}$ as seen from (8) by taking the Bjorken limit $q_{0} \rightarrow i \infty$. This would leave us with only the ETC term. Actually we should not have used the mean value theorem of integral as in the above, because $\psi$ is not positive definite. The spectral representation (9) gives not only the spectrum in $q^{2}(u=0)$ and in $k^{2}(u=p)$, separately, but also the
overlapping spectrum in $q^{2}$ and $\mathrm{k}^{2}$ simultaneously (doubles poles in $\mathrm{q}^{2}$ and $\mathrm{k}^{2}$ ), as $\psi(\mathrm{u}, \mathrm{s})$ will generally involve derivatives of delta function. Now taking only contributions from the vector mesons coupled to $j_{\mu}^{\mathrm{em}}$, (9) assumes a specialized form

$$
\begin{equation*}
m_{\pi} T_{\pi}=\sum_{V} \frac{e G_{V} f^{0} \pi V \gamma}{m_{V}^{2}-q^{2}}+(q-k)+\sum_{V, V^{\prime}} \frac{e^{2} G_{V} G_{V} V^{i} \pi V V^{\prime}}{\left(m_{V}^{2}-q^{2}\right)\left(m_{V^{\prime}}^{2}-k^{2}\right)} \tag{10}
\end{equation*}
$$

where $\pi V \gamma$ and $\pi V V^{\prime}$ interactions are measured in units of $m_{\pi}$, with $f_{\pi V \gamma}^{0}$ and $\mathrm{g}_{\pi \mathrm{VV}}$, being dimensionless constants (see Ref. 5 for their definitions). Putting $q^{2}$ on the mass shell $q^{2}=m_{V}^{2}$, and taking $k^{2}=0$, we obtain $\pi V \gamma$ coupling constant

$$
\begin{equation*}
f_{\pi V \gamma}=f_{\pi V \gamma^{\prime}}^{o}+e \sum_{V^{\prime}} G_{V^{\prime}} m_{V^{\prime}}^{-2} g_{\pi V V^{\prime}} \tag{11}
\end{equation*}
$$

We see from (11) that $\mathrm{f}_{\pi \mathrm{V} \gamma}^{\mathrm{o}}$ represents $\pi \mathrm{V} \gamma$ coupling constant at photon mass $\mathrm{k}^{2}=\infty$. The ETC of the spatial components of electromagnetic current is given by

$$
\begin{equation*}
\left[\mathrm{j}_{\mathrm{i}}^{\mathrm{em}}(\mathrm{x}), \mathrm{j}_{\mathrm{k}}^{\mathrm{em}}(0)\right]_{\mathrm{x}_{0}=0}=-2 \mathrm{i} \epsilon_{\mathrm{ikl}}\left[\tilde{\mathrm{j}}_{5 l}^{(3)}+\frac{1}{\sqrt{3}} \tilde{\mathrm{j}}_{5 \ell}^{(8)}+\left(-\frac{8}{3} \mathrm{j}_{5 \ell}^{(\mathrm{op})}+\frac{2 \sqrt{2}}{3} \mathrm{j}_{5 \ell}^{(0,8)}\right]\right. \tag{12}
\end{equation*}
$$

+ vector currents,
where $\tilde{j}_{5 \mu}^{(a)}$ is defined in (6). Applying Bjorken limit to (8) and (10) and (12) we obtain a sum rule

$$
\begin{equation*}
-\mathrm{ef}_{\pi} \mathrm{m}_{\pi}=\sum_{\mathrm{V}} \mathrm{G}_{\mathrm{V}} \mathrm{f}_{\pi \mathrm{V} \gamma}^{0} \tag{13}
\end{equation*}
$$

where we have to include $\omega^{\prime}$ in the summation as the sum rule is independent of mass ${ }^{12}$. Taking the symmetry limit $f_{(a b)(c d)(e f)}^{o} \propto d_{a c e} d_{b d f}$, we find from (4)

$$
\begin{equation*}
\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}=\mathrm{f}_{\pi \omega \gamma}^{\mathrm{o}}=\sqrt{3} / 2 \mathrm{f}_{\pi \omega^{\prime} \gamma^{\prime}}^{\mathrm{o}}, \mathrm{f}_{\pi \phi \gamma}^{\mathrm{o}}=0 . \tag{14}
\end{equation*}
$$

The deviation from the $\mathrm{SU}_{3}$ value $\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}=-\mathrm{f}_{\pi \omega \gamma}^{\mathrm{o}} \gamma^{13}$ is due to the contribution of the unitary singlet current to $\mathrm{f}_{\pi \rho \gamma}^{0}$. Inserting (5) and (14) into (13), we find

$$
\begin{equation*}
\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}=\mathrm{f}_{\pi \omega \gamma}^{\mathrm{O}}=-\mathrm{e} \mathrm{f}_{\pi} \mathrm{m}_{\pi} / 2 \mathrm{G}_{\rho}=-0.018 \tag{15}
\end{equation*}
$$

where we have used ${ }^{13} \mathrm{G}_{\rho} / \mathrm{m}_{\rho}^{2}=0.18$. We assume that $\omega^{t}$ mass is very large and neglect $\omega^{\prime}$ from the summation in (12) and in (11) for $q^{2}=k^{2}=0$. We then have

$$
\begin{align*}
\mathrm{f}_{\pi \omega \gamma} & =\mathrm{f}_{\pi \omega \gamma}^{\mathrm{o}}+\mathrm{e} \mathrm{G} \rho^{\mathrm{m}^{-2}} \mathrm{~g}_{\pi \rho \omega},  \tag{16}\\
\mathrm{f}_{\pi \rho \gamma} & =\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}+\mathrm{e} \mathrm{G} \omega^{\mathrm{m}^{-2}} \mathrm{~g}_{\pi \rho \omega}  \tag{17}\\
\mathrm{m}_{\pi}^{\mathrm{T}} \pi^{(0,0)} & =2 \text { e } \mathrm{G}_{\rho} \mathrm{m}^{-2}\left(\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}-\mathrm{f}_{\pi \omega \gamma} / 3\right) \tag{18}
\end{align*}
$$

Experimentally $m_{\pi} T_{\pi}=3.3 \times 10^{-3}$. Together with values given by Eqs. (5), (15) and (16), we determine from the above three equations:

$$
\begin{equation*}
\mathrm{f}_{\pi \omega \gamma}=-0.14, \quad \mathrm{f}_{\pi \rho \gamma}=-0.024, \quad \mathrm{~g}_{\pi \rho \omega}=-2.3 \tag{19}
\end{equation*}
$$

The upper limit for the magnitude of $\mathrm{f}_{\pi \omega \gamma}$ corresponding to the present experimental upper limit on $\Gamma\left(\omega \rightarrow \pi^{\circ}+\gamma\right)$ is $\sim 0.13$. We will not worry about this small discrepancy, as the symmetry breaking can change the values of $G_{V}$ and $f_{\pi V}^{0} \dot{\gamma}$ by as much as $20 \%$. It is instructive to write $\mathrm{T}_{\pi}(0,0)$, using (10) and (13) in the form

$$
\begin{equation*}
\mathrm{m}_{\pi^{T} \pi^{\prime}(0,0)=-2 \mathrm{e}^{2} \mathrm{f}_{\pi} \mathrm{m}_{\pi} / \mathrm{m}^{2}-2 \mathrm{e}\left(\mathrm{G}_{\omega^{\prime}} / \mathrm{m}^{2}\right) \mathrm{f}_{\pi \omega^{\prime} \gamma^{\prime}}^{0}+2 \mathrm{e}^{2} \frac{\mathrm{G}}{\mathrm{~m}^{2}} \quad \frac{\mathrm{G}_{\rho}}{\mathrm{m}^{2}} \mathrm{~g}_{\omega \rho \pi} . . . . . .} \tag{20}
\end{equation*}
$$

Using the parameters determined above, the three terms in (20) have values, $-4,+2.6$ and $+4.6\left(\times 10^{-3}\right)$, respectively. If we had assigned to $0^{-}$mesons charm singlet states $(a, 0)$, then $\mathrm{f}_{\pi \omega^{\prime} \gamma^{\prime}}^{0}=0$, and we would have no second term in (20). The sum of the first two terms in (20) is just $\sum_{V=\rho, \omega}\left(G_{V} / m^{2}\right) f_{\pi V \gamma}^{o}$, and
the second term simply represents the correction due to different values of $\mathrm{f}_{\pi \omega \gamma^{*}}^{\mathrm{O}} \quad\left(\mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}\right.$ has the same value in any case.) In our case $\mathrm{f}_{\pi \omega}^{\mathrm{o}} \gamma^{\text {is }-1 / 3}$ of the value one would obtain for $\pi^{0}=(3,0)$.

To calculate $\eta \rightarrow 2 \gamma$ decay rate, we assume the conventional mixing of octet $\eta_{8}$ and singlet $\eta_{0}$,

$$
\begin{equation*}
|\eta\rangle=\cos \theta^{\prime}\left|\eta_{8}>+\sin \theta^{\prime}\right| \eta_{0}> \tag{21}
\end{equation*}
$$

with $\theta^{\prime} \sim 10^{\circ}$. We further assume $\eta_{8}$ and $\eta_{0}$ have the wave function (4) with $\mathrm{a}=8$ and 0 , respectively. In view of the failure of the nonet mass formula for $0^{-}$mesons, this assignment for $\eta_{0}$ may not be justified. We take this model as just one simplest possibility to evalutate $\eta_{0}$ amplitude. For $\eta_{8}$ amplitude, we notice that we would have a $\mathrm{SU}_{3}$ symmetry value $\mathrm{T}_{8}=\mathrm{T}_{\pi} \sqrt{3}$, if $\phi$ meson mass $\mathrm{m}_{\phi}$ were close to $\omega-\phi$ average mass m. (Now we have $\phi$ contribution since $\mathrm{f}_{\eta_{8} \phi \gamma}^{\mathrm{o}} \neq 0$.) We obtain then

$$
\begin{align*}
\mathrm{T}_{8} \mathrm{~m}_{\pi} & =\mathrm{T}_{\pi} \mathrm{m}_{\pi} / \sqrt{3}-2 \mathrm{e} \frac{\mathrm{G}_{\phi}}{\mathrm{m}^{2}}\left(1-\frac{\mathrm{m}^{2}}{\mathrm{~m}_{\phi}^{2}}\right) \mathrm{f}_{\eta_{8} \phi \gamma^{o}}-\mathrm{e}^{2}\left(\frac{\mathrm{G}_{\phi}}{\mathrm{m}^{2}}\right)^{2}\left(1-\frac{\mathrm{m}^{4}}{\mathrm{~m}_{\phi}^{4}}\right) \mathrm{g}_{\eta_{8} \phi \gamma} \\
& =(1 / \sqrt{3})[3.3+1.1+1.9] \times 10^{-3}  \tag{22}\\
& =(1 / \sqrt{3}) 6.3 \times 10^{-3}
\end{align*}
$$

where we have used $\mathrm{f}_{\eta_{8} \phi \gamma}^{\mathrm{o}}=-2 \sqrt{2 / 3} \mathrm{f}_{\pi \rho \gamma}^{\mathrm{o}}$ and $\mathrm{g}_{\eta_{8} \phi \phi}=(2 / \sqrt{3}) \mathrm{g}_{\pi \rho \omega}$. We find that $\phi$ mass correction almost doubles the value of $\mathrm{T}_{8}$. (Incidentally, one obtains almost the same value if one directly sums over contributions from $\rho, \omega$ single and double poles, neglecting $\phi$ altogether.) If we use the quark model coupling constants, we obtain $^{14}(1 / \sqrt{3}) 4.8 \times 10^{-3}$ instead of (22).

For the unitary singlet component $\eta_{0}$, the ETC term is given by the last bracket in (12), which gives for $\eta_{0}$ the value $-2 \sqrt{2 / 3} \mathrm{f}$, compared to $\mathrm{f} \pi$ for $\pi^{\mathrm{o}}$.

Analogous to (20), we find

$$
\begin{align*}
\mathrm{T}_{0} \mathrm{~m}_{\pi} & =4 \sqrt{2 / 3} \mathrm{e}^{2} \mathrm{f}_{\pi} \mathrm{m}_{\pi} \mathrm{m}^{-2}-2 \mathrm{e} \mathrm{G}_{\omega^{\prime}} \mathrm{m}^{-2} \mathrm{f}_{\eta_{0} \omega^{\prime} \gamma}^{0}+\mathrm{e}^{2} \sum_{\mathrm{V}=\rho, \omega}\left(\mathrm{G}_{\mathrm{V}} \mathrm{~m}^{-2}\right)^{2} \mathrm{~g}_{\eta_{0} \mathrm{VV}} \\
& =(1 / \sqrt{3})[11.4-7.4+10.7] \times 10^{-3}  \tag{23}\\
& =(1 / \sqrt{3}) 14.7 \times 10^{-3},
\end{align*}
$$

where we have neglected $\phi$ terms, as its inclusion does not change the result. We notice an enormous enhancement of this amplitude. In the quark model, this number will be $(1 / \sqrt{3}) 8.1 \times 10^{-3}$. From (21), (22), and (23) we have

$$
\mathrm{m}_{\pi} \mathrm{T}_{\eta}=(1 / \sqrt{3}) 8.9 \times 10^{-3}
$$

which gives the decay rate

$$
\Gamma(\eta-2 \gamma)=(1 / 64 \pi)\left|\mathrm{T}_{\eta}\right|^{2} \mathrm{~m}_{\eta}^{3} \sim 1.2 \mathrm{keV}
$$

close to the experimental value. This compares with $\mathrm{m}_{\pi} \mathrm{T}_{\eta}=(1 / \sqrt{3}) 6.2 \times 10^{-3}$ and $\Gamma(\eta-2 \gamma) \sim 0.55 \mathrm{keV}$ for the quark model. ${ }^{15}$

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## FOOTNOTES AND REFERENCES

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9. In terms of $\tilde{f}_{\rho}$ of Sakurai (Ann. of Phys. $11,1(1960), G_{\rho}=m_{\rho}^{2} / f_{\rho}$.
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12. That such a state must exist can be seen by replacing in Eq. (9), (11), and (13) $\mathrm{j}_{\mu}^{\mathrm{em}}$ by $\mathrm{j}_{\mu}^{(0,8)}$ and $\pi^{\circ}$ by the unitary singlet $0^{-}$meson, which we call $\eta_{0^{\circ}}$ In the sum rule thus obtained, similar to (14), we have only $\omega^{\prime}$ in the sum, whereas the matrix element of $\left[j_{i}^{(0,8)}(x), j_{k}^{(0,8)}(0)\right]_{x_{0}=0}$ is non-vanishing.
13. This corresponds to $\mathrm{f}_{\rho}^{2} / 4 \pi \sim 2.4$. For the determination of $\mathrm{f}_{\rho}$, see
D. A. Geffen and T. Walsh, Phys. Rev. Letters 20, 1536 (1968);
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14. This number gives $\sqrt{3} \mathrm{~T}_{8} / \mathrm{T}_{\pi} \sim 1.5$ for the quark model. The quark model calculations by Van Royen and Weisskopf [R. Van Royen and V. F. Weisskopf, Nuovo Cimento, 50, 617, (1967)], gives a slightly smaller ratio.
15. The estimate of $\mathrm{T}_{0} / \mathrm{T}_{8}$ was made in a different way by Dalitz and Sutherland. R. H. Dalitz and D. G. Sutherland, Nuovo Cimento, 37, 1777, (1965). Our value is well within the limit set by them.

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