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NUCLEON ELECTROMAGNETIC FORM FACTORS AND THE ρ° , ω AND ϕ RESONANCES[†]

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ABSTRACT

A solution for the problem of expressing the four electromagnetic form factors of the nucleon in terms of the ρ^{0} , ω and ϕ resonances is proposed. The asymptotic behavior and analyticity properties of the solution are determined. The agreement with experiment is excellent.

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It is the main purpose of this work to propose a solution to the problem of expressing the electromagnetic form factors of the nucleon in terms of only the ρ^{0} , ω and ϕ resonances¹.

The general properties which we require the solution to have are:

- a) it is the sum of three terms representing the contribution of the ρ^{0} , ω and ϕ resonances respectively,
- b) analyticity,
- c) the coupling coefficient of each term should be a slowly varying function of the square of the four momentum transfer t in the space-like region t < 0 and which tends to a constant for |t| → ∞.

The simplest expression satisfying the requirements a), b), and c) is then the following:

$$G_{MP}(t) = \sum_{k=1}^{3} \frac{\epsilon_{k}(t)}{t - a_{k} - \gamma_{k} \sqrt{t_{k} - t}}$$
(1)

with

$$\epsilon_{k}(t) = \frac{\epsilon_{k}^{1} + \epsilon_{k}^{0} t}{t - a_{k} - \gamma_{k} \sqrt{t_{k} - t}}$$
(1a)

$$G_{EP}(t) = \sum_{k=1}^{3} \frac{\beta_{k}(t)}{t - a_{k} - \gamma_{k} \sqrt{t_{k} - t}}$$
(2)

with

$$\beta_{k}(t) = \frac{\beta_{k}^{l} + \beta_{k}^{o} t}{t - a_{k} - \gamma_{k} \sqrt{t_{k} - t}}$$
(2a)

 $G_{MP}(t)$ is the proton magnetic and $G_{EP}(t)$ is the proton electric form factor². $G_{MN}(t)$ and $G_{EN}(t)$ will indicate in the following the neutron magnetic and neutron electric form factors respectively.

The rest of our notation is the following: a_k , γ_k , t_k are the square of the resonance's masses, widths and threshold values respectively. The value k=l corresponds to the ρ^0 , k = 2 to the ω , and k=3 to the ϕ resonance. The values of t quoted in our work will always be in (GeV)²; masses, widths and thresholds will be given in GeV. Their values, taken from experiment, are:

k	^a k	k	t _k
1	0.585	0.12	0.078
2	0.608	0.012	0.176
3	1.040	0.003	0.980

Table	1
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The additivity condition a) together with the isotopic spin structure of the nucleon electromagnetic form factors imply that the expression for $G_{MN}(t)$ and $G_{EN}(t)$ corresponding to (1) and (2) can be obtained by simply changing the sign of the ρ^{0} term:

$$G_{MN}(t) = -\epsilon_{1}(t) f_{1}(t) + \epsilon_{2}(t) f_{2}(t) + \epsilon_{3}(t) f_{3}(t)$$
(3)

$$G_{EN}(t) = -\beta_1(t) f_1(t) + \beta_2(t) f_2(t) + \beta_3(t) t_3(t)$$
(4)

where we have put

$$f_{k}(t) = \frac{1}{t - a_{k} - \gamma_{k} \sqrt{t_{k} - t}}$$

We are faced with the problem of determining twelve parameters, six ϵ_k 's and six β_k 's, from experimental and theoretical boundary conditions. The well-known relations:

$$F_{1,2}^{V}(t) = \frac{1}{2} \left[F_{1,2}^{p}(t) - F_{1,2}^{n}(t) \right]$$

$$F_{1,2}^{S}(t) = \frac{1}{2} \left[F_{1,2}^{p}(t) + F_{1,2}^{n}(t) \right]$$
(5a)

$$F_{1}^{p}(t) = \left[G_{EP}(t) - \tau G_{MP}(t)\right] / (1 - \tau)$$

$$F_{1}^{n}(t) = \left[G_{EN}(t) - \tau G_{MN}(t)\right] / (1 - \tau)$$

$$F_{2}^{p}(t) = \left[G_{MP}(t) - G_{EP}(t)\right] / (1 - \tau)$$

$$F_{2}^{n}(t) = \left[G_{MN}(t) - G_{EN}(t)\right] / (1 - \tau)$$
(5b)

 $(\tau = t/4M^2)$, M is the mass of the proton) for the Dirac and Pauli form factors $F_{1,2}^{p,n}$ (*) and for the isovector and isoscalar form factors $F_{1,2}^{V,S}$, impose three linear relations among the twelve parameters. Indeed in order

(*) Normalized as $F_1^p(0) = 1$; $F_1^n(0) = 0$; $F_2^p(0) = 1.79$; $F_2^n(0) = -1.91$.

that the F's do not have a singularity at $t = 4M^2$ we must impose the conditions:

$$\beta_{k}^{o} = (\epsilon_{k}^{1} - \beta_{k}^{1})/4M^{2} + \epsilon_{k}^{o}$$
 R = 1, 2, 3 (A)

With these conditions we find:

$$F_{1}^{V}(t) = (\beta_{1}^{1} + \epsilon_{1}^{o} t) f_{1}^{2}(t)$$

$$F_{2}^{V}(t) = (\epsilon_{1}^{1} - \beta_{1}^{1}) f_{1}^{2}(t)$$

$$F_{1}^{S}(t) = (\beta_{2}^{1} + \epsilon_{2}^{o} t) f_{2}^{2}(t) + (\beta_{3}^{1} + \epsilon_{3}^{o} t) f_{3}^{2}(t)$$

$$F_{2}^{S}(t) = (\epsilon_{2}^{1} - \beta_{2}^{1}) f_{2}^{2}(t) + (\epsilon_{3}^{1} - \beta_{3}^{1}) f_{3}^{2}(t)$$
(6)

The normalization conditions at t = 0:

$$F_1^V(0) = \frac{1}{2};$$
 $F_2^V(0) = 1.85;$ $F_1^S(0) = \frac{1}{2};$ $F_2^S(0) = -0.06$ (B)

and the experimentally determined values of the derivatives of the form factors at t = 0: 3

$$\left(\frac{\mathrm{d}\,\mathrm{G}_{\mathrm{MP}}}{\mathrm{d}t}\right)_{t=0} = 7.98; \qquad \left(\frac{\mathrm{d}\,\mathrm{G}_{\mathrm{EP}}}{\mathrm{d}t}\right)_{t=0} = 2.95;$$

(C)

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$$\left(\frac{\mathrm{d}\,\mathrm{G}_{\mathrm{MN}}}{\mathrm{d}t}\right)_{t=0} = -5.61; \qquad \left(\frac{\mathrm{d}\,\mathrm{G}_{\mathrm{EN}}}{\mathrm{d}t}\right)_{t=0} = 0.497$$

give eight further linear relations among the twelve parameters; however, explicit calculation shows that the eleven conditions (A), (B), and (C) permit one to eliminate only ten of the twelve parameters^(*). We choose ϵ_2^{0} and ϵ_3^{0} as the two remaining independent parameters. Let us consider their physical meaning.

According to the usual definition of coupling constants it seems reasonable to define:

$$(\beta_1^{1} + \epsilon_1^{0} t) f_1(t) = g_{\gamma\rho}(t) g_{\rho NN}(t)$$

$$(\beta_2^{1} + \epsilon_2^{0} t) f_2(t) = g_{\gamma\omega}(t) g_{\omega NN}(t)$$

$$(\beta_3^{1} + \epsilon_3^{0} t) f_3(t) = g_{\gamma\phi}(t) g_{\phi NN}(t)$$
(D)

where the left hand side of (D) are just the coupling functions of $F_1^V(t)$ and of the ω and ϕ terms of $F_1^S(t)$ respectively, and $g_{\gamma\rho}(t) \dots g_{\phi NN}(t)$ represent the coupling functions of the vector mesons to the photon and to the nucleon respectively.

It now seems reasonable to assume the following asymptotic conditions:

^(*) ϵ_1^1 and β_1^1 are already determined by (B) and therefore the value of $F_2^V(0)$ predicted. Condition (C) for dF_2^V/dt is satisfied within 15%.

(I)
$$r_{12} = \lim_{|t| \to \infty} \left| \frac{g_{\gamma\rho}(t)}{g_{\gamma\omega}(t)} \right| \left| \frac{g_{\rho NN}(t)}{g_{\omega NN}(t)} \right| = 2.63$$

(II)
$$\mathbf{r}_{13} = \lim_{|t| \to \infty} \left| \frac{g_{\gamma \rho}(t)}{g_{\gamma \phi}(t)} \right| \left| \frac{g_{\rho NN}(t)}{g_{\phi NN}(t)} \right| = 1.63$$

Condition (I) and (II) are equivalent to the requirement of the asymptotic validity of SU₃ symmetry, the values of r_{12} and r_{13} being those predicted by SU₃ and a definite $\omega - \phi$ mixture of unitary singlet and octet⁴.

With these assumptions we can determine ϵ_2^{o} and ϵ_3^{o} - except possibly for their sign - and therefore all the parameters appearing in the form factors. Investigating all the four possible solutions with $\pm \epsilon_2^{o}$, $\pm \epsilon_3^{o}$ we find that the comparison with experiments favors $\epsilon_2^{o} > 0$, $\epsilon_3^{o} > 0$.

The values of the parameters obtained are therefore:

k	ϵ_k^{o}	$\epsilon_{\rm k}^{\rm l}$	β_k^{o}	β_k^{-1}
1	- 0.487	0.900	- 0.286	0.192
2	0.185	-0.117	0.151	0.003
3	0.299	0.817	0.379	0.535

Table 2

DISCUSSION OF THE RESULTS

We compare to experiment the prediction of our solution Eqs. (1), (2), (3), and (4) with the values of the parameters given in Table 2.⁵ Figs. 1), 2), 3), and 4) show that there is a generally good agreement between our results and the experimental data available in the space-like region t < 0. In the time-like region the form factors become complex for $t > t_1$, since the expression under the square root in the first term of (1), (2), (3), and (4) becomes negative⁶. The only experimental measurement in this region is an upper limit for the cross-section of the annihilation process $p + \overline{p} - e^+ + e^-$ at t = 6.8, which gives a value of $|G_{MP}|/2.79 \leq 0.04$.⁷

In Fig. 5 we have plotted $|G_{MP}(t)|/2.79$ for $0 \le t \le 10$. A discussion of the analyticity properties of the form factors can be found in the Appendix. We shall prove there the expressions (1), (2), (3), and (4) satisfy a dispersion relation.

A few comments are in order at this point:

a) Our solution is not a fit to the experimental data, since we have predetermined all our parameters from theoretical and experimental boundary conditions and then compared our prediction to experiment.

b) Our results give strong support to the vector dominance hypothesis^{1, 8}.

c) Just the ρ^0 , ω and ϕ resonances are sufficient in order to explain the known experimental facts about the electromagnetic form factors. In particular, no ρ' isovector resonance is necessary.

d) It can be seen directly from (1), (2), (3), (4) and Table 2 that the proposed form factors have the following asymptotic behavior: for $|t| \rightarrow \infty$ all the form factors G as well as $F_1^{p,n}$ and $F_1^{V,S}$ tend to zero as t^{-1} , while the form factors $F_2^{p,n}$ and $F_2^{V,S}$ tend to zero as t^{-2} .

e) The solution we propose shows that the electromagnetic form factors, as a consequence of their analyticity properties and of their asymptotic behavior obey unsubtracted dispersion relations. f) Our solution does not satisfy an exact "scaling law"

 $G_{EP}(t) = G_{MP}(t)/2.79 = G_{MN}(t)/-1.91$, but suggest that these relations have only an approximate, empirical validity.

g) The behavior of the coupling functions $g_{\gamma V}(t) g_{VNN}(t)$ is quite different in the space-like and in the time-like regions. For t < 0 they are real and vary slowly, while for $t > t_k$ they become complex and show a behavior similar to that of the index of refraction in optics.

h) If we assume that the coupling $g_{\rho NN}(t)$ is approximately constant for small, negative t values and if we take its value to be $\cong 5$ we have an estimate of $g_{\gamma\rho}$ for t < 0: $|g_{\gamma\rho}| \simeq 0.06$. This value of $g_{\gamma\rho}$ is smaller but not in disagreement with other estimates¹⁰, which give $g_{\gamma\rho} \cong 0.1$ with a 30-40% error.

i) It can be seen from Table 2 that there is a tendency for cancellation between the isovector and isoscalar parts in $G_{\rm MP}$ and a little also in $G_{\rm EP}$ at large t values, while the same effect occurs in $G_{\rm EN}$ and $G_{\rm MN}$ at small t.

j) The presence in our solution of poles of second order^(*) suggests the following physical interpretation of the photon nucleon interaction. The unitarity condition

$$\operatorname{Im} < N\overline{N} \mid j_{\mu}^{em} \mid 0 > = \sum_{n} < N\overline{N} \mid T \mid n > < n \mid j_{\mu}^{em} \mid 0 >$$

shows that the 2π , 3π , $K\overline{K}$ intermediate states contribute to the form factor via first an electromagnetic interaction which creates them and then via a strong interaction which converts them into NN. The vector dominance

(*) In the unphysical Riemann sheet.

hypothesis then suggests that the first step takes place mainly through a vector meson resonance. However, the strong interaction amplitude to has a pole at the vector meson mass. For example, if we consider the isovector contribution to Im G, one pole may be associated with the pion form factor and another with the pion-nucleon scattering amplitude.

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APPENDIX

The proof that our solution satisfies a dispersion relation consists in showing that an expression such as (1) does not have singularities in the upper half-plane of the complex variable t. Since it decreases sufficiently fast for $|t| \rightarrow \infty$ it then satisfies Cauchy's theorem from which follows that it also satisfies a dispersion relation.

To prove the analyticity we write for any one of the form factors:

$$\begin{aligned} G(t) &= \sum_{k} (\delta_{k}^{0} t + \delta_{k}^{1}) \frac{1}{(t - a_{k} - \gamma_{k} \sqrt{t_{k} - t})^{2}} \\ &= \sum_{k} (\delta_{k}^{0} t + \delta_{k}^{1}) \frac{(t - a_{k} + \gamma_{k} \sqrt{t_{k} - t})^{2}}{[(t - a_{k})^{2} + \gamma_{k}^{2}(t - t_{k})]^{2}} \\ &= \sum_{k} (\delta_{k}^{0} t + \delta_{k}^{1}) \frac{(t - a_{k} + \gamma_{k} \sqrt{t_{k} - t})^{2}}{[(t - \alpha_{+}^{k})(t - \alpha_{-}^{k})]^{2}} \\ \end{aligned}$$
where
$$\alpha_{\pm}^{k} = a_{k} - \frac{\gamma_{k}^{2}}{2} \pm i\gamma_{k} \sqrt{x_{k} - \frac{\gamma_{k}^{2}}{4}}$$

 $x_k = a_k - t_k$

The only singularity G(t) may have in the upper plane is therefore at $t = \alpha_+^k$. Let us write:

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$$G(t) = \sum_{k} \frac{(\delta_{k}^{0} t + \delta_{k}^{1})}{(t - \alpha_{-}^{k})^{2}} \left[1 + \frac{\gamma_{k} \sqrt{t_{k} - t} + i\gamma_{k} \sqrt{x_{u} - \frac{\gamma_{k}^{2}}{4} - \frac{\gamma_{k}^{2}}{2}}}{t - \alpha_{+}^{k}} \right]^{2}$$

Our choice of sign of the width term¹² in expression (1)....(4) guarantees that G(t) is finite at $t = \alpha_+^k$. Indeed if t moves in the upper halfplane from a point on the real axis at $t < t_k$ to a point on the same axis at $t > t_k$ (i.e. $\theta = \pi$ goes into $\theta = 0$) we see that we must choose the analytic continuation

$$\sqrt{t_k - t} \longrightarrow -i\sqrt{t - t_k}$$

since for $\theta = \pi$ we have chosen the + sign of the square root.

If we use the identity:

$$\sqrt{\alpha_+^k - t_k} = \sqrt{x_k - \frac{\gamma_k^2}{4}} + i \frac{\gamma_k}{2}$$

we find

$$\lim_{t \to \alpha_{+}^{k}} G(t) = \sum_{k} \frac{\delta_{k}^{0} \alpha_{+}^{k} + \delta_{k}^{1}}{(\alpha_{+} - \alpha_{-})^{2}} \left[1 + \frac{1}{2 i \gamma_{k} \sqrt{x_{k} - \frac{\gamma_{k}^{2}}{4} - \frac{\gamma_{k}^{2}}{2}}} \right]^{2}$$

This proves that $G(\alpha_{+}^{k})$ is finite. Similarly one can verify that $\frac{dG}{dt}\Big|_{t=\alpha_{+}^{k}}$ is also finite.

FIGURE CAPTIONS

Figure la - lb	The solid line is our theoretical solution for
	$G_{MP}^{}(t)$; the experimental data are taken from
	Ref. (5).
Figure 2	The solid line is our theoretical solution for
	$G_{EP}^{}(t);$ the experimental data are taken from
	Ref. (5).
Figure 3	The solid line is our theoretical solution for
	$G_{MN}(t)$; the experimental data are taken from
	Ref. (5).
Figure 4	The solid line is our theoretical solution for
	$G_{EN}^{(t)}$; the experimental data are taken from
	Ref. (5).
Figure 5a - 5b	The solid line is our theoretical solution for
	$G_{MN}(t)$ in the time-like region t 0; the upper
	limit is taken from Ref. (7).

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Fig. 1ь







Fig. 3





Fig. 5a



Fig. 5b