SLAC PUB 476
August 1968
(TH)

THE STRUCTURE OF HIGH ENERGY PROTON-PROTON SCATTERING*

Henry D.I. Abarbanel,† Sidney D. Drell, and Frederick J. Gilman

Stanford Linear Accelerator Center
Stanford University
Stanford, California

*Work supported by the U.S. Atomic Energy Commission

†Address after September 1, 1968: Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

(submitted to Physical Review)

ABSTRACT

A suggestion of how to correlate the electromagnetic form factors of the proton with p-p scattering is developed in detail. Postulating a new elementary local interaction of current-current form plus a diffractive term, we construct for p-p scattering an approximately unitary scattering amplitude for large fixed s and all values of t using the Fourier-Bessel transform of the scattering amplitude. The t dependence of the resulting cross section is closely correlated with the fourth power of the electromagnetic form factor of the proton as suggested first by Wu and Yang and agrees well with high energy data ($E_{lab} \approx$ 30 BeV) over many decades in values for $d\sigma/dt$. Differences from related models are discussed, as well as further applications and experimental implications of the theory.

I. Introduction

With heuristic arguments, Wu and Yang[1] predicted in 1965 that high energy, $s \gg M_N^2$, large momentum transfer, $-t \gg M_N^2$, elastic proton-proton scattering would reveal the same structure of the proton through its t dependence as that measured by the electromagnetic form factors in elastic electron-proton scattering.

Since that original suggestion appeared there have been important new experimental results in both p-p and e-p scattering extending into broad new domains of s and t. From these data there has emerged a suggestion[2] of how to correlate the electromagnetic form factors with the p-p cross sections, as illustrated in Fig. 1 which shows the cross section for p-p elastic scattering,[3]

$$X(s,t) \equiv (d\sigma/dt)/(d\sigma/dt)_{t=0},$$

plotted together with the fourth power of $G_{Mp}(t)$, the magnetic form factor measured in e-p scattering[4] normalized to $G_{Mp}(0) = 1$. This connection, although different in particulars from the earliest suggestions of universal functions that might represent all the high energy p-p data, supports in essence the original Wu-Yang proposal that the t dependence of p-p scattering and the fourth power of the electromagnetic form factor are proportional to one another.

The basis for the ideas presented in Ref. 2 and for the theory which is constructed in the present paper is the close coincidence, extending over almost 12 decades in range of values, of the p-p data at the laboratory energy $E_{lab} = 30$ BeV, or $s = 2M_N^2 + 2M_N E_{lab} \approx 60\ \mathrm{BeV}^2$, with the measured form factors, together with the following theoretical conjecture: As the incoming proton energy, or s, increases, the invariant differential cross section $d\sigma/dt$ for large t approaches a limit independent of s to within logarithms and the t dependence of this limit is proportional to $G_{Mp}^{4}(t)$, i.e.,

$$\lim_{s \to \infty} \frac{d\sigma}{dt}(s,t) \propto G_{Mp}^{4}(t) \; . \tag{1}$$

Fig. 1 suggests indeed that we have already witnessed, at least in a qualitative way, the emergence of this limit. Whether or not this is a true inference from Fig. 1 can clearly be tested directly, <u>and for our theory crucially</u>, before long at Serpukhov, at the CERN colliding proton ring facility, and at Weston.

The purpose of the present paper is to present a more complete theory of the conjectured behavior given by Eq. (1) starting with an input expression for the interaction forces or "driving terms" and deducing therefrom an approximately unitary S-matrix and scattering cross section. First we will review our earlier suggestion for a theoretical interpretation of the data in Fig. 1. In I we wrote an ansatz directly for the scattering amplitude on the basis of the same physical ideas that are used in this work to specify the form of the single-nucleon matrix elements of the interaction currents from which the p-p scattering amplitude is now <u>constructed</u>. Limitations of the earlier model as well as essential differences from the related theories proposed by others will also be explored.

In I we suggested the following correlation and interpretation of the data in Fig. 1: In the amplitude for p-p scattering there is a piece, the "diffractive tail," which dies precipitously for fixed t as s grows and, <u>in</u>

addition a point interaction of current-current from which depends on t alone and emerges as s becomes asymptotic. The differential cross section then appears as

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} \left[a\, G_{Mp}^{2}(t) + R(s,t)\right]^{2}, \tag{2}$$

where a is independent of s and t and R(s, t) vanishes as $s \to \infty$ for large, fixed -t.

For concreteness we chose for R(s, t) the canonical "Regge form"

$$R(s,t) = \beta(t)\, \frac{1 + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}\, s^{\alpha(t) - 1}, \tag{3}$$

although our ideas were and are weakly coupled to any special model for R. In a Reggeized world, $\alpha(t)$ refers to the usual vacuum trajectory. The experimental basis for choosing such an R(s, t) is the observed dramatic drop in X(s, t) by a factor of $\gtrsim 2$ for each 20% increase in s in the range 20-60 BeV^2. It is tempting to propose that $s^{\alpha(t)}$ accurately describes the approach to the high energy limit. Not only is this in accord with the data shown in Fig. 1 and more transparently by the straight line segments of Fig. 2 whose slopes measure $\alpha(t)$ at the labeled values of t, but it is also theoretically appealing. If one particular Regge trajectory has a slightly smaller slope than all others, then by the time we move out to large values of both s and -t it will dominate the others and a simplified parametrization of the elastic scattering amplitude such as proposed for R(s, t) is a natural consequence. The small slope for the Pomeranchuk or vacuum trajectory, compared to other known trajectories, which is suggested by p-p and π-p data at small t, is in agreement with this behavior. We emphasize that our main point of comparison between e-p and p-p scattering is not rigidly tied to a specific Regge model. More broadly stated, as $s \to \infty$ for fixed large $-t$, $R(s,t)$, which may be interpreted as the decreasing tail of the diffractive or unitarity contribution from the inelastic channels, falls below the postulated s independent contact term revealing the $G_{Mp}^4(t)$ structure.

An origin for the contact interaction was proposed as follows: Consider the reaction nucleon (p_1) + nucleon (p_2) $\to$ nucleon (p_1') + nucleon (p_2') in the region where $s \gg -t \gg M_N^2$. Writing out the T-matrix in terms of the Fermi invariants, we find that the pseudoscalar and scalar contributions are of order t/s or M_N^2/s compared to V, A, and T. If we imagine that in this kinematic region, where all masses are negligible compared with the relevant dynamical variables, the scattering occurs with no flip of the nucleon helicities, then the amplitude becomes to order t/s

$$T_{NN} = F_V \bar{u}(p_2') \gamma_\alpha u(p_2) \bar{u}(p_1') \gamma_\alpha u(p_1) + F_A \bar{u}(p_2') \gamma_\alpha \gamma_5 u(p_2) \bar{u}(p_1') \gamma_\alpha \gamma_5 u(p_1) \,. \tag{4}$$

This resembles one vector density probing another plus an axial density interacting with another. We proposed to take this resemblance seriously and suggested that the proper statement of the "contact interaction" which is exhibited in the p-p data is that for $s \gg -t \gg M_N^2$, F_V and F_A become proportional to the squares of the vector and axial-vector from factors one measures in the weak[5] and electromagnetic interactions.[6] The contact terms enter $d\sigma/dt$ as

$$|F_V|^2 + |F_A|^2 + 4\mathrm{Re}(F_V^* F_A)\, t/s \,. \tag{5}$$

If, further, the vector and axial-vector form factors become similar for large t, or if the contact interaction cannot distinguish between right handed and left handed protons so that the contact interaction is purely of the vector type and $F_A = 0$, then the structure $a^2 G^4_{Mp}(t)$ for X(s, t) emerges.[7]

Our statement of no helicity flip by the proton in the kinematic range when M_N^2 is negligible compared with both s and t has its parallel in both weak and electromagnetic processes. In the weak interactions this is trivial due to the special nature of the lepton coupling, but in the electromagnetic interactions it is suggested in a preliminary way by the data. It also follows from the theoretically popular scaling law for the proton's electromagnetic form factors. To see this we simply write out the Rosenbluth cross section for e-p scattering.

$$\left(\frac{d\sigma}{dt}\right)_{e-p} \propto (F_1(t))^2 - \frac{t}{4M_N^2}\,\mu^2\,(F_2(t))^2 + O\left(\frac{M^2 t}{s^2}\right)$$

and note that if $\sqrt{-t}\; F_2(t)/F_1(t) \rightarrow 0$ for large -t, then we are left only with the helicity non-flip term, $F_1{}^2$. The "scaling law" for electromagnetic form factors tells us that $F_1(t) - \frac{t}{4M^2}\,\mu\, F_2(t) \equiv G_E(t) = G_M(t)/\mu_T \equiv$ $[F_1(t) + \mu\, F_2(t)]/\mu_T$ where $\mu_T = 2.79$. If such a scaling is in fact experimentally verified it makes $F_2(t) \propto F_1(t)/t$ for large -t, and is thus an even stronger condition than is needed if we are to be left with only the helicity non-flip term as $s \rightarrow \infty$ at large -t. Experimentally, the largest t value at which F_1 has been measured is $-t \sim 3\ \mathrm{GeV}^2$ and by then the ratio of F_2/F_1 has dropped from 1 at t = 0 to $< \frac{1}{3}$.[8]

With these assumptions our picture of the large s, large t proton-proton scattering was completely drawn, the differential cross section being written

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} \left[a^2 G_{Mp}^4(t) + \gamma(t)\,(s/s_o)^{2[\alpha(t)-1]} + \text{interference terms}\right]. \tag{6}$$

The magnitude of the interference terms depends on the relative phases of the contact terms and R(s, t), given by the signature factor in the Regge case, as well as on the spin structure of the diffractive contributions. We need only consider the interference terms in the limited range of s and t where R(s, t) and $a\,G_{Mp}^2(t)$ are of comparable magnitude, and in our preliminary fits we ignored them, obtaining the following representative set of parameters:

$$\begin{aligned} \alpha(t) &= 1 + \alpha'(0)t + (\tfrac{1}{2})\alpha''(0)t^2\,, \quad \alpha'(0) = 0.5 \pm .1 \\ \alpha''(0) &= 0.02 \pm 0.005, \text{ and } a = 0.85 \pm 0.15 \end{aligned} \tag{7}$$

The small value of $\alpha'(0)$ is consistent with our earlier remarks. Within the uncertainties permitted by the unknown interference term, more complicated guesses are possible for these parameters.

There is an appealing simplicity to the idea that, in hadron processes, under a "diffractive tail" there should emerge a contact interaction of a current-current nature with the <u>same</u> currents whose transition form factors are being measured in weak and electromagnetic processes. However, before this idea of nature's simplicity in choice of currents and interactions can be promoted from a pure phenomenology and dignified (or encumbered?) with a more solid theoretical foundation, several questions must be addressed. The most fundamental one is the following: In e-p scattering one measures the matrix element of a conserved vector current for momentum transfer t between initial and final single physical nucleon states and summarizes the observations in terms of form factors. In contrast, in p-p scattering we have proposed a model containing both a strong interaction via vector currents, as in Eq. (4), as well as a strong diffraction term summarizing inelastic contributions via unitarity to elastic scattering. From out of this stew of strong interactions distorting the two proton wave functions via multiple vector and Regge type exchanges, how does purée of electromagnetic form factor emerge? More directly stated, if we construct a T-matrix starting with interacting currents such as in Eq. (4) as the "driving term" or input contact force and then add to this the inelastic or diffraction amplitudes, what is the t dependence of the resulting scattering amplitude fully unitarized? Does it still show a $G_{Mp}^{4}(t)$ variation in the differential cross section for large t?

There are several additional questions that can also be addressed. For example, what is going on at small t values? Eq. (2) with $aG_{Mp}(0) \sim 1$ tells us that the forward scattering amplitude naïvely obtained by extrapolating the contact term from large t has approximately equal real and imaginary parts, in contradiction with experiments that fix the ratio of real to imaginary parts of the forward amplitude to be much less than one, even at present energies. Although our original model was imagined to be applicable only when $-t >> M_N^2$, can our present approach remove this restriction and show how the observed behavior near t = 0 emerges? In this connection there is the very striking observation, emphasized by Feynman,[9] that the close proportionality of $d\sigma/dt$ and $G_{Mp}^{4}(t)$ remains valid all the way to very small t. Can we also shed light on this behavior? What about the famous "breaks" in $d\sigma/dt$? Finally, once we extend our theory to $t = 0$, what can we say about the total cross section and the status of the Pomeranchuk theorem? In particular, what is the resulting asymptotic limit of the contact interaction for $s \to \infty$?

Our program here will then be as follows: (1) We postulate that there is an elementary local interaction between two protons of the current-current form which operates in addition to the usual strong interaction dynamics leading to diffractive contributions which are customarily summarized in a Regge parametrization. (2) We can then introduce a precise form for this current-current interaction that embodies the Wu-Yang idea; namely, our input is just a product of single nucleon matrix elements whose structure is that of the electromagnetic current. It is introduced as an additional "force:" an inhomogeneous term in the fixed t dispersion relation in the energy s for p-p scattering. To this we add the usual forces leading to diffractive behavior. (3) In Section II we construct an approximately unitary scattering amplitude following the procedures developed by Blankenbecler and Goldberger[10] and Baker and Blankenbecler.[11] They introduce the Fourier-Bessel transform of the scattering amplitude, for in the high energy regime this leads to an exceedingly simple unitarity relation from which the elastic amplitude can be recovered by a judicious mixture of quadratures and computers.

The resulting theory, which we complete in Section III by computing in detail differs in two essential ways from related studies of the connection of p-p data and electromagnetic form factors. (1) We have introduced a local current-current interaction in addition to the diffraction scattering one would normally contemplate. In the models based on Yang's work,[12, 13, 14] the form of the diffraction term itself is identified with the electric charge density. More precisely, if one writes the partial amplitude at energy s and impact parameter b as $e^{2i\delta(b,s)} - 1$, the scattering phase $\delta(b, s)$ is interpreted in Refs. 12 and 13 as a path integral proportional to the overlap of the electric charge distributions of the colliding hadrons. (2) The S-matrix, as approximately unitarized in our approach with the Fourier-Bessel transform, also has desired analyticity properties--in particular, a unitarity cut. Formally, in scattering examples with elastic unitarity, this replaces this "eikonal" phase $\delta(b, s)$ by the form $\delta \rightarrow 2 \arctan \delta/2$.[15] This replacement was introduced by Blankenbecler and Goldberger in order to preserve desired analyticity as well as unitarity properties of the S matrix. Clearly these forms are indistinguishable for weak potentials, such as those with which the eikonal approximation has often been used, but differ dramatically for strongly interacting processes. We will exhibit an example in Section III which makes explicit the differences between these two procedures and shows the importance of preserving the analyticity properties in addition to unitarity of the S-matrix. We conclude this section by presenting our representations for the elastic p-p scattering cross-section and comparing with the original ansatz, Eq. (1). Our final results closely correlate with this form and hence with experimental data at energies ≈ 30 BeV over the full range of measured t values extending over many decades for $d\sigma/dt$.

Finally, in Section IV we will briefly recount the achievements of the earlier paragraphs, study the ultimate approach of our amplitude to the Pomeranchuk limit, and speculate on further applications of the theory and its experimental consequences. In particular, an intriguing connection between the contact interaction we have examined here and low energy nucleon-nucleon scattering is discussed.

II. Approximating Unitarity

In constructing our suggested representation for the p-p scattering amplitude we begin by assuming that there exists a local current-current interaction in addition to the usual t-channel particle exchanges and production matrix elements in p-p collisions. That is, we introduce a new two particle scattering matrix element with no physical singularities in s which we add to the usual driving terms or input forces B(s, t) that one might consider in constructing a unitary amplitude for elastic scattering. We then write for these driving or "Born" terms in the nucleon-nucleon T-matrix,[16]

$$
\begin{aligned}
&T_{NN}^{(\text{forces})}(N(p_1) + N(p_2) \rightarrow N(p_{1'}) + N(p_{2'})) \\
&= g^2 G^2(t)\, \bar{u}(p_{2'})\gamma_\mu u(p_2)\bar{u}(p_{1'})\gamma_\mu u(p_1) + B(s,t),
\end{aligned}
\tag{8}
$$

where g^2 measures the strength of the additional local coupling we are considering; G(t) is the form factor associated with the one nucleon matrix element of the vector current involved in the interaction;[6] s is the square of the total barycentric energy in the collision; and t is the four momentum transfer $t \equiv (p_1 - p_{1'})^2 \equiv (p_2 - p_{2'})^2$. B(s, t) includes any and all other driving forces leading, in the absence of our added current-current interaction, to the high energy diffractive scattering. We include in B(s, t) not only t-channel exchange contributions such as one pion exchange terms, but also the strong inelastic forces which, after acting twice via unitarity through multi-body channels, return the system back into the elastic p-p channel. A graphical representation of these contributions to B(s, t) is drawn in Figs. 3b and 3c, along with the current-current term of Eq. (8) in Fig. 3a. We are omitting terms required by crossing symmetry and the Pauli principle because we will subsequently examine a region of s and t ($s \gg -t$, $s \gg m_N^2$) where such effects may be safely ignored.

We are also suppressing inessential spinor factors. Our requirement of no helicity flip for large t as discussed in Section I and Ref. 2 is essential in order to introduce, by an argument that is essentially a statement of generalized CVC, the electromagnetic form factors to describe the structure of our direct interaction matrix element. We have no deep commitment to the Lorentz tensor structure of the driving terms B(s, t) in Eq. (8) leading to the usual diffraction behavior, and henceforth will suppress the spinor factors as inessential.

It is a basic physical assumption of our model that the form factor, G(t), appearing in the current matrix element above is to be identified with the electromagnetic form factor of the nucleon[6] as measured in elastic e-p scattering. Equivalently we may also think of this force with its form factor structure in t as arising from an <u>effective</u> Lagrangian interaction of the form $L_{eff}(x) = -g^2 J_\mu(x) J_\mu(x)$ as in weak interaction theory.

Given the driving forces we must now construct the properly unitary and analytic scattering amplitude for p-p scattering. Since many inelastic channels are open and important at high energies, giving rise to the diffraction pattern for elastic scattering, it requires in general Herculean labor to construct a unitary S-matrix. We will therefore attempt to approximate unitarity in a tractable manner following the route mapped some years ago by Blankenbecler and Goldberger.[10] They noted that if one writes a Fourier-

Bessel representation for the scattering amplitude T(s, t)

$$T(s,t) = \int_0^\infty bdb\, J_0(b\sqrt{-t})\, H(b^2, s) \ , \tag{9}$$

then for large values of the energy s the problem of implementing the unitarity condition on the partial amplitudes $H(b^2, s)$ for fixed impact parameter b can be managed with relative ease. In fact, a completely algebraic procedure for doing this was given by Baker and Blankenbecler[11] using a multi-channel formalism and a further strong assumption that we shall state shortly. B&B consider multiparticle unitarity for the amplitudes $T_{a'a}(s, t)$ for a particles to yield a' particles. (The 3(a+a') - 12 variables other than s and t are suppressed. t is still the momentum transfer between a nucleon in the initial state and a nucleon in the final state.) A Fourier-Bessel representation for $T_{a'a}(s, t)$ is written

$$T_{a'a}(s,t) = \int_0^\infty bdb\, J_0(b\sqrt{-t})\, H_{a'a}(b^2, s) \ , \tag{10}$$

and unitarity is given in the high energy limit as[17]

$$\mathrm{Im}\, H_{a'a}(b^2, s) = \sum_{i=2} H_{a'i}(s + i\epsilon,\ b^2)\rho_i(s)\, H_{ia}(s - i\epsilon, b^2) + 0(1/s) \ , \tag{11}$$

with $\rho_i(s)$ the appropriate kinematic density of states factor for the intermediate state with i particles. An approximate solution to (11) is now constructed by B&B by assuming, and it is both a strong assumption and the only tractable one available, that "the multiparticle matrix element is produced only through transitions to a fully interacting two-particle state."

We will accept this assumption as a working tool. In terms of the Fourier-Bessel amplitude defined in Eq. (9) for elastic scattering,

$$H_{22}(b^2, s) = \int_0^\infty q\, dq\, J_0(bq)\, T_{22}(s, t = -q^2)\ , \tag{12}$$

and of the driving terms in Eq. (8) (we drop the Lorentz tensor structure and spinor factors as noted earlier),

$$H_c(b^2, s) \equiv \int_0^\infty q\, dq\, J_0(bq)\, \{g^2 G^2(t = -q^2)\}\ , \tag{13}$$

$$H_D(b^2, s) \equiv \int_0^\infty q\, dq\, J_0(bq)\, B(s, t = -q^2)\ , \tag{14}$$

we write the approximate solution to the unitarity relation constructed by Baker and Blankenbecler as

$$H_{22}(b^2, s) = \frac{H_c(b^2, s) + H_D(b^2, s)}{1 - I(s)[H_c(b^2, s) + H_D(b^2, s)]} \tag{15}$$

As defined, $H_c(b^2, s)$ is given directly by the Fourier-Bessel transform of the electromagnetic form factors, and $H_D(b^2, s)$, which is the reflection back on the $p+p \rightarrow p+p$ elastic transition of all the inelastic channels as well as the boson exchanges as illustrated in Fig. 3, will be given a suitable paramaterization below. The factor $I(s)$ is an integral over the two body phase space factor

$$I(s) = \frac{1}{\pi} \int_{s_0}^\infty ds' \frac{\rho_2(s')}{s' - s}\ , \tag{16}$$

and the lower limit s_0 is introduced to cut off the phase space integral at an energy $s_0 >> 4\, M_N^2$ below which this high energy approximate solution of the unitarity condition, Eq. (11), ceases to be valid. Only the two body phase space appears, and never three or more body phase space, as a result of the approximation stated above of building up the multiparticle scattering amplitude through two body intermediate states. Eq. (15) is recognized as no more than the summation of a geometric series formed by successive iterations of the driving terms in Eqs. (13) and (14), as illustrated in Fig. 3, using two body unitarity. It is a consequence of this[17] that unitarity leads to the simple algebraic form of Eq. (15) for the partial amplitude $H_{22}(b^2, s)$ at "impact parameter" b.

The form of I(s) for $s > s_o$ is

$$I(s) = \text{constant} \times [\frac{1}{\pi} \log \frac{s}{s_o} + i] \ , \tag{17}$$

with the logarithm coming from the principal part integration in (16). For $s < 25\ s_o$, the imaginary or absorptive part is the primary contribution to I(s). Taking $4M_N^2 << s_o \lesssim 20\ M_N^2$, we see that we may approximate I(s) by $I(s) = i \times$ real constant for all energies $s \lesssim 500\ M_N^2$ or laboratory energies up to $E_L \lesssim 250$ BeV.

For larger energies yet there will be logarithmically growing corrections to this approximation as the energy is increased. Such a growth would certainly be imperceptible in the p-p data as presently available; however, we return to consider these logarithms in Section IV as we discuss the approach to infinite energy behavior and the emergence of the Pomeranchuk theorem.

The elastic scattering amplitude for proton-proton scattering is now constructed by integrating

$$T(s,t) = \int_0^\infty bdb\, J_o(b\sqrt{-t}) \left[\frac{H_c(b^2,s) + H_D(b^2,s)}{1 - \frac{i}{\sqrt{4\pi}}\left[H_c(b^2,s) + H_D(b^2,s)\right]}\right], \tag{18}$$

where the real constant from Eq. (17) has been determined to be $1/\sqrt{4\pi}$ from the normalization requirement

$$\frac{d\sigma_{p-p}}{dt} = |T(s,t)|^2 . \tag{19}$$

It proves convenient to define a slightly renormalized amplitude

$$\frac{iT(s,t)}{\sqrt{4\pi}} = \int_0^\infty bdb\, J_o(b\sqrt{-t}) \left[\frac{h_c(b^2,s) + h_D(b^2,s)}{1 - h_c(b^2,s) - h_D(b^2,s)}\right] , \tag{20}$$

where

$$h_c(b^2,s) = (i/\sqrt{4\pi})\, H_c(b^2,s) ,$$

and

$$h_D(b^2,s) = (i/\sqrt{4\pi}) H_D(b^2,s) .$$

Now rewrite $T(s,t)$ in the form

$$\frac{iT(s,t)}{\sqrt{4\pi}} = \int_0^\infty bdb\, J_o(b\sqrt{-t}) \left[\frac{h_D(b^2,s)}{1 - h_D(b^2,s)} + \frac{h_c(b^2,s)}{(1-h_D(b^2,s))(1 - h_c(b^2,s)-h_D(b^2,s))}\right], \tag{21}$$

where the first term is what would survive if there were no contact interaction, that is, if the entire amplitude came from "diffraction" scattering. The second term may be viewed as our unitarized contact interaction.

For this contact term we compute Eq. (13) with the electromagnetic form factor represented by the dipole fit to the data

$$G(t) = (1 - t/\kappa^2)^{-2} \tag{22}$$

with $\kappa^2 = 0.71\ (\mathrm{BeV})^2$. The difference between (22) and the measured G(t) can only result in some inconsequential, for our arguments, numerical changes in the output, namely $\frac{d\sigma}{dt}$ for p-p scattering, and will not change the main features of our predictions as given below. This form also allows us to have a closed analytic expression for $h_c(b^2, s)$, which is essentially the Fourier-Bessel transform of $G^2(t)$,

$$h_c(b^2, s) = \frac{iA}{\sqrt{4\pi}} \frac{(b\kappa)^3 K_3(b\kappa)}{48} = \frac{iA}{\sqrt{4\pi}} \int_0^\infty q dq\, J_o(bq) G^2(t=-q^2) . \qquad (23)$$

A is a constant that characterizes the strength of the contact interaction; its relation to g^2 is $g^2 = A\sqrt{4\pi}/\kappa^2$.

We may switch to dimensionless variables now by defining $x =' \kappa b$ and $y = \sqrt{-t}/\kappa$. Eq. (23) reads

$$h_c(x) = \frac{iA}{\sqrt{4\pi}} \frac{x^3 K_3(x)}{48} , \qquad (24)$$

and the scattering amplitude is

$$\frac{iT(s,t)}{\sqrt{4\pi}} = \frac{1}{\kappa^2} \int_0^\infty x dx\, J_o(xy) \left[\frac{h_D(x,s)}{1-h_D(x,s)} + \frac{h_c(x,s)}{(1-h_D(x,s))(1-h_c(x,s)-h_D(x,s))} \right] \equiv \frac{F(s,t)}{\kappa^2} \qquad (25)$$

and

$$\frac{d\sigma}{dt} = \frac{4\pi}{\kappa^4} \mid F(s,t) \mid^2 \approx 8\pi \mid F\ (s,t) \mid^2/(\mathrm{BeV})^4 . \qquad (26)$$

It only remains to give a convenient parametrization for $h_D(x, s)$ which represents in an acceptable approximate manner our ignorance of the diffractive or inelastic channel contribution to the elastic process. Motivated by the experimental knowledge that <u>for small momentum transfers,</u> differential cross sections fall exponentially in t and are essentially independent of s for p-p scattering (again, up to logarithms), we choose to give $h_D(x, s)$ the s independent form

$$h_D(x) = \frac{(\alpha + i\beta)\, e^{-x^2/R^2}}{1 + (\alpha + i\beta)\, e^{-x^2/R^2}} \tag{27}$$

so that

$$\frac{h_D(x)}{1 - h_D(x)} = -(\alpha + i\beta)\, e^{-x^2/R^2} \quad . \tag{28}$$

Some s dependence may always be added to this if one desires. For example, Regge pole enthusiasts would recommend that R^2 be proportional to log s, so the resulting diffraction peak in $\frac{d\sigma}{dt}$ would "shrink." We will return to this possibility in Section IV in discussing the approach to the Pomeranchuk limit, and for the present proceed with the thought in mind that we are working at some <u>fixed</u> large energy s.

We may interpret the parameters in Eq. (28) by recognizing α as the imaginary contribution to T(s, t) and thus a measure of the absorption due to the inelasticity at large energies. Common sense and a bit of unitarity led us to require it to be positive. R^2 is the width of the diffraction peak, more or less, and is clearly a positive number. Typical widths of diffraction peaks led us to expect it to be on the order of 10 in units of κ^{-2}. The meaning of β, the real part of the diffractive amplitude, is less transparent. It reflects the fact that at finite energies, like those found at accelerators, our "diffractive" like processes are not purely imaginary.

The four parameters A, α, β, and R^2 will be determined in the following section by certain physical requirements on T(s, t). At that point we shall be prepared to evaluate T(s, t) for all t (at our imagined large fixed value of s) from

$$T(s,t) = \frac{\sqrt{4\pi}}{\kappa^2} \int_0^\infty x\,dx\, J_o(xy) \left\{ i(\alpha + i\beta)\, e^{-x^2/R^2} \right.$$

$$\left. + \frac{A}{\sqrt{4\pi}} \frac{x^3 K_3(x)}{48} \left[1 - (\alpha + i\beta)\, e^{-x^2/R^2}\right]^2 \left[1 - \frac{iAx^3 K_3(x)\left(1-(\alpha+i\beta)\, e^{-x^2/R^2}\right)}{48\sqrt{4\pi}}\right]^{-1} \right\} \qquad (29)$$

III. Quadratures and Computers

Our unitarized Fourier-Bessel representation for T(s, t), Eq. (29), contains four unknowns which we fix by the following physical requirements:

(1) The correct value of $d\sigma/dt$ at $t = 0$ must be reproduced; that is,

$$\left.\frac{d\sigma}{dt}\right|_{t=0} = |T(s,o)|^2 \simeq 80 \text{ mb/BeV}^2 . \qquad (30)$$

(2) The observed slope of $d\sigma/dt$ must be reproduced. This gives the "diffraction radius" and is primarily determined by our R^2. We chose, for definiteness, a slope of 10 BeV^{-2}, suggested by the present high energy data, so that

$$\frac{d\sigma}{dt} = \left.\frac{d\sigma}{dt}\right|_{t=0} e^{10t} \text{ for } t \simeq 0 . \qquad (31)$$

(3) The real part of T(s, o) must be much smaller than the imaginary part. This is in accordance with the observations of Foley _et al._[18] In particular, we take small here to mean zero, although ReT(s, o)/ImT(s, o) ~ 20% is suggested by the data at present "high" energies. Our program can certainly

accommodate that view of "small." Therefore, we demand that

$$\mathrm{ReT}(s,o)/\mathrm{ImT}(s,o) = 0 . \tag{32}$$

(4) Finally, we require that for large -t, say above 20 BeV^2, the differential cross section approaches a constant times $G_{Mp}^4(t)$, as given by the dipole fit in Eq. (22). More precisely, we demand that for large -t, and the large energies we are always considering here,

$$\frac{d\sigma}{dt} \xrightarrow[\text{large } -t]{} a^2 \, G_{Mp}^4(t) , \tag{33}$$

where a is a constant independent of s and t and on the order of one. This is the substance of our observations in Ref. 2 on the manner in which the Wu-Yang asymptotic behavior emerges.

In order to implement this last condition we need to know the asymptotic t dependence of the Fourier-Bessel transform of a given function. We find by integration by parts that

$$\begin{aligned} \int_o^\infty x dx \, J_o(xy) \, f(x) &= \left(\frac{1}{y}\frac{d}{dy}\right)^3 \int_o^\infty x dx \, J_o(xy) \left(\frac{1}{x}\frac{d}{dx}\right)^3 f(x) \\ &\rightarrow \left(\frac{1}{y}\frac{d}{dy}\right)^3 \left[\frac{x}{y^2}\frac{d}{dx}\left(\frac{1}{x}\frac{d}{dx}\right)^3 f(x)\right]_{x=0} \end{aligned} \tag{34}$$

plus surface terms and integrals which vanish as $y \rightarrow \infty$ faster than $1/y^8$, the asymptotic behavior expected from our particular choice of

$$f(x) = \frac{\sqrt{4\pi}}{\kappa^2} \left\{ i(\alpha+i\beta) \, e^{-x^2/R^2} + \frac{\frac{A}{\sqrt{4\pi}} \frac{x^3 K_3(x)}{48} \left[1 - (\alpha+i\beta) \, e^{-x^2/R^2}\right]^2}{1 - \frac{iA}{\sqrt{4\pi}} \frac{x^3 K_3(x)}{48} \left[1 - (\alpha+i\beta) \, e^{-x^2/R^2}\right]} \right\}$$

from Eq. (29). Although we have used the "dipole" approximation of Eq. (22) to the proton form factor for simplicity in constructing closed expressions, our results are not essentially dependent on this approximation. Using Eq. (34) we find that T(s, t) in Eq. (29) has the large -t behavior (recall $t = \kappa^2 y^2$):

$$T(s,t) \xrightarrow[-t \to \infty]{} \frac{\sqrt{4\pi}}{\kappa^2} \frac{48}{y^8} \frac{\frac{A}{\sqrt{4\pi}} \cdot \frac{1}{48} \left[1 - (\alpha + i\beta)\right]^2}{\left\{1 - \frac{iA}{\sqrt{4\pi}} \frac{1}{6} \left[1 - (\alpha + i\beta)\right]\right\}^2} \tag{35}$$

We note that the diffractive terms proportional to α and β have made the originally purely real contact term pick up a non-zero phase at large -t.

We are now in a position to actually construct T(s, t). Choosing a value of a^2 in Eq. (33) we imposed the four conditions given above by doing four-parameter searches on a computer with successively finer mesh. For any reasonable value of a^2 we found that we could always find a solution.[19] Some typical values of α, β, R^2, and A for given values of a^2 are to be found in Table I. The resulting amplitude was then constructed by numerical integration of Eq. (29). Because of the rapid oscillations of $J_o(xy)$, leading to almost cancelling contributions to the integral for large y (i.e., large -t), it was necessary to do the integral numerically by integrating between the zeros of the Bessel function $J_o(xy)$ and printing out the result of each such sub-integration. This also provides a convenient check that the final (very small) amplitude for large y (i.e., -t) is not of the same order of magnitude as the last significant figure carried by the computer.

In Fig. 4 we have the computed $(d\sigma/dt) / (d\sigma/dt)_{t=o}$ curves for some typical values of a^2, as well as $G_{Mp}^4(t)$. Our "best fit"[20] is shown in Fig. 5 where $X(s,t) \equiv (d\sigma/dt)/(d\sigma/dt)_{t=o}$ as computed from Eq. (29) is exhibited with $G^4_{Mp}(t)$, as given by the dipole fit, and e^{10t}. The asymptotic strength of $X(s,t)$ for large $-t$ is $0.4\,G^4_{Mp}(t)$. The parameters α, β, R^2, and A are 0.53, +.35, 12.7, and +12.92 respectively. This corresponds to a g^2 in the driving terms, Eq. (8), of

$$g^2 = (1.6 \text{ fermi})^2 . \tag{36}$$

Further light is shed on the properties of our form of $T(s,t)$ by detailed comparison of $d\sigma/dt$ for $-t \leq 2.5 \text{ BeV}^2$ with the dipole form of $G^4_{Mp}(t)$, as is seen in Fig. 6. The differential cross section falls as e^{10t} near $t=o$, as it was constrained to do, but for $-t \gtrsim 1.5 \text{ BeV}^2$ or so it has already turned over to closely approximate the fourth power of the form factor. In between, at $-t \approx 1.0 \text{ (BeV)}^2$, there is what might be called a "break" in $d\sigma/dt$ where the contact interaction emerges to take over a dominant role from the precipitiously decreasing diffractive contribution. Note also that for $-t \geq 0.5 (\text{BeV})^2$ $d\sigma/dt$ falls below $G^4_{Mp}(t)$ then crosses over near $-t = 1.0\,(\text{BeV})^2$ to rise above it, only to return below again at very large values of $-t$. This is in fact qualitatively similar to the observed behavior of $d\sigma/dt$ for high energy elastic p-p scattering at small $-t$. Attempts to fit these data with exponentials alone have suggested appearance of a break or knee in the curve at $-t \approx 2$ where the two different slopes joined. Fig. 5 fits the observed p-p differential cross section at $s \approx 60 \text{ (BeV)}^2$ to within a factor of $\approx$ 2-3 over the measured range out to $-t \simeq 15 \text{ (BeV)}^2$. This is evident by comparing with Fig. 1 and noting the close coincidence of our computed $d\sigma/dt$ to the form $G^4(t)$. The main point to be emphasized is that in the high energy region the unitarized result differs in form from $G^4(t)$ by less than a factor of two over many decades in values for the momentum transfer $-t$ and, hence, is a good representation of the data.

This is a confirmation of the basic ideas presented in I. Whether this behavior remains correct at Serphukov and higher energies is now the crucial question.

For completeness, rather than any particular implication for experiments, we give in Fig. 7 the real and imaginary parts of $T(s, t)$ as a function of t. It is amusing to observe that both Re $T(s, t)$ and Im $T(s, t)$ have zeros, but that these zeros are arranged, by unitarity, to fall where their effect on $d\sigma/dt$ is not noticeable, resulting in a smooth behavior for the differential cross section. This is in strong contrast to the results presented in Ref. 12 and 13. This difference shows the importance of protecting, at least approximately in the high energy limit, desired analyticity properties in s as we have done, in contrast to the eikonal method. Had we adopted the eikonal approach the amplitude within brackets in Eq. (20) would have been replaced by the transcription

$$\left[\frac{h_c + h_D}{1 - h_c - h_D}\right] \rightarrow \left\{e^{h_c + h_D} - 1\right\}_{\text{eikonal}} , \tag{37}$$

or equivalently the complex "scattering phase" $\delta_e(b, s) \equiv -i(h_c + h_D)$ is replaced in our procedure by

$$\delta_e \rightarrow 2 \operatorname{arc\,tan} \tfrac{1}{2} \delta_e$$

as remarked by Blankenbecler and Goldberger.[10] The unitarity cut in particular is absent from the eikonal amplitude. To illustrate the effect of this substitution we have taken for simplicity a model given in the paper of Durand and Lipes[13] and performed this substitution on it. In particular, we took their Model A in which the scattering phase $\delta(b, s)$ is given as a path integral over a purely absorptive density proportional to $i(b\kappa)^3 K_3(b\kappa)$. This is the solid line in Fig. 8 and agrees with Fig. 1 of Ref. 13. Next we made the substitution $\delta \rightarrow 2 \arctan(\delta/2)$, indicated above, and the resulting $d\sigma/dt$ is given by the dashed curve of Fig. 8. The striking zeros of the eikonal amplitude have gone away and only one little wiggle remains. With the addition of a real part to the potential and spin dependent pieces of the cross section as in Ref. 13, even this little wiggle can be washed out. Similar effects may be anticipated for the other models.

IV. Further Observations and Consequences

We have now completed the major task of this paper: the construction of an approximately unitarized high energy representation for the elastic p-p scattering amplitude starting from a basic input force given by our current-current driving term

$$T^{\text{Driving}} = g^2 G^2(t)\, \bar{u}\gamma_\mu\, u\bar{u}\gamma_\mu\, u\ , \tag{38}$$

plus diffractive contributions. Unitarity was implemented essentially through the N/D formalism of Blankenbecler, Baker, and Goldberger which writes

$$T_{NN} \approx \int_0^\infty b\,db\, J_0(b\sqrt{-t})\, [H^{\text{Driving}}/(1 - I(s)\, H^{\text{Driving}})]\quad . \tag{39}$$

The extra handle of unitarity provided us with enough leverage to be able to extend our basic ideas down to small momentum transfers--a regime we had avoided before. Some of the interesting features of this extension have now been spelled out both in Section III and in the accompanying graphs.

The value of g^2 which we have extracted from our analysis indicates that the interaction we have been discussing is a strong interaction. If we take out the dimensions of g^2 by expressing it in units of BeV^{-2}, then

$$g^2/4\pi \simeq 5.1\quad , \tag{40}$$

which is certainly strong. Although we have been discussing very high energy scattering, one might ask whether such a new strong interaction can be accommodated by present phenomenological analyses[21] of low energy nucleon-nucleon scattering in terms of forces generated by the exchange of mesons. Recalling the Born approximation arising from our interaction

$$T_{NN}(N(p_1) + N(p_2) \rightarrow N(p_1') + N(p_2')) = \tag{41}$$
$$g^2 G^2_{Mp}(t)\, \bar{u}(p_2')\, \gamma_\alpha\, U(p_2)\, \bar{u}(p_1')\, \gamma_\alpha u(p_1) + \text{"u channel" terms,}$$

we see that it has a "range" dictated by the form factors for the hadronic structure and spin properties determined by the non-relativistic limit of the tensor products of the spinors and γ-matrices. Given the dipole fit to G(t) in Eq. (22), the effective radius or range of the force in Eq. (38) is given by a mass of $\frac{4}{.7\ \mathrm{BeV}^2} \approx \left(\frac{1}{3m_\pi}\right)^2 \simeq \left(\frac{1}{420\ \mathrm{MeV}}\right)^2$. This is very close to the mass of the so-called σ-meson discussed in Ref. 21 and introduced to provide a needed force intermediate in range between the pion and rho meson, but with isospin zero. The non-relativistic limit of Eq. (38) gives a spin independent force as also desired. The strength of the coupling we find is smaller than the ones favored by these authors by about a factor of three, so that a σ meson may still be necessary to fit the low energy nucleon-nucleon scattering data, but our additional interaction is at least not in contradiction with such data in range, strength, or sign.[19]

The nuclear physics requirement that the extra force be in the I = 0 state may shed some light on the isospin properties of our interaction--a matter we have not discussed here.[5] This is most relevant when we turn to collisions of other hadrons--and in particular to π-p elastic scattering.

Since a pion can only couple to a vector current with $I = 1$ (viz. the ρ but not ω or ϕ mesons), if our current is assigned $I = 0$ only, it will not contribute directly to π-p scattering at high energies since the contact term $H_c(b^2, s)$ in Eq. (13) would be identically zero.

Further, we may address ourselves to the region of ultra high energies. This is the realm where the phase space integral, I(s), in Eq. (18), is no longer well approximated by a purely imaginary constant, but grows as $\log(s/s_o)$. Let us also add an s dependence to the diffraction term in $H(b^2, s)$ by letting α fall as $1/\log(s)$ and R^2 grow as $\log(s)$ for large s in Eq. (28). This is needed to give a shrinking forward peak, appropriate to Regge asymptotic behavior, for diffraction scattering and to lead to a constant contribution to the amplitude at $t = 0$ (corresponding to a constant total cross section in our normalization). Thus we write

$$\frac{H_D(b^2, s)}{1 - I(s)\, H_D(b^2, s)} = \lambda(s)\, e^{i\phi(s)/2}\, e^{-b^2/(R^2 \log s)} , \tag{42}$$

and imagine that as $s \to \infty$, $\lambda(s) \to 1/\log s$ and $\phi(s) \to \pi$. The unitarized elastic partial amplitude now looks like

$$\begin{aligned} H_{22}(b^2, s) = {} & \lambda(s)\, e^{i\phi(s)/2}\, e^{-b^2/(R^2 \log s)} \\ & + \frac{H_c(b^2, s)\, [1 + I(s)\, \lambda(s)\, e^{i\phi(s)/2}\, e^{-b^2/(R^2 \log s)}]^2}{1 - I(s)\, H_c(b^2, s)\, [1 + I(s)\, \lambda(s)\, e^{i\phi(s)/2}\, e^{-b^2/(R^2 \log s)}]} \end{aligned} \tag{43}$$

For large s, I(s) $\lambda(s)$ is a constant. We are interested in the asymptotic behavior of

$$T(s, t) = \int_0^\infty b\,db\, J_0(b\sqrt{-t})\, H_{22}(b^2, s)$$

for fixed $t \neq 0$ as $s \to \infty$. As constructed,the diffraction term, (the first one in Eq. (22)), vanishes in the Regge manner. The detailed nature of the contribution to $T(s, t)$ of the contact term in the ultra high energy limit depends somewhat on the exact asymptotic behavior of $G_{Mp}(t)$. For example, if $G_{Mp}(t)$ is a dipole, then the behavior of this term is $(\log \log s)^2/\log s$; if $G_{Mp}(t)$ is an exponential in $\sqrt{-t}$, then its behavior is $1/(\log s)^{1/3}$; if $G_{Mp}(t)$ is an exponential in t, then asymptotic decreases of this term is $\log(\log s)/\log s$. In each case the part of the amplitude coming from the contact term goes away quite slowly, but it does go away. Thus the elastic scattering cross section vanishes for $s \to \infty$, and we are able to recover pure diffraction scattering in the ultra high energy regime. This enables us, independent of the isotopic or unitary spin properties of our current-current interaction, to enforce Pomeranchuk theorems such as $\sigma_T(pp) \to \sigma_T(\bar{p}p)$ at infinite energies, although only in a logarithmic and not in a power law manner.[22] Whether or not it will prove to be feasible to trace such a gentle approach of the p-p total cross section to its asymptotic limit is a matter for future experimental analysis.

Finally, we remind the reader of the possibility that the local current-current interaction we have been discussing should show up in other high energy hadron collisions. We have discussed pion-nucleon scattering above and outlined some others of these in our earlier paper.[2] The observation, or lack of it, of the contact interaction in other reactions provides a strong handle for determining its isospin properties.

Before we consider recommending such difficult experiments, however, we must reiterate our earlier statement that elastic proton-proton scattering done at the energies available to Serpukhov, Weston, or the CERN Intersecting Storage Rings will provide crucial tests of the theory of p-p scattering presented here based on the assumption of a new and strong interaction between hadrons of the current-current type.

ACKNOWLEDGMENTS

We thank S. J. Brodsky for his critical and unflagging help with computer programming, and H. P. Noyes for discussions of low energy nucleon-nucleon scattering.

FOOTNOTES AND REFERENCES

1. T. T. Wu and C. N. Yang, Phys. Rev. 137, B708 (1965).

2. H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. Letters 20, 280 (1968); hereafter referred to as I.

3. G. Cocconi *et al.*, Phys. Rev. 138, B165 (1965); J. V. Allaby *et al.*, Phys. Letters 23, 389 (1966); and J. V. Allaby *et al.*, Phys. Letters 25B, 156 (1967).

4. D. H. Coward *et al.*, Phys. Rev. Letters 20, 292 (1968) and reference therein.

5. At this point the isotopic and unitary spin properties of the current in the contact interaction are not specified beyond saying that its diagonal matrix element between proton states exists. The possibility that the contact interaction in fact corresponds to the SU(3) symmetry breaking interaction, thus restricting its unitary spin properties, has been suggested by Y. Ne'eman, Phys. Rev. (to be published).

6. Specifically we mean the form factors $F_1(t)$ and $g_A(t)$ which are the coefficients of γ_α and $\gamma_\alpha\gamma_5$ respectively. If the scaling law $G_{Ep}(t) = G_{Mp}(t)/G_{Mp}(0)$ holds, as assumed by Coward *et al.*, Ref. 4, then $F_{1p}(t)$ becomes proprotional to $G_{Mp}(t)$ for large t.

7. Such an indistinguishability of right and left handed protons, or helicity independence, as $s \to \infty$, t fixed, has previously been discussed by R. Torgerson, Phys. Rev. 143, 1194 (1966), who calls this strong γ_5 invariance. The previous case of just no helicity flip terms as $s \to \infty$ is called weak γ_5 invariance in his notation.

8. W. Bartel et al., Phys. Letters 25, B236 (1967).

9. R. P. Feynman, Proceedings of the XIIIth International Conference on High Energy Physics (University of California Press, Berkeley, California, 1967), p. 98.

10. R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).

11. M. Baker and R. Blankenbecler, Phys. Rev. 128, 415 (1962).

12. T. T. Chou and C. N. Yang, in Proceedings of the Second International Conference on High Energy Physics and Nuclear Structure, Rehovoth, Israel, 1967, edited by G. Alexander (North-Holland Publishing Company, Amsterdam, The Netherlands, 1967), pp. 348-359; T. T. Chou and C. N. Yang, Phys. Rev. (in press); T. T. Chou and C. N. Yang, Phys. Rev. Letters 20, 1213 (1968).

13. L. Durand and R. Lipes, Phys. Rev. Letters 20, 637 (1968).

14. C. Chiu and J. Finkelstein, circulated CERN preprint, CERN-TH-892.

15. A rather brief, clear account of the transition $\delta \rightarrow 2 \arctan \delta/2$ is given in the book by M. L. Goldberger and K. M. Watson, Collision Theory (John Wiley and Sons, Inc., New York, 1964).

16. Here T_{NN} is the Feynman amplitude which is related to the differential cross section in the barycentric system by $\frac{d\sigma}{d\Omega} = \frac{M_N^4}{4\pi^2 s} \frac{1}{4} \sum_{\text{spins}} |T_{NN}|^2$.

At high $s >> -t$, the contribution to $d\sigma/dt$ of a vector contact term as in Eq. 8 is $\frac{d\sigma}{dt} = \frac{g^4 G^4(t)}{4\pi}$.

17. This simple form of unitarity comes when the sum over inelastic states includes all states for which s is much larger than the invariant masses of the two body "clumps" making up the states. Therefore, the phase space integral is effectively over states of "low threshold" compared to s.

18. K. Foley et al., Phys. Rev. Letters 19, 857 (1967).

19. Conditions (1), ... , (4) on α, β, A, and R^2 allow one to change $\beta \rightarrow -\beta$, $A \rightarrow -A$ and leave the scattering amplitude unchanged. The sign of A, namely the question of whether the contact force is attractive or repulsive, is thus left open at this point.

20. We did not try to achieve a best fit in any precise statistical sense; for example, by minimizing some χ^2.

21. A. Scotti and D. Y. Wong, Phys. Rev. 138, B145 (1965); J. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. 142, 1000 (1960); and D. V. Bugg, Nuc. Phys. B5, 29 (1968). In discussions with J. D. Jackson, P. Signell, and F. von Hippel, we also learned that it may be possible to account for a large part of the "σ-meson" effects by a realistic model of the I = 0 uncorrelated two pion states. It is worthwhile to remark here that the quoted g^2 was gotten by comparing $d\sigma/d\Omega$ from the vector contact term to that of the quoted authors at $s = 4M_N^2$, $t = 0$. Since the shapes of the potentials coming from a simple Klein-Gordon propagator for the σ-meson and the $G^2(t)$ form we consider are quite distinct, one might expect our "potential" to have different effects on the phase shifts. One might even be able to find the smaller strength quite acceptable.

22. Although we certainly avoid logarithmically any mathematical contradiction with Pomeranchuk-like theorems asserting the equality of the pp and $\bar{p}p$ total cross sections, we may in fact also avoid such contradictions through a power law approach of total cross sections if β changes sign appropriately together with the contact interaction in going from pp to $\bar{p}p$. Since the t-channel isotopic or unitary spin properties of β (as well as the contact interaction) are not now known to us, no definitive statement can be made.

TABLE CAPTION

Table I--Values of the parameters of α, β, R^2, and A found by computer search for a set of choices for a^2, where $\frac{d\sigma}{dt} \rightarrow a^2 G_{Mp}^4(t)$ for large $-t$. $a^2 = 0.40$ is our best fit. The sign of A corresponds to an <u>attractive</u> input contact force as suggested by the low energy discussion in Section IV.

TABLE I

a^2	α	β	R^2	A
.25	.50	+.30	13.0	+11.41
.35	.51	+.33	13.0	+12.43
.40	.53	+.35	12.7	+12.92
.50	.57	+.38	12.3	+13.85
.70	.58	+.40	12.2	+15.0

FIGURE CAPTIONS

Fig. 1--The normalized differential cross section $X(s,t) = (d\sigma/dt) / (d\sigma/dt)_{t=o}$ for p-p scattering and the fourth power of $G_{Mp}(t)/G_{Mp}(0)$ plotted against t. The experimental points are labeled by the corresponding values of s, the square of the c.m. energy, and are taken from Ref. 3. Equal s contours are shown by dotted lines.

Fig. 2--The normalized differential cross section $X(s,t)$ for p-p scattering and the fourth power of $G_{Mp}(t)/G_{Mp}(0)$ plotted against s for $-t = 10.0$, 11.1, and 15 $(\mathrm{BeV})^2$. If $X(s,t)$ were purely of the form $\beta(t)\, s^{\alpha(t)}$, the plotted points for given $-t$ would lie on the straight lines. The deviation from these lines we attribute to the emergence of the contact term.

Fig. 3--3a. Apictorial representation of the local current-current interaction.

3b. A picture of the "usual" t-channel exchanges leading to diffraction scattering.

3c. Graphical representation of inelastic channel contributions to the elastic p-p scattering.

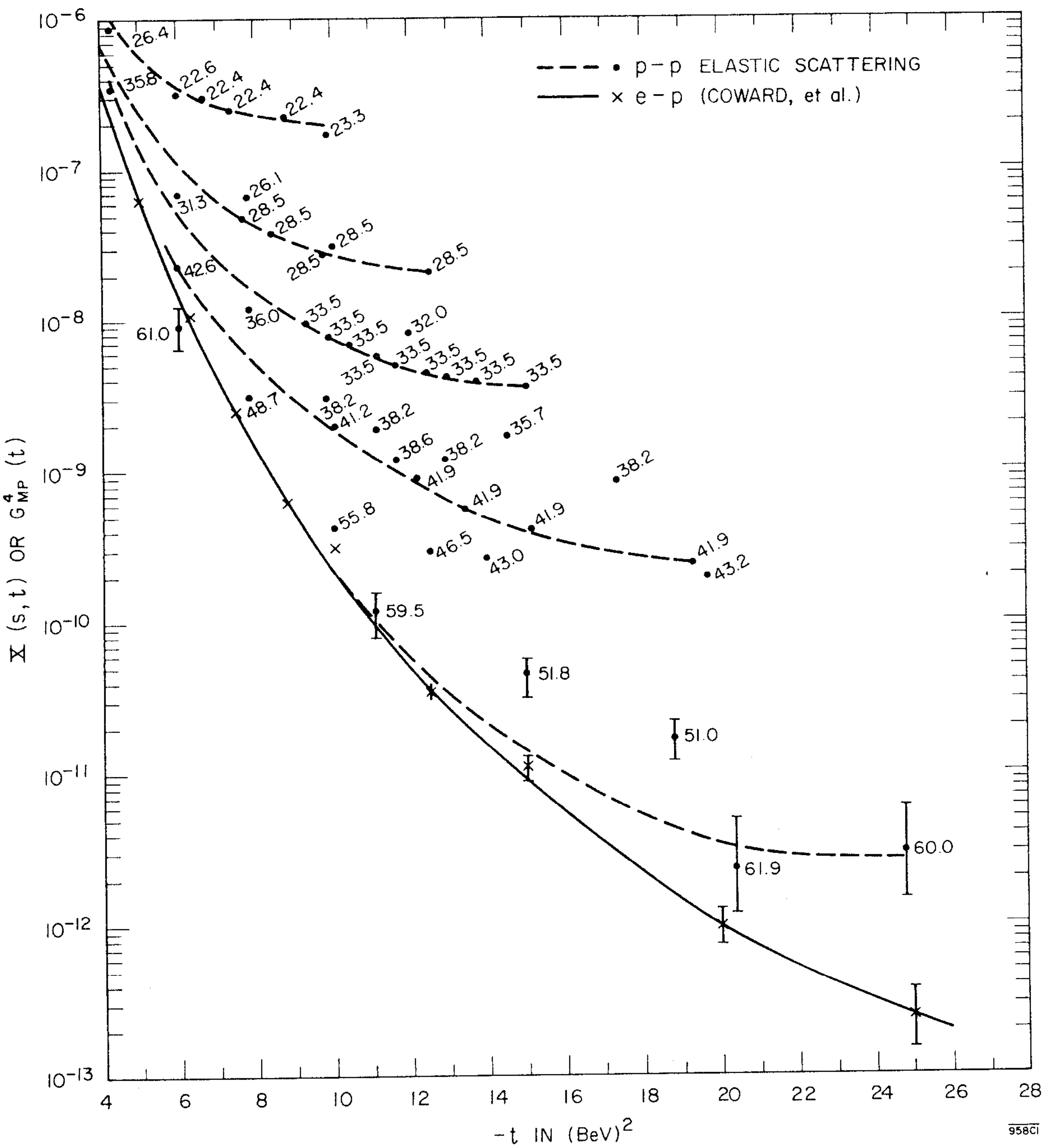
Fig. 4--The normalized differential cross section $X(s,t)$ as computed from our unitarized T-matrix for various choices of the asymptotic condition $\frac{d\sigma}{dt} \rightarrow a^2\, G^4_{Mp}(t)$. Also plotted is the fourth power of the dipole fit to the proton form factor: $G_{Mp}(t) = (1 + |t|/\kappa^2)^{-2}$.

Fig. 5--The normalized differential cross section $X(s,t)$ as computed from our unitarized T-matrix for the asymptotic condition $\frac{d\sigma}{dt} \rightarrow 0.4\, G^4_{Mp}(t)$. This is our best fit. Also plotted is the fourth power of the dipole fit to the proton form factor: $G_{Mp}(t) = (1 + |t|/\kappa^2)^{-2}$ which would have been the prediction of the non-unitarized hypothesis of I.

Fig. 6--The computed small -t behavior of $\frac{d\sigma}{dt}$ for the best fit: $\frac{d\sigma}{dt} \rightarrow 0.4\ G_{Mp}^{4}(t)$ for large -t. Note the "break" as $\frac{d\sigma}{dt}$ turns from the diffractive behavior e^{10t} to the dominance of the contact interaction above $-t \approx 1.0\ (\mathrm{BeV})^2$.

Fig. 7--The real and imaginary parts of the elastic p-p scattering amplitude.

Fig. 8--Modification of Model A of Ref. 13. The solid line reproduces a result found in Fig. 1 of Ref. 13. The dashed curve takes $\delta \rightarrow 2 \arctan (\delta/2)$.



p-p ELASTIC SCATTERING
e-p (COWARD, et al.)
X (s, t) OR G^4_{MP} (t)
-t IN (BeV)^2
958C1

Fig. 1

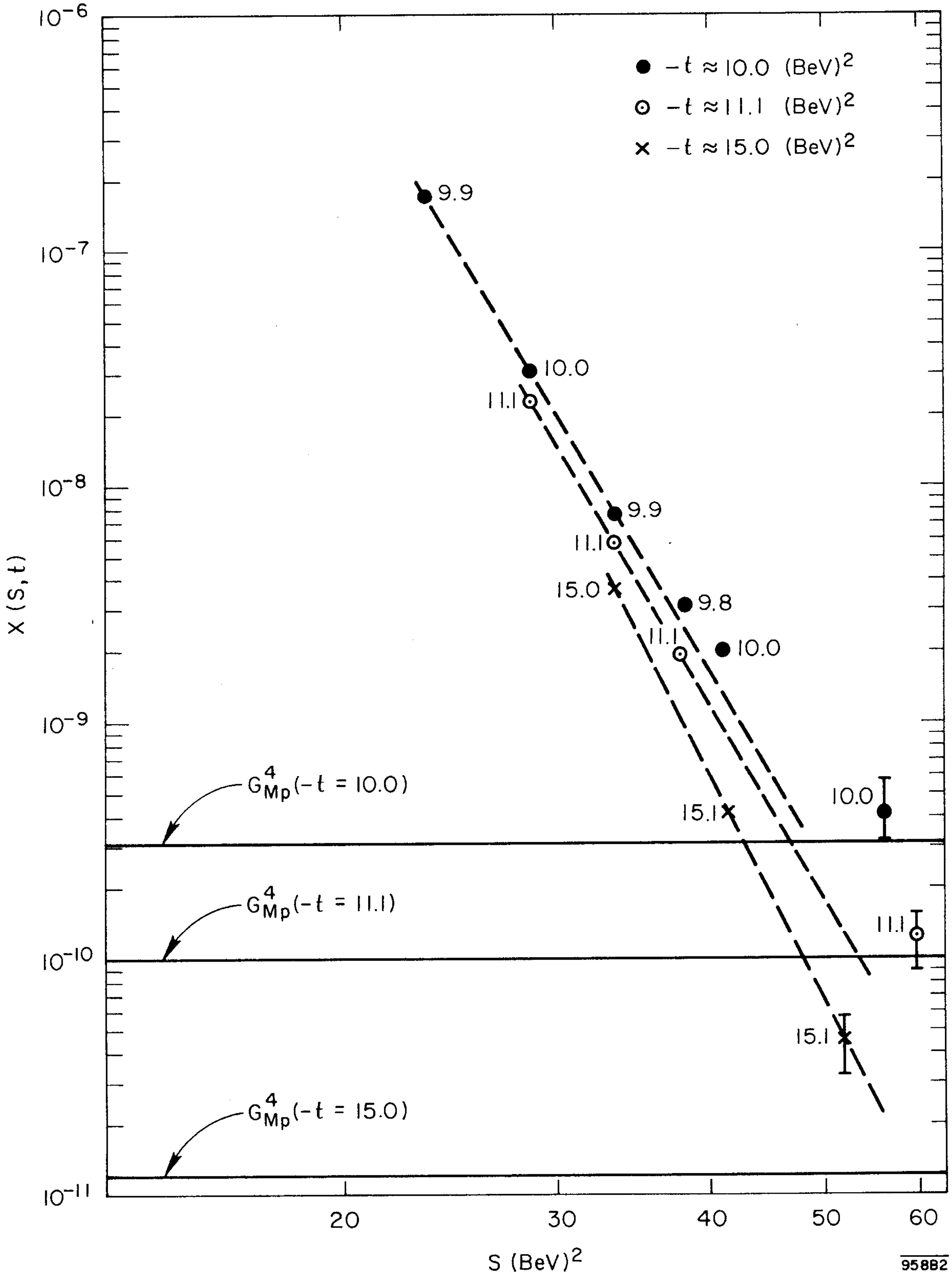

10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
X (S, t)
● $-t \approx 10.0$ (BeV)^2
⊙ $-t \approx 11.1$ (BeV)^2
× $-t \approx 15.0$ (BeV)^2
9.9
10.0
11.1
9.9
11.1
15.0
9.8
11.1
10.0
15.1
10.0
11.1
15.1
$G_{Mp}^4(-t = 10.0)$
$G_{Mp}^4(-t = 11.1)$
$G_{Mp}^4(-t = 15.0)$
20
30
40
50
60
S (BeV)^2
95882

Fig. 2

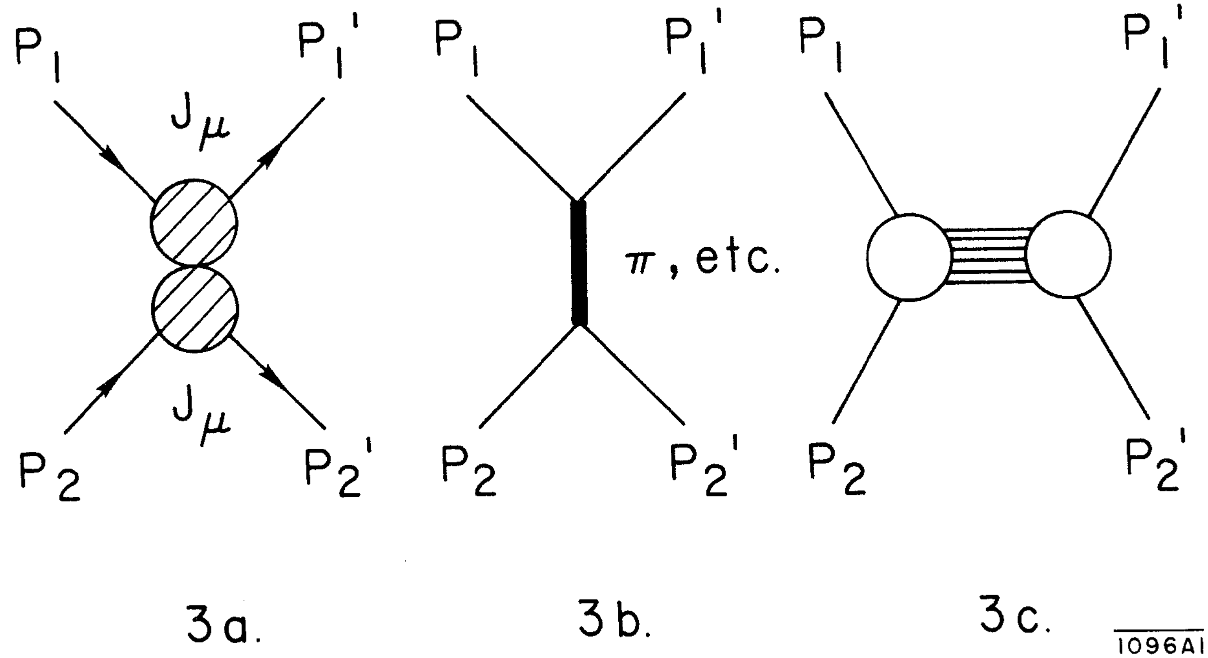
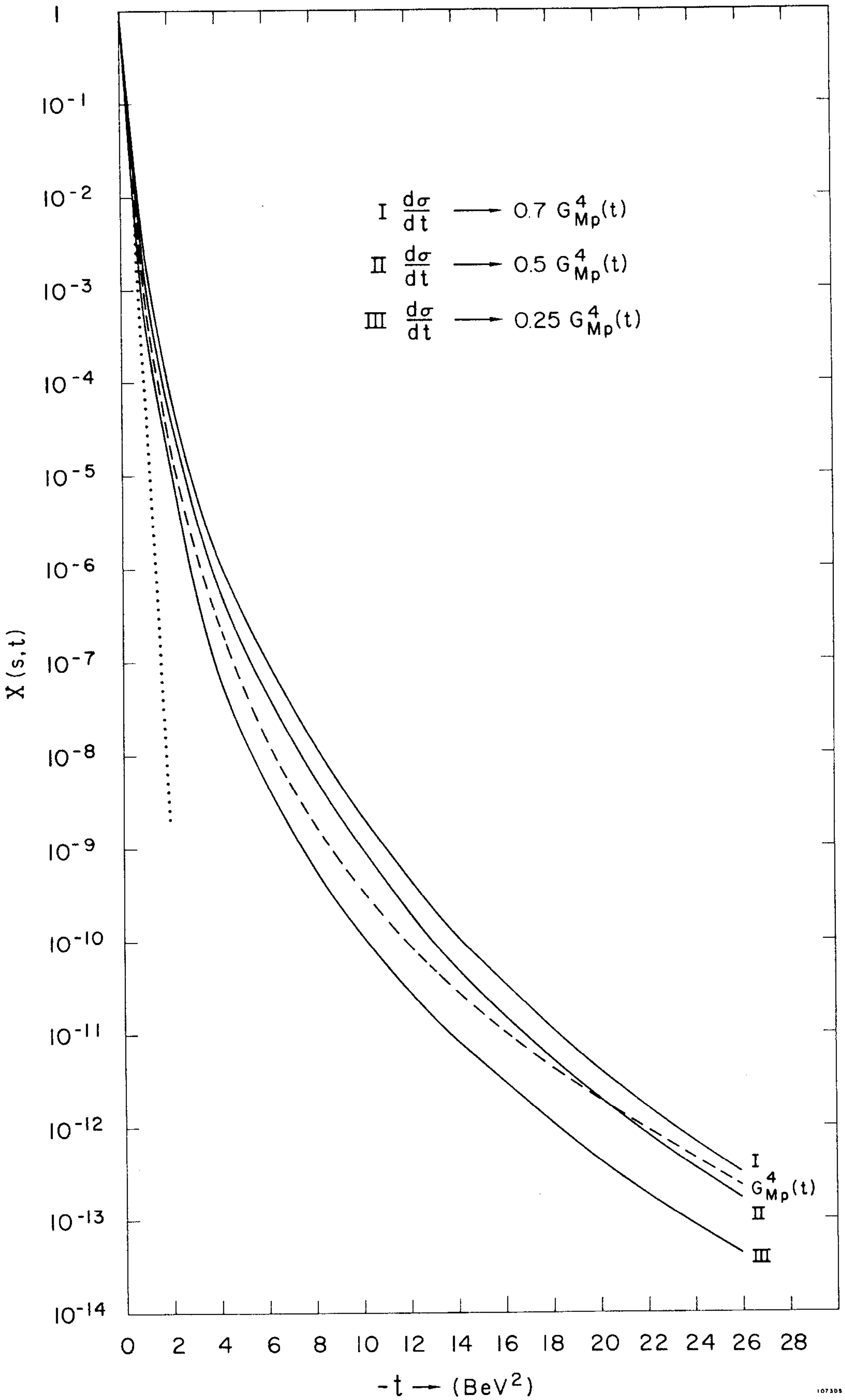
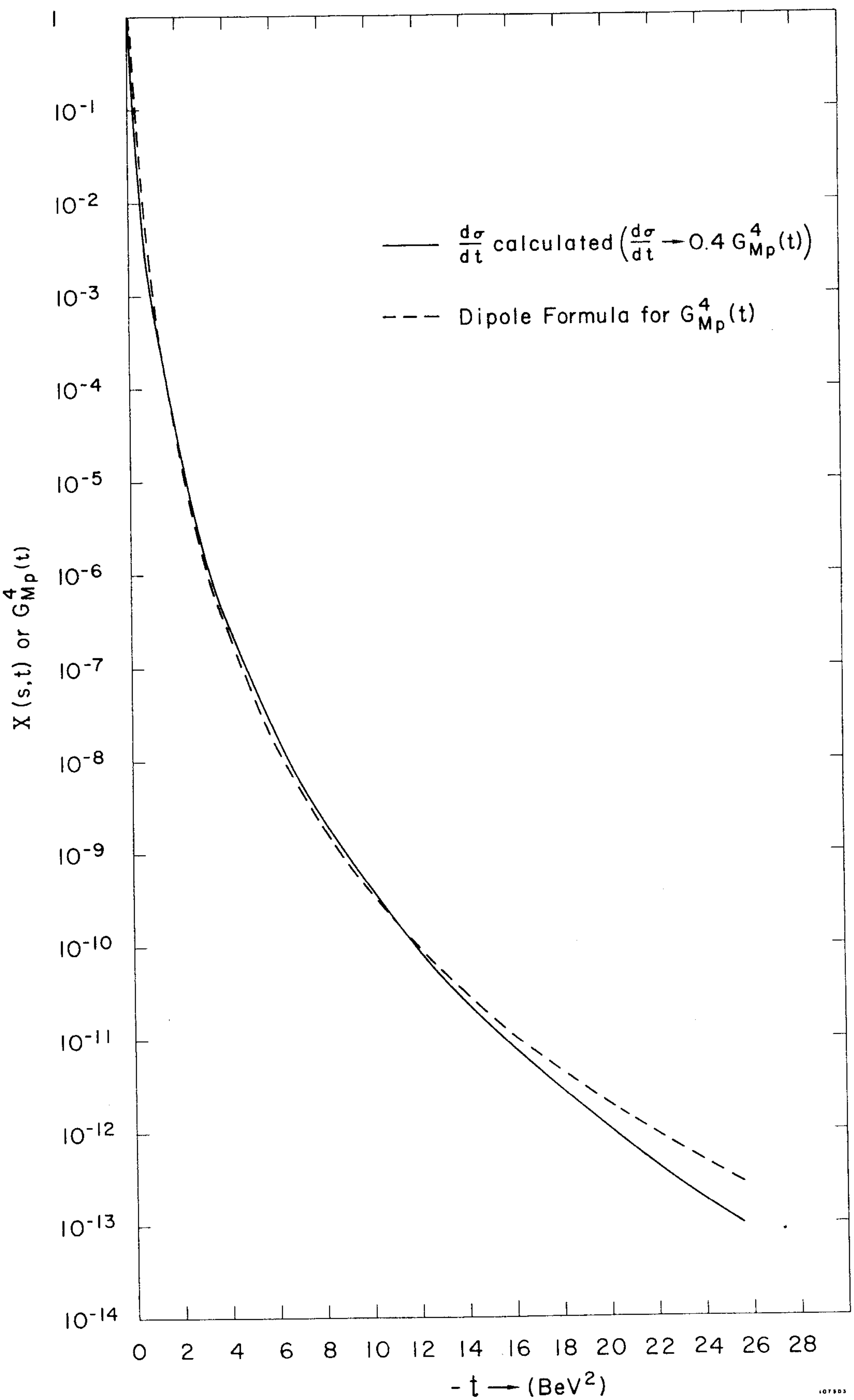


Fig. 3



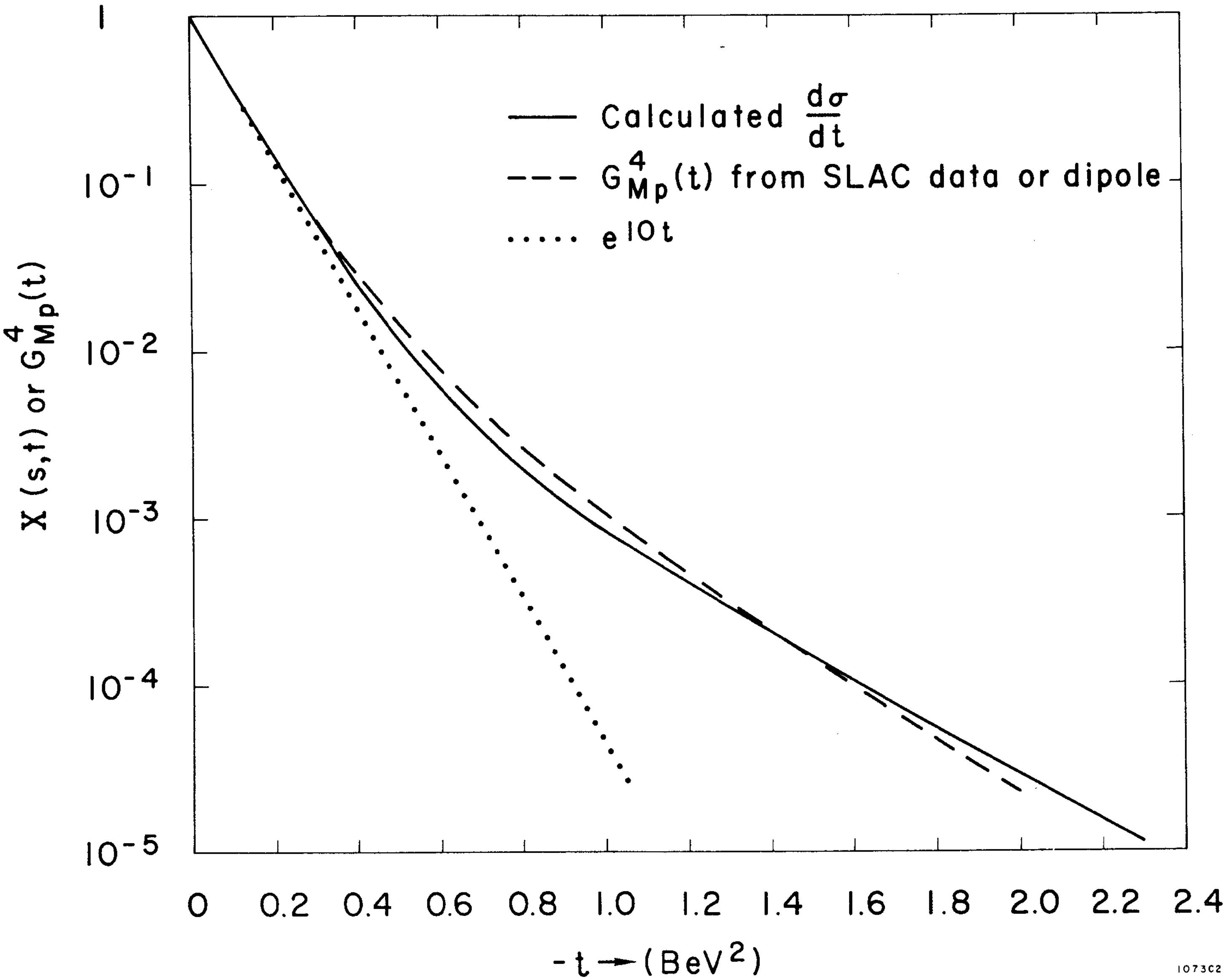
I $\frac{d\sigma}{dt} \longrightarrow 0.7\, G_{Mp}^4(t)$
II $\frac{d\sigma}{dt} \longrightarrow 0.5\, G_{Mp}^4(t)$
III $\frac{d\sigma}{dt} \longrightarrow 0.25\, G_{Mp}^4(t)$
I
$G_{Mp}^4(t)$
II
III
$X(s,t)$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
$-t \rightarrow (\mathrm{BeV}^2)$

Fig. 4



$\frac{d\sigma}{dt}$ calculated $\left(\frac{d\sigma}{dt} \to 0.4\, G^4_{Mp}(t)\right)$
Dipole Formula for $G^4_{Mp}(t)$
$X(s,t)$ or $G^4_{Mp}(t)$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
$-t \longrightarrow (\mathrm{BeV}^2)$

Fig. 5



Calculated $\frac{d\sigma}{dt}$
$G^4_{Mp}(t)$ from SLAC data or dipole
e^{10t}
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
X (s,t) or $G^4_{Mp}(t)$
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
$-t \rightarrow (\mathrm{BeV}^2)$
1073C2

Fig. 6

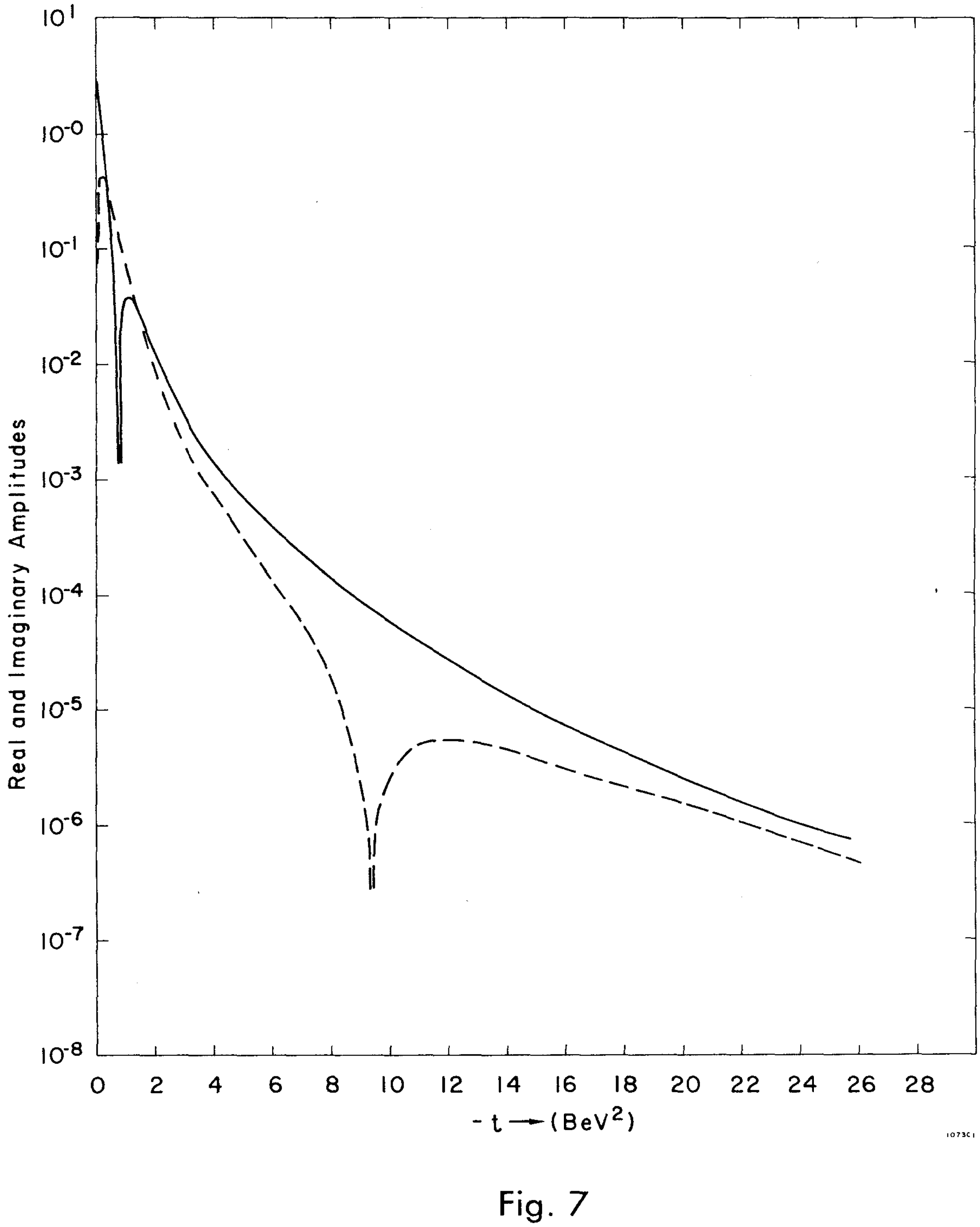
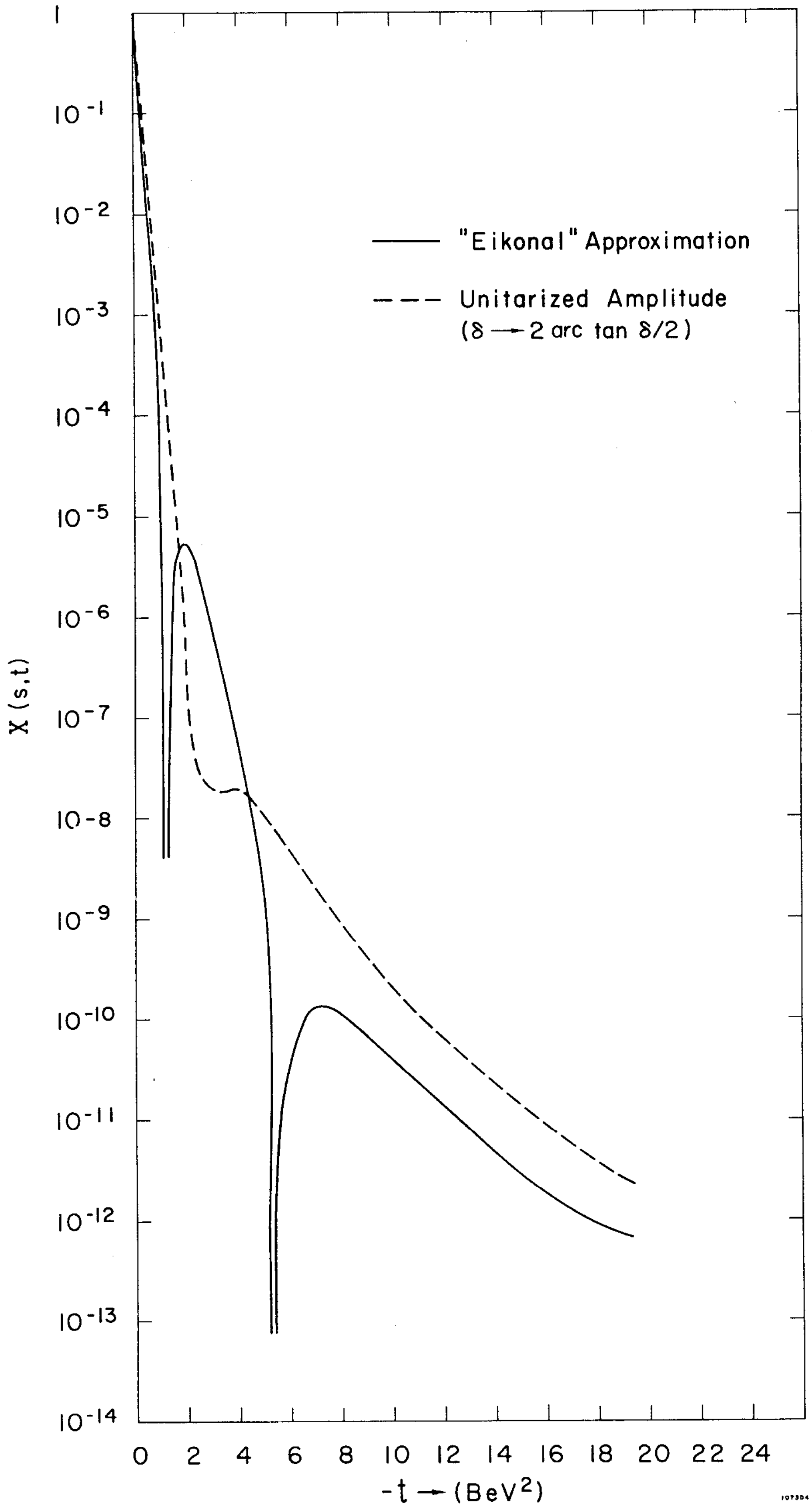

Real and Imaginary Amplitudes
10^1
10^-0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
- t → (BeV^2)
1073C1

Fig. 7



"Eikonal" Approximation
Unitarized Amplitude
(δ → 2 arc tan δ/2)
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
X (s,t)
0 2 4 6 8 10 12 14 16 18 20 22 24
-t → (BeV²)

Fig. 8