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CURRENT - CURRENT INTERACTION AND

A CALCULATION OF THE NON LEPTONIC BARYON DECAYS*

by

Shmuel Nussinov**

Palmer Physical Laboratory Princeton University

and

Giuliano Preparata*** †

Palmer Physical Laboratory Princeton University

and

Stanford Linear Accelerator Center Stanford University

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** Address as of October 1, 1968, Dept. of Physics, Tel Aviv University, Tel Aviv, Israel.

Address as of September 1, 1968, Lyman Laboratory, Harvard University, Cambridge, Massachusetts.

[†]Fulbright Scholar.

*ABSTRACT

Arguments are given why the non-leptonic weak interaction should, in a quark model with neutral vector boson strong interaction (gluon model), be calculable in terms of low-energy contributions, which can be estimated from the knowledge of semi-leptonic processes. Fair agreement with experiments seems to support this possibility. The suggestion is also made that the "gluon" model could be very helpful in understanding many properties of e.m. and weak interactions.

I. INTRODUCTION

The universal current – current hamiltonian for the weak interactions¹ has been extremely useful in explaining leptonic and semileptonic processes. An equally satisfactory understanding of the non leptonic decays in this framework, however, has not been achieved yet.

Interesting results² have, on the other hand, been obtained by introducing a few low-lying intermediate states between the currents, in the current-current hamiltonian, and using the information available from semileptonic processes. The picture that emerges from such a "saturation" scheme is, as we will review, consistent with experiments. This success is quite surprising. In fact, even if the current-current form is basically "correct," the local product of currents may be too singular to allow meaningful tests via a crude "saturation" approximation. Our experience with the calculation of electromagnetic mass-splittings may also serve as grounds for pessimism. It has been shown that one contribution of low-lying states to the Cottingham formula³ fails to reproduce even the correct sign of the $\Delta I = 1$ e.m. mass splittings. ^{4, 5} Such a failure is relevant to the present discussion, because the S-wave decays in the soft pion limit⁶ are related to the matrix elements <B' $|H_W^{P, C}| B>*$, which are very similar (except for the missing photon propagator) to <B' $|H_e, m_i|^{B>.}$

It has been recognized that additional "tadpole" terms⁷, reflecting high energy contributions, must be present and account for most of the $\Delta I = 1$ mass splittings^{4,5}; and it has been suggested⁷ that the $\Delta I = \frac{1}{2}$ rule in non-leptonic decays should emerge through a similar tadpole mechanism, thus casting severe doubts on low-energy saturation.

-2-

^{*} The Suzuki-Sugawara analysis assumes certain commutation relations between the weak hamiltonian and the axial charges, which are true both in the JJ and in the intermediate vector boson pictures.

A possible interpretation of the "tadpoles" has been suggested by Bjorken.⁸ By applying his method to the virtual "Compton-like" amplitudes, one finds in general divergent integrals, both in non-leptonic and e.m. amplitudes. We do not think that the occurrence of such divergencies is disastrous. Motivated by renormalization theory, we take the attitude that when these divergencies occur, they are going to supply us with <u>uncalculable</u> "renormalization" constants. On the other hand, if such divergencies are not present, the possibility of calculating such amplitudes in terms of low-energy contributions seems to be likely. We would like to emphasize that this is the basic attitude taken in the present investigation.

In Section II we show that there exists at least one model of the strong interactions where the "divergent" terms have operator coefficients whose matrix elements vanish between the physical states of the weak decays. Such a privileged model is the gluon-model, i.e. a quark-model where the interaction is mediated by a massive neutral vector meson coupled to the conserved baryon current, and the $SU_3 \otimes SU_3$ chiral invariance of the theory is broken only through mass terms in the Lagrangian^{*}. It is worth noticing that this is the only renormalizable model of the strong interactions which guarantees either finiteness or universality of the radiative corrections to semileptonic processes.⁹

II. BJORKEN'S METHOD AND EVALUATION OF DIVERGENT PARTS

II. 1. Intermediate vector boson weak interaction.

We write the weak Lagrangian in the form:

$$L_{W} = g J^{\mu}(x) W_{\mu}(x)$$

(1)

Some properties of this model have been considered by M. Gell-Mann, Phys. Rev. 125, 1064 (1962) and by J.D. Bjorken, ref. 8.

-3-

where J_. is the Cabibbo current

$$J_{\mu}(\mathbf{x}) = \cos\theta \left(\nabla_{\mu}^{\pi^{+}}(\mathbf{x}) + A_{\mu}^{\pi^{+}}(\mathbf{x}) \right) + \sin\theta \left(\nabla_{\mu}^{\mathbf{k}^{+}}(\mathbf{x}) + A_{\mu}^{\mathbf{k}^{+}}(\mathbf{x}) \right)$$
(2)

and $W_{\mu}(x)$ is the vector boson field whose mass m_W relates the dimensionless coupling constant g to the Fermi coupling constant, via

$$\frac{g^2}{m_W^2} = \frac{G}{\sqrt{2}}$$
(3)

This Lagrangian leads to the non leptonic amplitude

$$T (B \to B'\pi) = \frac{g^2}{2} \int \frac{d^4k}{(2\pi)^4} \left(g_{\mu\nu} - \frac{k_{\mu\nu}}{m_W^2} \right) \frac{T_{\mu\nu}}{-k^2 + m_W^2}$$
(4)

where

$$\Gamma_{\mu\nu} = -i \int d^4 x \ e^{ikx} < B'\pi \left| T^* \left(J^+_{\mu}(x) J_{\nu}(0) \right) \right| B>$$
(5)

and T^{*} denotes the covariant amplitude which represents one response of the S-matrix to the second order weak vector perturbation.*

We now apply to $T_{\mu\nu}$ the Bjorken's analysis. We analyze first the $k^{\mu}k^{\nu}T_{\mu\nu}$ part.¹⁰ By using the chiral algebra we have

$$k^{\mu}k^{\nu}T_{\mu\nu} = ik_{\nu} < B'\pi \left| \widetilde{J}_{\nu}(o) \right| B > -\int d^{3}x e^{-i\vec{k}\cdot\vec{x}} < B'\pi \left| \left[J_{0}(x), D^{+}(o) \right]_{0} \right| B > -\int d^{4}x e^{iKx} < B'\pi \left| T^{*} \left(D^{+}(x), D(o) \right) \right| B > \qquad (6)$$

where $\widetilde{J}_{\nu}(0)$ is a combination of neutral vector and axial currents, and $D(x) = \partial_{\mu} J^{\mu}(x)$. The first term integrates to zero by a symmetrical integration over k. The second term yields a quadratic divergence in (4):

$$\frac{g^2}{m_W^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{m_W^2 - k^2} \int d^3x e^{-i\vec{k}\vec{x}} \langle B'\pi | \left[J_0(\vec{x}, 0), D^+(0) \right] | B \rangle$$
(7)

T* consists in general of the time order product of the currents and additional "Schwinger" terms. Here and in the following we assume that no $\Delta S = 1$ "Schwinger" terms are present so that we can ignore them throughout our discussion.

Logarithmic divergencies in (5) may arise from the third term in (6) and from the $g_{\mu\nu}$ piece in (4), and according to the Bjorken's analysis will be given by

$$\frac{(-ig^{2})}{(2\pi)^{4}} \int \frac{d^{4}k}{m_{W}^{2}-k^{2}} \frac{1}{k^{2}} \left\{ \frac{1}{m_{W}^{2}} \int \langle B'\pi | \left[D^{+}(x), \left[H, D(o) \right] \right] | B > d^{3}x + \int d^{3}x < B'\pi | \left[J^{+}_{\mu}(x), \left[H, J^{\mu}(o) \right] \right] | B > \right\}$$
(8)

where H is the hamiltonian of the system.

We now evaluate (7) and (8) in the framework of the above mentioned "gluonmodel," which is characterized by the Lagrangian

$$\mathbf{L} = \overline{\mathbf{q}} (-\mathbf{i} \, \mathbf{\beta} + \mathbf{g} \, \mathbf{\beta} + \mathbf{M}) \mathbf{q} + \mathbf{L}_{\mathbf{B}}$$
(9)

where L_B refers to the vector boson B_{μ} part, and M is a numerical quark mass matrix. In such a model the Cabibbo current has the form:

$$J_{\mu}(\mathbf{x}) = \bar{\mathbf{q}}(\mathbf{x})\gamma_{\mu} (1 + \gamma_5)\lambda^{\dagger}\mathbf{q}(\mathbf{x})$$
(10)

where

$$\lambda^{+} = \begin{pmatrix} \circ & \cos\theta & \sin\theta \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{pmatrix}$$

and

$$D(x) = i \overline{q}(x) \mathcal{M} q(x)$$
(11)

where script letters here and in the following denote products of Gell-Mann's matrices with 1, and
$$\gamma_5$$
.

The equal time commutator in (7) is now

$$\left[J_{0}(\vec{x}, 0), D^{\dagger}(\vec{0}, 0)\right] = \delta^{3}(\vec{x}) i \vec{q} \mathcal{M}' q \qquad (12)$$

In particular the part relevant to non leptonic decays ($\Delta S = 1$) is

$$\overline{q}\mathcal{M}^{(\Delta S=1)}q = \overline{q}(\alpha\lambda_6 + \beta\lambda_7\gamma_5)q$$
(13)

where α and β are constants.

We analyze next the logarithmically divergent part (8), and consider first the matrix element

$$\int d^{3}x < B'\pi \left| \left[D^{+}(\vec{x}, o), \left[H, D(o) \right] \right] \right| B > .$$

Using the hamiltonian H corresponding to (9), and the expression (11) for D(x), we find

$$\int d^{3}x < B'\pi \left| \left[D^{+}(x), \left[H, D(o) \right] \right]_{X_{0}} = o \right| B^{>} = \int d^{3}x < B'\pi \left| \left\{ \overline{q} \mathcal{N}\gamma^{i}(-i\overline{V}_{i} + gB_{i})q + \overline{q} \mathcal{N}'q \right\} \right| B^{>}$$
(14)

The first term in (14) can be written in the form

$$\overline{\mathbf{q}} \, \mathcal{N} \gamma^{\mathbf{i}} (-\mathbf{i} \vec{\nabla}_{\mathbf{i}} + \mathbf{g}_{\mathbf{B}}) \mathbf{q} = -\overline{\mathbf{q}} \, \mathcal{N} (-\mathbf{i} \, \vec{\partial} + \mathbf{g} \, \vec{\mathcal{B}}) \mathbf{q} + \eta_{\mu} \eta_{\nu} \, \overline{\mathbf{q}} \, \mathcal{N} \, \gamma^{\mu} (-\mathbf{i} \, \vec{\partial}^{\nu} + \mathbf{g}_{\mathbf{B}}^{\nu}) \mathbf{q} \quad (15)$$
where $\eta_{\mu} \equiv (\vec{0}, 1)$.

The covariant form corresponding to (15) is obtained by the substitution⁸ $\eta_{\mu} \eta_{\nu} \stackrel{k}{\rightarrow} \frac{\mu}{k^2} \frac{\nu}{\nu}$ yielding the following contribution to (8),

$$-\frac{ig^{2}}{(2\pi)^{4}}\int_{k}\frac{d^{4}k}{k^{2}(m_{W}^{2}-k^{2})}\frac{3}{4m_{W}^{2}} < B'\pi |\bar{q}_{\mathcal{M}}''q|B>$$
(16)

A similar calculation applies to the second term in (8).

The crucial observation is that within the framework of this model the S = 1 scalar and pseudoscalar densities can be expressed as four-divergences of the corresponding current operators.

Matrix elements of these densities therefore vanish between states of equal energy and momentum (provided such operators are, as they indeed are, nonsingular). As a consequence we find that the coefficients of both the quadratic and logarithmic divergencies Eqs. (13) and 16) vanish for the physical decay process.

- 6 -

This is different from what one finds, within this same approach, for the second order e.m. mass shifts. There the coefficient of the leading logarithmic divergence⁽⁸⁾ is

$$\begin{bmatrix} \mathbf{J}_{\mu}^{\text{em}}, \begin{bmatrix} \mathbf{H}, \mathbf{J}_{\text{em}}^{\mu} \end{bmatrix}] \propto \mathbf{\bar{q}} \quad \mathbf{Q}^{2} \mathbf{q}$$
(16.1)

when Q is the 3×3 charge matrix. This density <u>cannot</u> be written as a fourdivergence, and therefore its relevant matrix elements will in general be nonvanishing. Indeed if we wish to attribute the prominent $\Delta I = 1$ mass differences to such tad-pole terms, these matrix elements should be quite large, as we will discuss later on.

II. 2. Current-current interaction.

We may obtain the current-current interaction formally from (4) by letting $m_{yy}^2 \rightarrow \infty^*$, giving

$$T (B \to B'\pi) = \frac{G}{\sqrt{2}} \int \frac{d^4k}{(2\pi)^4} T_{\mu\mu}$$
 (17)

In addition to the quadratic divergence in (17) which by the above argument may be absent there are logarithmic divergencies. If the Bjorken's analysis can be pushed this far, the coefficient of this divergence is $\begin{bmatrix} J^+_{\mu}, \frac{d^3}{dt^3} & J_{\mu} \end{bmatrix}$. Evaluation of this commutator gives in addition to quark densities (13) expressions of the form

$$g^{2}m \bar{q} \left(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}\right) \sigma_{\mu} \nu \mathcal{T}^{q}$$
⁽¹⁸⁾

which are quite different from a quark density, and their matrix elements may well be much smaller than those of the e.m. tadpole (16.1). This, together

We have however to warn that this procedure may be meaningless due to the possible bad behavior of the theory at small distances.

with the fact that the leading quadratic divergence is absent leaves open the possibility that the unknown appropriately cut-off high energy contribution to the non-leptonic amplitude is relatively small compared with the calculable low-energy contribution. This may serve as a motivation for the analysis of the low energy part of the weak amplitudes to which we now proceed.

III. LOW-ENERGY CONTRIBUTIONS

Since we are interested in the region of small virtual momenta (k² \ll experimental lower limit of m_W^2) Eq. (17) is an adequate starting point for the calculation.

In order to evaluate the contribution of the low-lying states in (17), it is most useful to write down a Cottingham-like formula³,

$$T_{low}(B \to B \pi) = \frac{G}{\sqrt{2}} - \frac{1}{32\pi^3} \int_{0}^{\infty} d(-k^2) \int_{0}^{\mu} \int_{-k^2 + \nu^2}^{\mu} \int_{-1}^{+1} dz \operatorname{Im} T(k^2, \nu, z)$$
(18)

where

Im T(k²,
$$\nu$$
, z) = $\frac{(2\pi)^4}{2} \sum_{n} \delta^4(p_n - p - k) < B'\pi |J^+_{\mu}(0)|n > < n |J_{\mu}(0)|B > (19)$

and $\nu = \frac{\mathbf{k} \cdot \mathbf{P}}{\mathbf{m}_{B}} \mathbf{z} = \frac{\mathbf{k} \cdot \mathbf{q}}{|\mathbf{k}||\mathbf{q}|}$ where \mathbf{q} is the pion momentum in the rest frame of the decaying baryon B.

 $\nu_{\rm max}$ is a cut-off energy defined by the condition

$$(k + p_B)^2 \le M^2$$
 (20)

and M² will be specified below.

In (19) there are three different types of intermediate states n:

 n_s containing no disconnected parts (Fig. 2a), n_u containing a disconnected pion (Fig. 2b), n_v containing a disconnected baryon (Fig. 2c).

- 8 -

In the following we will keep only the two lowest SU_3 multiplets in the s,u, and v channels. This is effectively achieved by choosing M of Eq. (20) to be slightly above $m_v + m_B \simeq 1800$ MeV. The final result will in fact not be particularly sensitive to the value of M^2 .

Within this approximation the typical contributions to Im T of Eq. (19) involve weak currents form factors and weak meson production amplitudes. Following earlier calculations² we take the weak currents baryon matrix elements from the fit to the Cabibbo theory and use universal dipole form factors. Lacking detailed experimental information about weak meson production amplitudes we use the soft pion limit.

In previous estimates of S-wave decays^{6,2} PCAC and the soft pion limit was used at the outset, restricting the saturation procedure to the matrix elements of the weak Hamiltonian between single baryon states. In the frame work of the present model the PCAC extrapolation may be dangerous, since in the soft pion limit the non-leptonic Hamiltonian carries effectively a momentum transfer q and the matrix elements of the quark densities (13,16) do not vanish any more. In practice, as we shall show later, the difference between this and our approach of using PCAC in the weak meson production amplitudes is relatively small.

We consider the general weak meson production amplitudes $T^{a}_{\mu}(k,q)$ (sec Fig. 3). If we assume that $T_{\mu}(k,q) - T^{B}_{\mu}(k,q)$ (where T^{B}_{μ} is the born amplitude) is a smooth function of q when $q \rightarrow 0^{11}$, we may write the amplitude

$$T^{a}_{\mu}(k,q) \simeq T^{aB}_{\mu}(k,q) + \left[T^{a}_{\mu}(k,0) - T^{aB}_{\mu}(k,0)\right]$$
 (21)

-9-

Making use of PCAC and the chiral SU₃ SU₃ algebra we have

$$T^{a}_{\mu}(\mathbf{k},0) - T^{a}_{\mu}(\mathbf{k},0)^{B} = -\frac{\sqrt{2}}{f_{\pi}} < B'(p') \left| \tilde{J}^{a}_{\mu}(0) \right| A(p) >$$

$$+ \lim_{q \to 0} \left[\frac{\sqrt{2}}{f_{\pi}} iq^{\alpha} \int d^{4} x e^{iqx} < B \left| T^{*}(A^{a}_{\alpha}(\mathbf{x}) J_{\mu}(0)) \right| B > - T^{a}_{\mu}(\mathbf{k},q)^{B} \right] \qquad (22)$$

$$(0) \text{ is the result of the equal time commutator } \left[A^{a}_{\alpha}(\mathbf{x}), J_{\alpha}(0) \right] . \quad \text{If we}$$

where $\tilde{J}^{a}_{\mu}(0)$ is the result of the equal time commutator $\begin{bmatrix} A^{a}_{0}(x), J_{\mu}(0) \end{bmatrix}$. If we write $T^{a}_{\mu}(k,q)^{B}$ using a derivative coupling the last term in (22) is identically zero when $q \rightarrow 0$. So that in the soft pion limit we have

$$T^{a}_{\mu}(k,q) = T^{a}_{\mu}(k,q)^{B} - \frac{\sqrt{2}}{f_{\pi}} < B \left| \widetilde{J}^{a}_{\mu}(0) \right| A >$$
(23)

Equipped with Eq. (23) for the weak pion-production amplitude and with the usual weak currents form factors we now evaluate the contribution to the non leptonic amplitude from $n_s = n_u = baryon$ octet and decuplet diagrams.

For the S-wave decays the dominant contribution comes from the equal-time commutator term in Eq. (23), so that we recover formally an S-wave amplitude identical with that obtained by direct application of $PCAC^3$ to the nonleptonic Hamiltonian:

$$S(B \to B'\pi^{a}) \cong \frac{\sqrt{2}}{f_{\pi}} < B' \middle| \widetilde{H}^{a} \middle| B >$$
(24)

where \widetilde{H}_W is an effective non leptonic Hamiltonian evaluated by saturating a current-current Hamiltonian by the low-lying and decuplet states. In SU₃-symmetry limit one can write:

with D, F, T referring to the D and F octet, and 27 coupling respectively. An estimate of $\langle B' | \widetilde{H}_{a} | B \rangle$ was done by Hara², who obtained*

$$D = -3,2 \ 10^{-5} \ \text{MeV}$$

F = 3,8 10⁻⁵ MeV (26)
T = -.1 10⁻⁵ MeV

Eqs. (24) and (27) yield a resonable predication of all S-wave decays. (See Table I).

In particular the $\Delta I = 1/2$ selection rule seems to emerge in a dynamical way, due to mutual cancellation of octet and decuplet contributions.

The corrections to Eqs. (24) and (26) which arise from the Born term in Eq. (23), and the so far neglected n_v diagrams, have been estimated. We find that such contributions give at most 20-30% corrections.

We turn now to consider the P-wave non leptonic amplitudes. Neglecting again the n_v -type diagrams, and the equal-time commutator term in the weak-production amplitude (23), we obtain from the Born diagrams effectively the results which have been previously obtained in Ref. 12, where we have to use for the 'spurion' matrix elements the values of Eqs. (26). As it is shown in Table I this gives a substantially correct picture of the P-wave amplitudes.**

Uncertainties in F, D, and T of Eqs. (26) arise from the insufficient experimental information on the vertices $\langle B | J_{\mu} | B \rangle$ and $\langle B | J_{\mu} | \Delta \rangle$. The forms used by Hara for these vertices are rather simple and appealing. In particular the universal dipole form factor $(m_V^2 / k^2 - m_V^2)^2$ with $m_V^2 \simeq 0.71 (BeV)^2$ was used in all cases. We found only small variations ($\simeq 15\%$) when choosing different form factors, incorporating the correct static values (including the radii).

The results for the P-wave decays are not as significant as those for the S-wave, due mainly to some subtle cancellations in the Born diagrams, which on the other hand are particularly sensitive to mass SU_3 -breaking effects.

We found that the neglected pieces (ETC and n_V -diagrams) give small corrections without altering the picture. It is however, interesting to notice that the possible effect of a P_{11} resonance in the P wave weak production amplitude will add a contribution which is qualitatively of the right structure to improve agreement with experiment. Also here the $\Delta I = 1/2$ rule is dynamically brought in through the matrix elements of the weak "Hamiltonian" between baryon states.

CONCLUSIONS

We have shown that in a particular field theoretical model, a justification can be given to the "saturation" approach to the non-leptonic interaction. A review of its implications has shown that, within the approximations made, it describes correctly the main features of the non-leptonic baryon decays.

That this situation is significantly different from what we have in the case of e.m. mass difference, we think is supported by the following argument. Let's consider the S-wave amplitudes; if the deviations between the calculated and experimental values for the S-wave decays are interpreted in terms of additional "tad-pole" contributions we find for the magnitude of the tad-poles

$$|F_W| + |D_W| \simeq 1.8 \ 10^{-5} \ MeV.$$

The analysis of e.m. $\Delta I = 1$ mass difference indicates⁵ that the low-energy contributions need be augmented by a tad-pole term with a magnitude

(F + D) e.m. = -2.08 MeVF/D = -1.8

giving

 $|\mathbf{F}| + |\mathbf{D}| = 6.3 \text{ MeV}$

- 12 -

If we adopt Bjorken's interpretation⁸ of the tad-pole contributions, we would in general expect a ratio

$$\mathbf{r}_{\text{th}} = \frac{\left(\left|\mathbf{D}\right| + \left|\mathbf{F}\right|\right)_{W}}{\left(\left|\mathbf{D}\right| + \left|\mathbf{F}\right|\right)_{e.\text{ m.}}} = \frac{G\sin\theta\cos\theta}{\sqrt{2}} \qquad \frac{\int_{k}^{\Lambda_{Wk}} dk^{2} \left[g_{VV}^{+} g_{AA}^{-}\right]}{\int_{k}^{\Lambda_{e.m.}^{2}} dk^{2} \left[g_{VV}^{+} g_{AA}^{-}\right]}$$

and $g_{VV} = g_{AA} = g_{EM}$ as a consequence of the "universality" of tad-poles. Using for the cut-offs the values $\Lambda_{wk} \simeq 10$ BeV and $\Lambda_{.e.m.} \simeq 100$ BeV, we find that r_{th} is smaller than " r_{exp} " by almost an order of magnitude, and the situation is of course much worsened if we increase the value of Λ_{wk}^2 . In spite of the crudeness of the argument we think that this is a fairly meaningful indication of the difference between the e.m. and the weak case, which seems to be incorporated in the model discussed previously.

The question now is: What have we learnt from all this? We think optimistically that from the preceding discussion may emerge the basic adequacy of the current-current picture for low-energy non leptonic interactions, and the interesting role played by the "Gluon" model in supplying us with information going beyond the realm of "current algebra". We think that investigating other features of such a model could be helpful in understanding the weak and e.m. interactions of the hadrons.

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- 13 -

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Decay	A.10 ⁶ exp.*	Calculated	B.10 ⁶ exp.*	Calculated
λ_0	0.335 ± 0.004	0.285	2.3 ± 0.1	1.48
Σ_{+}^{+}	0.001 ± 0.006	. 0	4.2 ± 0.08	2.6
Σ_0^+	0.338 ± 0.030	0.40	2.6 ± 0.4	1.85
Σ_	0.405 ± 0.003	0.57	-3.4 ± 8.5 10^{-2}	0
[]	0.440 ± 0.006	0.50	1.47 ± 0.12	1.28

We define $\mathcal{M} = \overline{u}(p')$ (A - B γ_5) u(p) as the decay amplitude. The amplitudes satisfy the $\Delta I = 1/2$ rule.

* The experimental figures have been taken from N. Cabibbo, Proceedings of the XIIIth International Conference on High Energy Physics, Berkeley (1966).