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QUANTUM ELECTRODYNAMICS: THEORY

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I. INTRODUCTION

At the last International High Energy Physics Conference in 1966 the speaker on this same topic of quantum electrodynamics concluded with these remarks: "By now I have strayed very far from the usual energy domain of $\sim 10^{+10}$ eV for a high energy conference to $\sim 10^{-9}$ eV in discussing a missing few tenths of a MHz in the Lamb shift. In fact, both domains are important in probing the detailed behavior of QED at small distances. The high-energy road of large momentum transfer experiments and the low-energy road of atomic measurements with extreme precision are two complementary routes for making progress toward the same goal. Moreover, there is no unique or theoretically compelling figure of comparison between experiments in these two domains as to which is probing QED to a smaller distance or to a higher momentum transfer."[1] I call attention to these words and to the fact that I spoke them so that you won't think that I am here addressing an Atomic Physics Conference by mistake, having wandered out of the wrong end of the SLAC 2-mile accelerator. Quite the contrary — quantum electrodynamics which is used as the model and as the only working theory of all elementary particle physics first matured 20 years ago under the stimulus of beautiful experiments in atomic physics, and it is here in precision fine structure and hyperfine structure measurements that we still find much of the action.

Quantum electrodynamics, the quantum theory of photons and electrons, has a simple conceptual basis. Shortly after the birth of quantum mechanics it was constructed very simply by applying the ordinary rules of quantum mechanics both to the electromagnetic field amplitudes, $\vec{E}(\underline{x}, t)$ and $\vec{B}(\underline{x}, t)$, whose space time development is given by the Maxwell equations, and to the electron field amplitude (or wave-function), whose space-time development is determined by the Dirac equation. Thus, as had originally happened

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to the position and momentum coordinates of a single particle, the field amplitudes also became operators whose matrix elements are observable.

We may wonder whether this prescription is only an idealization that can be adopted in the sense of a correspondence principle. It may be a sufficiently precise description when the theory is being tested by low-resolution probes that "see" the average behavior of the system over a volume of dimensions of the order of the electron Compton wave length, $\sim \hbar/m_e c \sim 3.9 \times 10^{-11}$ cm. However, if we look with a higher resolution microscope at dimensions comparable to, say, the nucleon Compton wavelength, $\hbar/Mc \sim 2 \times 10^{-14}$ cm, an elementary space-time structure or granularity may reveal itself. This is in fact what occurs for most physical systems. Sound waves or vibrating membranes, for example, are described by wave fields. However, such a wave description is an idealization valid only for distances larger than a characteristic length that measures the structure of the medium (the interatomic separation ~ 1Å or 10^{-8} cm). At smaller distances there are indeed profound modifications in these theories.

On the scale of atomic dimensions no comparable granularity is observed for the electromagnetic field. In fact, it was just the absence of any evidence for the existence of an "ether" or of any need for a mechanistic interpretation of the radiation field that led to Einstein's theory of special relativity. Now we take it for granted that both photon and electron fields satisfy differential wave equations and exhibit local interactions. We should recognize, however, the enormity of the extrapolation of this concept from atomic ($\sim 10^{-8}$ cm) to electron ($\sim 10^{-11}$ cm) and eventually to nuclear ($\sim 10^{-14}$ cm) dimensions, and we must ask whether this description may falter ever so slightly along the way.

How can we find out what is going on in this region? One way is to take the high-energy-road of experiments with very large momentum transfers q that probe distances R of the order of $R \sim \hbar/q \sim 10^{-14}$ cm for $q \sim 2$ GeV/c to an accuracy of several percent. Such high momentum transfers can best be realized in colliding beam experiments, e.g. electron-electron or electron-positron scattering. Colliding beams are necessary because otherwise very energetic incident electrons appear as massive projectiles in the relativistic sense striking light target electrons. In such a case, a multi-BeV electron beam incident on a target electron (essentially at rest in its atomic orbit) loses most of its energy by having to conserve center of mass motion, and momentum transfers only up to 100 MeV/c are realizable at present. To avoid this one can also do experiments such as wide-angle electron (or muon) pair photoproduction in which the target proton is used to anchor the center of mass. The unknown proton structure form factors can be factored out by comparison between these and elastic scattering processes. An alternate route is along the low-energy road of very high precision atomic and resonance experiments (in particular, very precise measurements of the Lamb shift and hyperfine structure) and the free electron (or muon) gyromagnetic ratio. Today I want to travel this low-energy road.

Consider first the hydrogen atom spectrum — or the energy levels of any one-electron atom or ion.[2] The exact eigensolutions and energy eigenvalues for a Dirac electron in a pure Coulomb field have been known for forty years since the 1928 works of Darwin and Gordon. Although these solutions are all-too-rare examples of exact solutions to problems of physical interest, they do not describe the hydrogen atom in nature. The situation is as shown in Figure 1. Indeed, in the real world electrons absorb and radiate light, or photons, and their interaction with the radiation field may not be altogether ignored. The effect of this interaction is to spread the electron's charge out over an effective size, or radius, somewhat larger than 1/10 of an electron Compton wave length or $\sim 5 \times 10^{-12}$ cm. When the electron approaches to within this distance of the proton, as occasionally happens for an electron bound in an S-state orbit, their interaction deviates from and becomes weaker than a pure $-e^2/r$ attraction, and thus there is an upward shift of S-state energies.

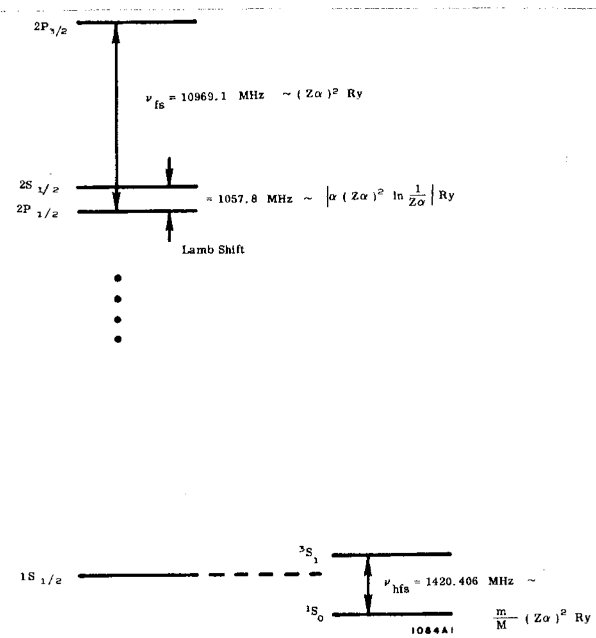

Figure 1. The Hydrogen Atom Spectrum (n = 1, 2)

There is no theoretical mystery in this shift. Once the laws of quantum theory are applied to the radiation field, we have to contend with zero point fluctuations, and hence with nonvanishing values of the mean square electric and magnetic field strengths, since $\vec{E}$ & $\vec{B}$ are not mutually commuting or simultaneously measurable observables. Under the influence of these fluctuations, present even if the atom is isolated in an ideal hohlraum at absolute zero temperature, the electron will dance about, and the mean square radius of this zitterbewegung is[3]

$$< \delta r^2 > \approx (\hbar/mc)^2 \frac{2\alpha}{\pi} \ln \frac{1}{Z\alpha} \sim (5 \times 10^{-12} \text{ cm})^2 \quad \text{for } Z = 1,$$

leading to an energy shift (see Figure 2)

$$\Delta E \propto \left[\alpha (Z\alpha)^2 \ln \frac{1}{Z\alpha} \right] \text{Ry}.$$

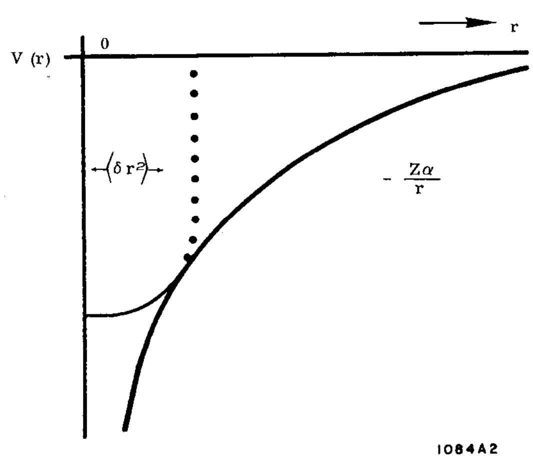

Figure 2. Radiative Correction to the Coulomb Potential

There are additional corrections to the hydrogen spectrum coming from finite proton mass and proton current contributions, from the corrections to the electron's g-value due to its quantum electrodynamic interactions as described above, and from the vacuum polarization correction to the Coulomb force law. The latter contribution arises as follows: In writing the equations of interacting electrons and protons, one introduces a parameter e_0 — the bare charge of the electron. This would be the observed electron charge e if there were no radiation field interactions — but it is not the observed or physical charge when we measure the electron, say, as it scatters in an externally applied electric field such as a pair of condensor plates provides. One actually "sees" the electron through a cloud of virtual particle-antiparticle pairs, especially electron-positron pairs, which surround the electron at all times as a result of its interaction with the radiation field. As one penetrates through this shielding cloud or surrounding dielectric medium one sees deviations from Coulomb's law. (See Figure 3.)

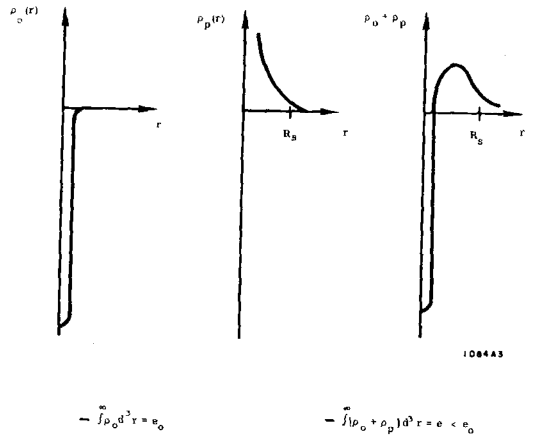

Figure 3. Electron Charge Distribution due to Vacuum Polarization

In fact, the potential is more attractive than the pure Coulomb result when we probe inside the shielding radius of $R_S \sim 10^{-11}$ cm, and this effect tends to lower the S-state energy levels in hydrogen in contrast to the other radiative effects. Again its origin is no mystery. The strength of the shielding cloud can be computed in terms of the amplitudes for the electromagnetic current to produce any physical state — such as $e^- e^+$ pairs, and its correction to the hydrogen spectrum can be computed with high precision. Whether or not the potential is not only stronger but even more singular than $1/r$ is an open question.

In spite of unanswered questions, such as those generally collected under the rubric: is QED in fact a finite theory — do the renormalization effects from bare to physical amplitudes change the masses, charges, and state amplitude by an infinite amount? — a relativistic theory of QED has been developed under the stimulus of the very beautiful precision atomic experiments starting with Lamb and Retherford, Foley and Kusch in 1947. Because of the pioneering theoretical efforts of Feynman, Schwinger, and Tomonaga, and heroic labors by many others, QED possesses a systematic unambiguous calculational scheme which has met with brilliant quantitative successes when faced with all experimental challenges.[4]

II. THE PRECISION TESTS OF QUANTUM ELECTRODYNAMICS

A historical and still critical test of QED is the Lamb shift in hydrogenic atoms since almost the entire level shift is due to radiative effects. The comparison of the theoretical predictions (as tabulated by Yennie and Erickson[5]) with experiment is given in Table I. The various contributions to the theoretical value for the $2S_{\frac{1}{2}} - 2P_{\frac{1}{2}}$ separation in hydrogen are listed in Table II. The various experimental results give cross checks on the Z and n dependence of the theoretical prediction, as well as the absolute magnitude of the level shift.

The measurements and analyses of the Lamb shift are so precise that the proton structure itself is now barely being detected. The proton has a charge radius due to its interactions with charged mesons and other hadronic entities participating in the strong, nuclear interactions. This size is some 50 times smaller than the electron's or $\sim 0.8 \times 10^{-13}$ cm, as determined by high energy large angle electron-proton scattering. To give an idea of the sensitivity in the Lamb measurement, the size of the proton changes the fine structure transition frequency by $\sim$ 0.13 MHz, or the energy interval by $\sim 10^{-9}$ eV, which is the present limit of detectability.

Since we are presently on the edge of a discrepancy between theory and experiment in H and D we may call attention to the possibility of increasing the proton size contribution, which would bring theory and experiment closer together.[6] As determined by electron proton scattering, the proton radius is the slope of the proton's form factor with momentum transfer in the measurable region $|\vec{q}| \gtrsim 100$ MeV/c, whereas as appearing in the calculation of atomic energy levels it is the slope at $|\vec{q}| \to 0$. All our understanding of hadron dynamics, all our concepts of strong interaction physics with strong, short range interactions tell us that the extrapolation from $|\vec{q}| = 0$ to $|\vec{q}| \sim 100$ MeV/c can be made smoothly, surely, and linearly with confidence. Yet, rare is the theorist who does not blink before real data, and should the discrepancy endure, perhaps the more precise studies with lepton scattering and muon x-rays will yield further clues in the proton structure at $|\vec{q}| \to 0$.

When we turn away from s states to the fine structure splitting, i.e., the $2P_{1/2} - 2P_{3/2}$ energy splitting due to spin-orbit interactions between levels with the same orbital angular momentum quantum numbers but different spin orientation, life is theoretically much simpler and cleaner. Since the p wave orbital wave function of the electron has a node at the origin, the electron and proton essentially never "see" into each others private drawing rooms of charge clouds, and the fine isoteric effects due to QED which are probed by the Lamb shift are very small. The theoretical prediction thus differs little from the Sommerfeld fine structure formula. In return for this simplicity, the precision analysis gives us α, the fine structure constant, from the equation

$$\Delta E(2P_{3/2} - 2P_{1/2}) = \frac{1}{16}\alpha^2 \mathrm{Ry}_\infty c \left(\frac{m_r}{m}\right)^3 \cdot \left[g_s \left(\frac{m}{m_r}\right) - 1 + \frac{5}{8}\alpha^2 + \frac{\alpha^3}{\pi} \log \alpha^2 \right]$$

where g_s = measured electron gyromagnetic ratio $= 2(1+a)$, $m_r^{-1} = m^{-1} + M^{-1}$.

The first precise determination of the fine structure interval was obtained by Dayhoff, Triebwasser, and Lamb[7] by combining their measurement of the $2P_{3/2} - 2S_{1/2}$ interval in D with the previous determination of the Lamb shift. The result, in terms of α^{-1} is

$$\alpha_{FS}^{-1} = 137.0388\ (12) \quad \text{(limit of error)} \quad \text{Dayhoff, et al.}$$

A direct level crossing determination of the fine structure interval in H has been reported to this conference by Baird, Metcalf, Brandenberger, and Gondaira[8] of Brown University. The result for the fine structure constant is

$$\alpha_{FS}^{-1} = 137.0353\ (8) \qquad (1\sigma) \qquad \text{Baird, et al.}$$

In addition, a new precision measurement of the $2P_{3/2} - 2S_{1/2}$ interval in H has been performed by Kaufman, Leventhal, and Lea[9] of Yale. Adding their result to either of the experimental results for the Lamb shift (see Table I) in H yields, within experimental error, the same fine structure interval, and hence the same α^{-1}, as obtained by the Brown group.

The determination of the fine structure interval is of considerable importance since α is the least well-known of the basic atomic constants. Although atomic physicists and solid state physicists often seem worlds apart — the one looking into details within one atom while the other studies cooperative effects of many atoms in regular crystals forming periodic structures with conduction bands — progress in understanding and experimenting with the superconducting state has given an important new input to the determination of α. By measuring the frequency ν of the ac current across a Josephson junction produced when two weakly coupled superconductors (separated by a thin insulating layer which permits some tunneling of the freely moving Cooper pairs of charge 2e) are maintained at a known potential difference V, one determines e/h through the (presumably exact) relationship

$$h\nu = (2e)\,V.$$

What one actually measures[10] is the current-voltage characteristic curve of Josephson junctions when they are irradiated by an applied microwave radiation of known frequency ν, observing steps in the induced voltage as a function of applied current at V and multiples thereof.

A value of α can be derived from the Josephson relation using the equation

$$Ry_\infty = \tfrac{1}{2}\,\alpha^2\, mc/\hbar \ \text{cm}^{-1}$$

or

$$\alpha^{-1} = \sqrt{\frac{mc}{2Ry_\infty \hbar}} = \left[\frac{c}{4Ry_\infty \gamma_p}\ \frac{\mu_p}{\mu_0}\ \frac{2e}{h}\right]^{\frac{1}{2}}$$

where c, the velocity of light, Ry_∞, the Rydberg constant for infinite mass, and μ_p/μ_0, the magnetic moment of the proton in units of the Bohr magneton are known to better than 1 ppm, and γ_p, the gyromagnetic ratio of the proton, is known to ± 3 ppm. The (now famous) result obtained in this manner for α^{-1} is[10]

$$\alpha_{ss}^{-1} = 137.0359 \quad (4) \quad (1\sigma)$$

This result is further strengthened by the recent work of J. Clarke.[11] He observed no voltage difference to 1 part in 10^{11} when two very dissimilar Josephson junctions were irradiated with radiation of the same frequency. It thus seems unlikely that the Josephson formula is missing material-dependent or macroscopic correction factors.

Although a resolution of the question of what is α has only a minor effect on the comparison of theory and experiment of the Lamb shift — for α we are talking about changes in 10 - 20 ppm whereas in the Lamb shift we are concerned with a potential discrepancy of 100 - 200 ppm — it is of fundamental concern when we turn to the other precision experiments. We shall just run through these briefly:

The hyperfine splitting (hfs) in atomic hydrogen, i.e., the energy shift due to interaction of the electron with the proton's magnetic moment,[12] is an important and historic link between the usually disconnected fields of high energy and precision atomic physics. This is because the hfs is quite sensitive to details of proton structure which are usually seen only in high energy electron-proton elastic and inelastic scattering experiments. The possibility that there exists a discrepancy between very accurate experimental measurements and theoretical calculations has stimulated considerable work on the subject. The way things stand at the moment is as follows.

In comparing the very precise experimental number on the triplet-singlet splitting in the hydrogen atom ground state[13]

$$\nu_{expt} = 1420.405\,751\,800 \quad (28)\ \text{MHz} \quad \text{Crampton, et al.}$$

with the theoretical formula,[14] the greatest error is introduced by the uncertainty in α and the uncertain magnitude of the proton structure corrections. If the proton is treated as a rigid structure, like a hard baseball or golfball, its interactions can be <u>completely</u> summarized in terms of its ground state charge and magnetic distribution, i.e., its form factors which are measured (in Born approximation) in the electron-proton elastic scattering experiments. On this basis we arrive at a comparison between theory and experiment that can be expressed in the form[15]

$$\begin{aligned}\frac{\nu_{rigid}}{\nu_{expt}} &= [137.0388\ \alpha]^2[1-(43\pm2)\times 10^{-6}] \\ &= [137.0359\ \alpha]^2[1-(\ 0\pm2)\times 10^{-6}]\end{aligned}$$

In assessing whether or not this is to be interpreted as potentially a fundamental challenge to QED, depending on what the true α turns out to be, one must still resolve a theoretical question, since it is clear that the rigid - "baseball" model of the proton is a gross over-simplification. In fact, the proton is a highly polarizable structure, as is evidenced by the large magnitude of the photo- and electroproduction cross sections of mesons from protons, as well as the many resonances contributing.

How large are these polarizability contributions to ν? The finite size correction of the rigid proton is responsible for a reduction in the hfs interval of some 35 ppm. In the opposite limit where the proton is highly polarizable, the orbital electron and the proton mutually adjust to each other, so that the electron orbit is distorted and recentered about the instantaneous charge position of the polarized proton structure as a result of their mutual Coulomb attraction. This Born-Oppenheimer approximation would apply if the proton excitation frequencies were small compared to the frequency of circulation of the nearby part of the electron amplitude at the proton surface. If this were the case, the electron could adjust its wave function around the instantaneous charge-current distribution so that the proton appears to be a point charge. The deuteron is a very loosely bound system and the above criterion is well-satisfied in the analysis of the deuterium hfs, as was first shown by A. Bohr.[16]

Now the proton is less polarizable than the deuteron, its excited states lying at least 140 MeV above its ground state. Nevertheless, there are important excitations, so that in part, at least, the electron can follow the instantaneous charge distributions. This will increase the calculated hfs — since the proton charge-current distributions seen by the electron will be more like a point than its rigid baseball limit. It in fact has been argued[17] that hitherto uncalculated parts of the proton structure correction involving the detailed behavior of nonresonant channels of the electropion production amplitude may very well contribute up to $\approx + 5$ to 10 ppm to the theoretical hfs. Attempts to calculate these polarizability contributions with dispersion theory run into the very same difficulties as attempts to calculate accurately the neutron-proton mass difference. It is necessary to know fine details of the strong interactions, and there is no one dominant resonant channel such as the 3-3 resonance to rely on.[18]

I view the hfs analysis as follows: With the more modern values of α, theory and experiment meet on "easy street". With the older value, 30 or 40 ppm looks serious enough, but I for one would have no convictions that the entire towering structure of QED could be brought down with an analysis based on assumptions of "reasonable" proton structure or behavior if interactions at close range with the electromagnetic currents of the hadrons are involved. Since an accurate theoretical result is impossible at this time, I am led to conclude that the hfs in H is not reliable for the determination of α to better than 10's of ppm. I view our understanding of the hadrons as facing more crises on its own home front than to permit it to go abroad to challenge QED.

We can sidestep these problems of hadron structure by turning to the muonium ($\mu^+ - e^-$) atom whose muon nucleus does not share in strong interactions. To the best of our present understanding — as well as to the disappointment of many — the muon is nothing but a heavy electron — $\sim$ 207 times heavier, but with no other discernible feature. Therefore, the pure electrodynamic system of a muonium atom, which has been studied in pioneering experiments by the preceding speaker[19], is of central importance. The hfs in the ground state — i.e., the triplet-singlet splitting has been measured to be

$$\Delta\nu = 4463.16 \pm .06 \text{ MHz} \qquad (1\sigma)$$

which differs from the H atom hfs line by approximately the ratio of muon to proton magnetic moments $\mu_\mu/\mu_p \cong (m_p/m_\mu)/2.79$. From $\Delta\nu$ a value of the fine structure constant is deduced:[19]

$$\alpha_\mu^{-1} = 137.0383 \quad (\pm\ 19 \text{ ppm limit of error})$$

The main uncertainty of the interpretation of this result is a theoretical one first discussed by Ruderman[20]. The proton to muon magnetic moment ratio must be corrected to allow for the fact that the chemical environment of a μ^+ in water (or aqueous HCl) in which the muons are stopped is different from that for the proton and so, therefore, is the diamagnetic shielding correction. In water this chemical shift reduces the applied magnetic field on a proton by 26 ppm. However, because of its lighter mass and the resulting higher zero point energy, the μ^+ forms a different type of bond. Rather than displacing a proton and entering into an H_2O bond it is more likely to "sit" in the intermolecular space with less shielding of the applied field, roughly estimated by Ruderman to be $\sim$ 10 ppm. In writing the above result for α^{-1} a difference of some 8 ppm in the chemical shift for protons and μ^+ was allowed. Progress here awaits a more precise chemical analysis to match up with the muonium hfs experimental accuracy.

There is an analogous fine structure splitting of the ground state of positronium.[21] The $^3s_1$ state lies above the $^1s_0$ level, in part due to the dipole-dipole interaction as in the hydrogen hfs, and in part due to the virtual annihilation of the $e^- e^+$ pair in a $^3s_1$ state into a single quantum. The comparison of theory and experiment is thus a test of QED — or an independent determination of α. However, the present limits of the experimental accuracy (60 ppm for 1 s.d.) and of the theoretical calculations (α^3 Ry or α corrections to the normal fine structure) are still too large to permit an independent stringent comparison.

For a final test of quantum electrodynamics we turn to precision measurements and calculations of the gyromagnetic ratio of a free electron (or muon). The calculations of second and fourth order radiative corrections[22] and an approximate evaluation of the sixth order contribution from dispersion theory[23] lead to

$$\left(\frac{g-2}{2}\right)_e = \frac{\alpha}{2\pi} - 0.328479 \left(\frac{\alpha}{\pi}\right)^2 + 0.13 \left(\frac{\alpha}{\pi}\right)^3$$

where the last tern is $< 2 \times 10^{-9}$. The experimental possibility of measuring small corrections to the normal Dirac g-value of 2 arises from the happy fact that an electron orbit in a uniform $\vec{B}$ field has a Larmor frequency that differs from the spin precession frequency only in proportion to $((g-2)/2) \cong 10^{-3}$, and the angle between momentum and spin vectors can be determined to a few ppm. Wilkinson and Crane[24] made a precise determination of this precession in 1963 by polarizing and subsequently analysing the spin direction of electrons using Mott scattering. A very recent refinement in their analysis by Rich[25] has led to the following results, which we express in relation to the theoretical formula using various values of α as deduced earlier:

$$\left(\frac{g-2}{2}\right)_{e(\text{expt})} = .001\,159\,557 \quad (30)$$

$$= \frac{\alpha}{2\pi} - 0.328479 \left(\frac{\alpha}{\pi}\right)^2 - \begin{Bmatrix} 4.5\ (2.5) \\ 6.5\ (2.5) \\ 7.0\ (2.5) \end{Bmatrix} \left(\frac{\alpha}{\pi}\right)^3 \text{ for } \alpha^{-1} = \begin{Bmatrix} 137.0388\ \ (6) \\ 137.0359\ \ (4) \\ 137.0351\ \ (8) \end{Bmatrix}$$

The agreement is not entirely soothing. A similar situation obtains in the analysis of the muon g-2 value which is being pursued with great vigor through beautiful experiments by Farley and collaborators at CERN[26]. This is really a high energy experiment using a storage ring to store muons of ~ 1.2 GeV/c momentum which are decay products of pions produced in multi-GeV proton collisions. These relativistic muons have time dilated lifetimes of some 27 μ sec. during which their spin vector precesses relative to their momentum through some 20 precession cycles. Coupled with very precise measurements of the $\vec{B}$ field along the orbit, this experiment has yielded g-2 values to accuracies of $\sim 5 \times 10^{-7}$. The difference between theory[27] and experiment presently appears as something like a 2 to 3 standard deviation effect (and of opposite sign to the possible electron discrepancy); this must still be closely followed and further evaluated before leading to any definite conclusions. At this level of precision for muons one is within an order of magnitude of the anticipated contributions due to the weak and strong interactions such as vacuum polarization due to pion pairs. What is so very unique about the muon moment is that we have before us the very exciting prospect of observing in an isolated electrodynamic system deviations from purely QED behavior due to coupling with the world of strong- and weak-interaction physics. In this aspect the muon g-2 value is more exciting than that of the electron, for which the contributions of these interactions are scaled down by the square of the electron-to-muon mass ratio.

Let me close with a final theoretical point illustrating the value of close ties joining the intellectual communities of high and low-energy physics. In this example, we on the high energy road have an offering to atomic physics concerning electromagnetic interactions of bound systems with external fields.

Very general physical principles of physics — special relativity, the principles of quantum mechanics, differential current conservation, local action and causality — have led to theorems on the threshold behavior of physical amplitudes[28] and to the construction of sum rules[29] analogous to the Thomas, Reiche, Kuhn sum rule in atomic spectroscopy by joining low energy theorems with dispersion relations. The validity of these theorems is of basic importance in elementary particle physics, and when it was discovered recently that some of them were apparently not satisfied by the hydrogen atom — considered as a composite system of definite total mass, charge, and spin — there was some consternation. To make a long story short, the trouble arose in terms of order $1/m^2$ when computing with a Foldy-Wouthuysen Hamiltonian for a system whose constituents have spin:

$$H_{fw}^{elm} = \sum_{s=a,b}\left[\frac{-\vec{p}_s\cdot e_s\vec{A}_s}{m_s} + \frac{e_s^2\vec{A}_s^2}{2m_s} + e_s\Phi_s - \mu_s\vec{\sigma}_s\cdot\vec{B}_s - \frac{1}{2m_s}\left(2\mu_s - \frac{e_s}{2m_s}\right)\vec{\sigma}_s\cdot\vec{E}_s\times(\vec{p}_s - e_s\vec{A}_s) - \frac{e_s}{8m_s^2}\,\mathrm{d}\vec{\mathrm{iv}}_s\cdot\vec{E}_s\right] + 0(1/m^3)$$

It has been assumed almost universally in the literature of atomic and nuclear physics that the interaction of a loosely bound system with an external electromagnetic field is given by such an interaction Hamiltonian that is additive in terms of the properties of the individual particles. In fact, such a Hamiltonian incorrectly describes the spin interactions of the bound system with an external electric field, and was indeed responsible for the conflict with the low energy theorems and hallowed sum rules.[30,31]

The crucial error in deriving F-W additivity is neglecting the spin transformation of the composite state wave function associated with the center of mass motion. These correction terms lead to an additional contribution to the interaction with an external field[30]

$$H_{corr}^{elm} = \frac{1}{4(m_a+m_b)}\left[\frac{\vec{\sigma}_a}{m_a} - \frac{\vec{\sigma}_b}{m_b}\right]\cdot\left[e_b\vec{E}_b\times(\vec{p}_a - e_a\vec{A}_a) - e_a\vec{E}_a\times(\vec{p}_b - e_b\vec{A}_b)\right];$$

when this is added to the F-W Hamiltonian, all sum rules and low energy theorems for Compton scattering from a hydrogen atom are restored. Their presence is important because matrix elements in an external potential which transfers momentum to the bound system require knowledge of the bound state wave function at different, nonvanishing total momenta.

With one's faith thus shaken in using additivity of a non-relativistic Hamiltonian the whole question of additivity of Dirac Hamiltonians has been re-examined critically.[32] It should be noted that this involves approximating a many-time formalism where each particle has not only its own $\vec{x}$, but also t, with a single time description.

This is important in connection with the problems I have discussed earlier. The comparison of theory with the experimental measurements of the Lamb shift and fine structure intervals requires the precise extrapolation of the experimental results to zero magnetic field (for example, from ~ 3500 gauss in the new H atom fine structure level crossing experiment[8]). Thus, care in the calculation of the Zeeman effect is as essential as it is in the calculation of the zero field levels themselves. The work of Brodsky and Primack[32] has verified that the usual reductions of the relativistic Bethe-Salpeter equation are accurate to better than 1 ppm for the Zeeman spectrum.

TABLE I

THE LAMB SHIFT IN HYDROGENIC ATOMS: COMPARISON OF THEORY AND EXPERIMENT[a]

	H	D	D - H
experiment	1057.86 ± 0.10 (R)[c] 1057.77 ± 0.10 (L)[b]	1059.24 ± 0.10 (R)[c] 1059.00 ± 0.10 (L)[b]	1.38 ± 0.15 (R) 1.23 ± 0.15 (L)
theory	1057.56 ± 0.08	1058.82 ± 0.14	1.26 ± 0.08
discrepancy	0.30 ± 0.18 (R) 0.21 ± 0.18 (L)	0.42 ± 0.24 (R) 0.18 ± 0.24 (L)	0.12 ± 0.23 (R) - 0.03 ± 0.23 (L)

	He^+(n=2)	He^+(n=3)	He^+(n=4)	Li^{++}
experiment	14040.2 ± 4.5[d]	4181.7 ± 1.0[e]	1766.0 ± 7.5[f]	63031 ± 327[g]
theory	14040.0 ± 4.0	4182.7 ± 1.2	1768.4 ± 0.5	62743 ± 43
discrepancy	0.2 ± 8.5	1.0 ± 2.2	- 2.4 ± 8.0	288 ± 370

(a) The experimental errors represent 3σ or limits of error. The theoretical range represents estimated upper limits of uncalculated higher order terms. The theoretical results are based on expressions given in Ref. (5) as evaluated by G. W. Erickson (unpublished). We wish to thank Professor Erickson for providing us with this compilation.

(b) E. S. Dayhoff, S. Triebwasser, and W. E. Lamb, Jr., Phys. Rev. 89, 106 (1953).

(c) R. T. Robiscoe, Phys. Rev. 138, 4 (1968), and unpublished.

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TABLE II

VARIOUS CONTRIBUTIONS TO THE LAMB SHIFT IN H†

DESCRIPTION	ORDER	MAGNITUDE (MHz)
4^{th} ORDER — SELF-ENERGY	$\alpha(Z\alpha)^4 m\{\log Z\alpha, 1\}$	1079.32 ± .01
	$\alpha(Z\alpha)^5 m$	7.14
	$\alpha(Z\alpha)^6 m\{\log^2 Z\alpha, \log Z\alpha, 1\}$	- .38 ± .04
4^{th} ORDER — VAC. POL.	$\alpha(Z\alpha)^4 m$	- 27.13
6^{th} ORDER — SELF-ENERGY	$\alpha^2(Z\alpha)^4 m$	0.00
	$\alpha^2(Z\alpha)^5 m$	± .02
6^{th} ORDER — VAC. POL.	$\alpha^2(Z\alpha)^4 m$	- 0.24
REDUCED MASS CORRECTIONS	$\alpha(Z\alpha)^4 \frac{m}{M} m\{\log Z\alpha, 1\}$	- 1.64
RECOIL	$(Z\alpha)^5 \frac{m}{M} m\{\log Z\alpha, 1\}$	0.36 ± .01
PROTON SIZE	$(Z\alpha)^4 (mR_N)^2 m$	0.13
		1057.56 ± .08

† Based on Table II of Ref. (5), including the revisions of the $\alpha^2(Z\alpha)^4 m$ contribution given by Soto[5]. We take $\alpha^{-1} = 137.0359$.

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