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TIME REVERSAL INVARIANCE AND THE DECAY $\pi^{\pm} \longrightarrow e^{\pm}\bar{\nu}_e e^+ e^-$ *

by

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ABSTRACT

A model of radiative pion decay which incorporates a violation of time reversal invariance is studied. An asymmetry of up to 12% in the angular distribution of particles in the final state is predicted.

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TIME REVERSAL INVARIANCE AND THE DECAY $\pi^{\pm} \to e^{\pm}\bar{\nu}_e e^+ e^-$

Radiative kaon decay, with subsequent conversion of the gamma ray, has attracted interest recently as a tool for the study of time reversal invariance.[1] It is the purpose of this paper to study the analogous decay of the pion in the context of a particular model that demonstrates a violation of the symmetry. If, in the model proposed, time reversal invariance is violated in a "maximal" way, there is an asymmetry between the charged particles in the final state of up to 12%.

The apparent violation of time reversal (or CP) invariance measured in K^o decay is of such a magnitude as to suggest that it arises from a collusion of the weak and electromagnetic interactions.[2] The scheme presented here is concerned with a situation wherein the weak currents are coupled similarly to the electric and magnetic fields, $\underset{\sim}{E}$ and $\underset{\sim}{B}$. Such a situation is described by the effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{eGa}{\sqrt{2}\, M^2}\, j_\mu^\pi\, j_\nu^{lept.} \left(F^{\mu\nu} + \frac{1}{2}\xi\, \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \right) .$$

j_μ^π is proportional to the pion four-momentum, $j_\nu^{lept.}$ is the usual V - A lepton current, $F^{\mu\nu}$ is the electromagnetic field tensor, $e^2 = 4\pi/137$, G is the pion β-decay coupling constant, a is defined by the rate for electronic pion decay,[3]

$$\Gamma(\pi \to e\bar{\nu}_e) = \frac{G^2|a|^2}{8\pi}\, m_\pi\, m_e^2 \left[1 - \left(\frac{m_e}{m_\pi} \right)^2 \right]^2 ,$$

where m_π and m_e are pion and electron masses, respectively, M is a parameter with the units of mass that make the dimensions come out right, and ξ is a parameter that measures the amount of $\underset{\sim}{B}$ relative to $\underset{\sim}{E}$ in the coupling. "Maximal" violation of the time reversal invariance occurs when ξ is real. The inner bremsstrahlung Hamiltonian must be added to $\mathcal{H}_{eff}$ for general consideration of the radiative decay.

The contribution of $\mathcal{H}_{eff}$ to the decay rate, $\Gamma(\pi \to e\bar{\nu}_e e^+e^-)$, is given by the amplitude,

$$A = \frac{e^2 Ga}{\sqrt{2}M^2} \bar{u}(p_e)\left[p_i^\mu \not{k} - k \cdot p_i \gamma^\mu + \xi\ \epsilon^{\mu\lambda\rho\sigma} p_{i\lambda}\gamma_\rho k_\sigma\right]$$

$$(1-\gamma_5)\, v(p_\nu)\, \frac{1}{k^2}\, \bar{u}(p_-)\gamma_\mu v(p_+)\ ,$$

where p_i, p_e, p_ν, k, p_+, p_- are the four-momenta of the pion, electron, neutrino, internal photon, positron and electron, respectively. The electromagnetic field tensor is calculated from the conversion pair current, and final state exchange terms neglected. Terms in the amplitude proportional to the real part, ξ_R, of the parameter ξ violate time reversal invariance (M is taken to be real) and result in an asymmetry between the particles in the final state that is in principle subject to experimental detection. In the pion rest frame, with the plane defined by the three momenta of the conversion pair taken as reference, the decay rate to electrons, of the electron-neutrino pair, that emerge above this plane, less the rate of decay to electrons that emerge below, D, is calculated. The geometry is illustrated in Fig. 1.

The total decay rate resulting from the amplitude, A, can be expressed as

$$\Gamma = \frac{e^4 G^2 |a|^2}{2^{11}\pi^8 M^4 m_\pi^2} \int \frac{dx^2}{x^2}\, dy^2\, du\, dv\, d\phi_e\, |\underset{\sim}{k}|$$

$$\left[\frac{4m_\pi^2 x^2}{\left(m_\pi^2 + x^2 - y^2\right)^2}\left(1-u^2\right)\mathcal{M}_3^2 + u^2\mathcal{M}_1^2 + \mathcal{M}_2^2 - 4\,\frac{m_\pi x u (1-u^2)^{1/2}}{m_\pi^2 + x^2 - y^2}\,\mathrm{Re}\,\mathcal{M}_1\mathcal{M}_3^*\right],$$

where

$$\mathcal{M}_1^2 = m_\pi^2 y^2\Big[k_0^2\left(\sin^2\phi_e + v^2\cos^2\phi_e\right) + k^2|\xi|^2\left(\cos^2\phi_e + v^2\sin^2\phi_e\right)$$

$$- 2kk_0\,\xi_R\,(1-v^2)\cos\phi_e\sin\phi_e - 2vkk_0\,\xi_I\Big],$$

$$\mathcal{M}_2^2 = m_\pi^2 y^2 \Big[k_0^2 \left(\cos^2 \phi_e + v^2 \sin^2 \phi_e\right) + k^2 |\xi|^2 \left(\sin^2 \phi_e + v^2 \cos^2 \phi_e\right)$$

$$+ 2kk_0 \xi_R (1 - v^2) \cos \phi_e \sin \phi_e - 2vkk_0 \xi_i \Big],$$

$$\mathcal{M}_3^2 = m_\pi^2 k_0^2 t_0^2 (1 - v^2)$$

$$\mathrm{Re}\, \mathcal{M}_1 \mathcal{M}_3^* = - m_\pi^2 \left[2k_0^2 \eta s t \cos \phi_e + 2kk_0 \xi_R \eta s t \sin \phi_e - 2kk_0 \xi_I s t_0 \cos \phi_e \right],$$

after the manner of Ref. 5. In the above, $k_\mu = (k_0, \underset{\sim}{k})$, $k = |\underset{\sim}{k}|$, $x^2 = k_\mu k^\mu$, $y^2 = (p_e + p_\nu)^2$, $u = \lambda |k|/k_0$, $v = \eta |\underset{\sim}{t}|/(m_\pi - k_0)$. The variables u and v measure the energy partition in the conversion and electron-neutrino pairs, respectively. In the following, electrons are assumed massless whenever this does not lead to the vanishing of the quantity of interest. Otherwise, only the lowest order term in the electron mass is kept. Then, from the above, the decay rate difference is found to be

$$D = \frac{-e^4 G^2 |a|^2 \xi_R}{2^{10} \pi^6 M^4 m_\pi^3} \int dx\, y\, dy^2\, du\, dv \;\; u(1 - u^2)^{1/2}\, v(1 - v^2)^{1/2}$$

$$\left[\left(m_\pi^2 + x^2 - y^2\right)^2 - 4 m_\pi^2 x^2 \right] \left(m_\pi^2 - x^2 + y^2\right) .$$

When the integrations are done over all energy partitions, in either pair, $D \equiv 0$. However, for events such that $E_+ > E_-$; $E_e > E_\nu$, one obtains

$$\frac{D}{\Gamma} = -0.075 \frac{\xi_R}{1 + |\xi|^2} \equiv \mathscr{A}$$

where $\Gamma(\pi \to e \bar{\nu}_e e^+ e^-)$ is the total decay rate pursuant to $\mathcal{H}_{\mathrm{eff}}$. Depommier et al. (Ref. 4) quote

$$|\xi| = 2.5 \text{ or } 0.476 \text{ for } \tau_{\pi^0} = (1.05 \pm 0.18) \cdot 10^{-16} \text{ sec.},$$

so that, for $\xi = \xi_R$, $\mathscr{A} \simeq -0.026$ or -0.029. Since the sign of D depends on the choice of energy partitions, a total asymmetry of up to 12% may be detected.

The total decay rate $\Gamma(\pi \rightarrow e\bar{\nu}_e e^+ e^-)$ may be calculated in straightforward fashion from the amplitude previously given. If all terms beyond the lowest order in electron mass are neglected, and the assumption of pairs with small opening angle (on mass, shell photons) is made, the result is

$$\Gamma(\pi \rightarrow e\bar{\nu}_e e^+ e^-) = (1.1 \times 10^{-5}) \frac{e^4 G^2 |a|^2 m_\pi^7}{2\pi^2 M^4} \left(1 + |\xi|^2\right)\left(\log \frac{m_\pi^2}{m_e^2} - \frac{5}{3}\right)$$

which leads to a branching ratio

$$\frac{\Gamma(\pi \rightarrow e\bar{\nu}_e e^+ e^-)}{\Gamma(\pi \rightarrow \mu\nu)} = (.5 \times 10^{-6}) \left(\frac{m_\pi}{M}\right)^4 \left(1 + |\xi|^2\right) \equiv R .$$

The process $\pi \rightarrow e\bar{\nu}_e e^+ e^-$ is related by CVC to the decay of the neutral pion,[7] and we can use the measured lifetime to determine $(m_\pi/M) \cong 0.1$ or 0.224, in the order corresponding to the values given for $|\xi|$ above. This leads to $R \simeq 3.6 \times 10^{-10}$ or 15×10^{-10}. We then need at least $10^{13} - 10^{14}$ pion decays to detect the asymmetry with reasonable statistics.

Neglect of the inner bremsstrahlung (and interference) terms in the above may be justified by inspection of the differential decay rates. If the inner bremsstrahlung amplitude

$$B = \frac{e^2 G a m_e}{\sqrt{2}} \bar{u}(p_e) \left[\frac{2p_e^\mu + \gamma^\mu \not{k}}{k^2 + 2k \cdot p_e} + \frac{2p_i^\mu}{k^2 - 2k \cdot p_i}\right] (1 - \gamma_5) v(p_\nu) \frac{1}{k^2} \bar{u}(p_-) \gamma_\mu v(p_+)$$

is included, the total differential rate, pursuant to the amplitude A + B, is the sum of three parts:

$$\left.\frac{d^2\Gamma}{dk_0 d\Omega_k}\right|_A = \frac{e^4 G^2 |a|^2 m_\pi}{12(2\pi)^6 M^4}\left(\log\frac{m_\pi^2}{m_e^2} - \frac{5}{3}\right)\frac{k_0^3\left(m_\pi^2 - 2m_\pi k_0\right)^2}{\left[m_\pi - k_0\left(1-\cos\theta_e\right)\right]^3}$$

$$\left\{\left[m_\pi - k_0\left(1-\cos\theta_e\right) - \frac{\left(m_\pi^2 - 2m_\pi k_0\right)\sin^2\theta_e}{2\left[m_\pi - k_0\left(1-\cos\theta_e\right)\right]}\right]\left(1+|\xi|^2\right) - \xi_I\left[k_0 + \left(m_\pi - k_0\right)\cos\theta_e\right]\right\}$$

$$\left.\frac{d^2\Gamma}{dk_0 d\Omega_k}\right|_B = \frac{e^4 G^2 |a|^2 m_e^2 m_\pi}{12(2\pi)^6}\left(\log\frac{m_\pi^2}{m_e^2} - \frac{5}{3}\right)\frac{\left(\frac{1}{2}m_\pi^2 - m_\pi k_0 + k_0^2\right)\left(1+\cos\theta_e\right)}{k_0\left[m_\pi - k_0\left(1-\cos\theta_e\right)\right]^2\left(1-\cos\theta_e\right)}$$

$$\left.\frac{d^2\Gamma}{dk_0 d\Omega_k}\right|_{AB} = \frac{e^4 G^2 |a|^2 m_e^2 m_\pi^2}{6(2\pi)^6 M^2}\left(\log\frac{m_\pi^2}{m_e^2} - \frac{5}{3}\right)\frac{k_0\left(1+\cos\theta_e\right)}{\left(1-\cos\theta_e\right)\left[m_\pi - k_0\left(1-\cos\theta_e\right)\right]^2}$$

$$\left[\frac{\left(m_\pi - 2k_0\right)^2\left(1-\cos\theta_e\right)}{2\left[m_\pi - k_0\left(1-\cos\theta_e\right)\right]} + k_0\left(1-\xi_I\right)\right]$$

which refer to $\mathcal{H}_{eff}$, inner bremsstrahlung, and interference contributions, respectively.[8] The angle θ_e is measured between $\underset{\sim}{p}_e$, the electron three-momentum, and $\underset{\sim}{k}$, the three-momentum of the virtual photon. The first two parts are plotted below. It is clear that we wish to look at events for which $\theta \approx \pi$.

Finally, some attention must be given to competing processes that result in three electrons in the final state. In particular, $\pi \to \pi^0 e\bar{\nu} \to e^+e^- \gamma e\bar{\nu}$, and $\pi \to \mu\nu \to e\nu\bar{\nu}\gamma\nu \to e\nu\bar{\nu}\, e^+e^-\nu$. In the first process, the small Q value of the neutral pion mode ensures that such events will be characterized by a final state in which one electron has low energy and the remaining two share an energy of roughly half a pion mass. Moreover, we expect a branching ratioo to this mode of $\sim 10^{-10}$, so such events should be excludable. The second process is potentially more troublesome. We expect $\Gamma(\mu \rightarrow e\nu\bar{\nu}\gamma)/\Gamma(\mu \rightarrow e\nu\bar{\nu}) \sim 10^{-5}$, [6] so that the branching ratio to pair final states is $\sim 10^{-7}$. Such events may be avoidable by concentrating on the region $\theta \approx \pi$, because of the usual bremsstrahlung forward peaking.

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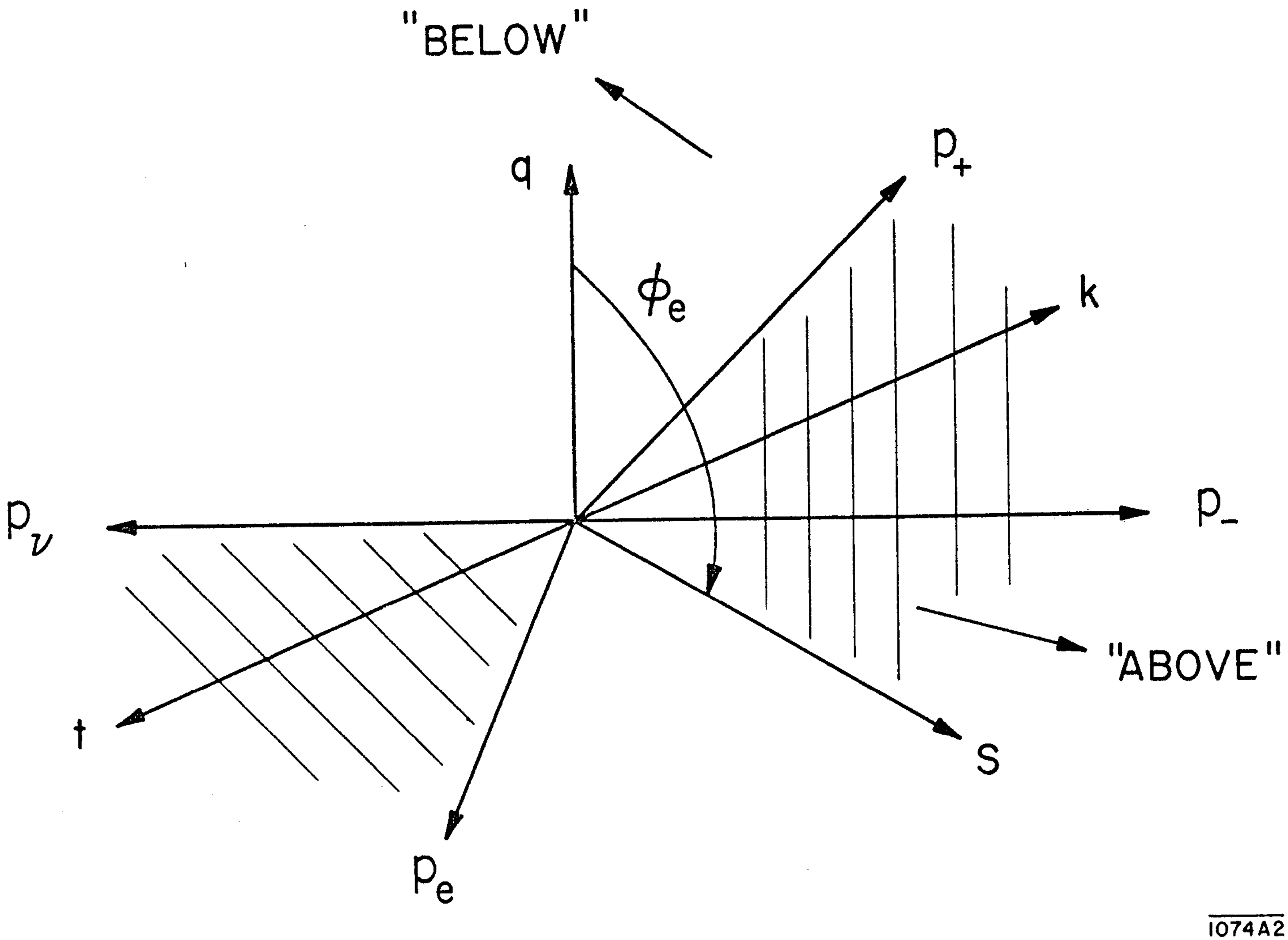

Fig. 1

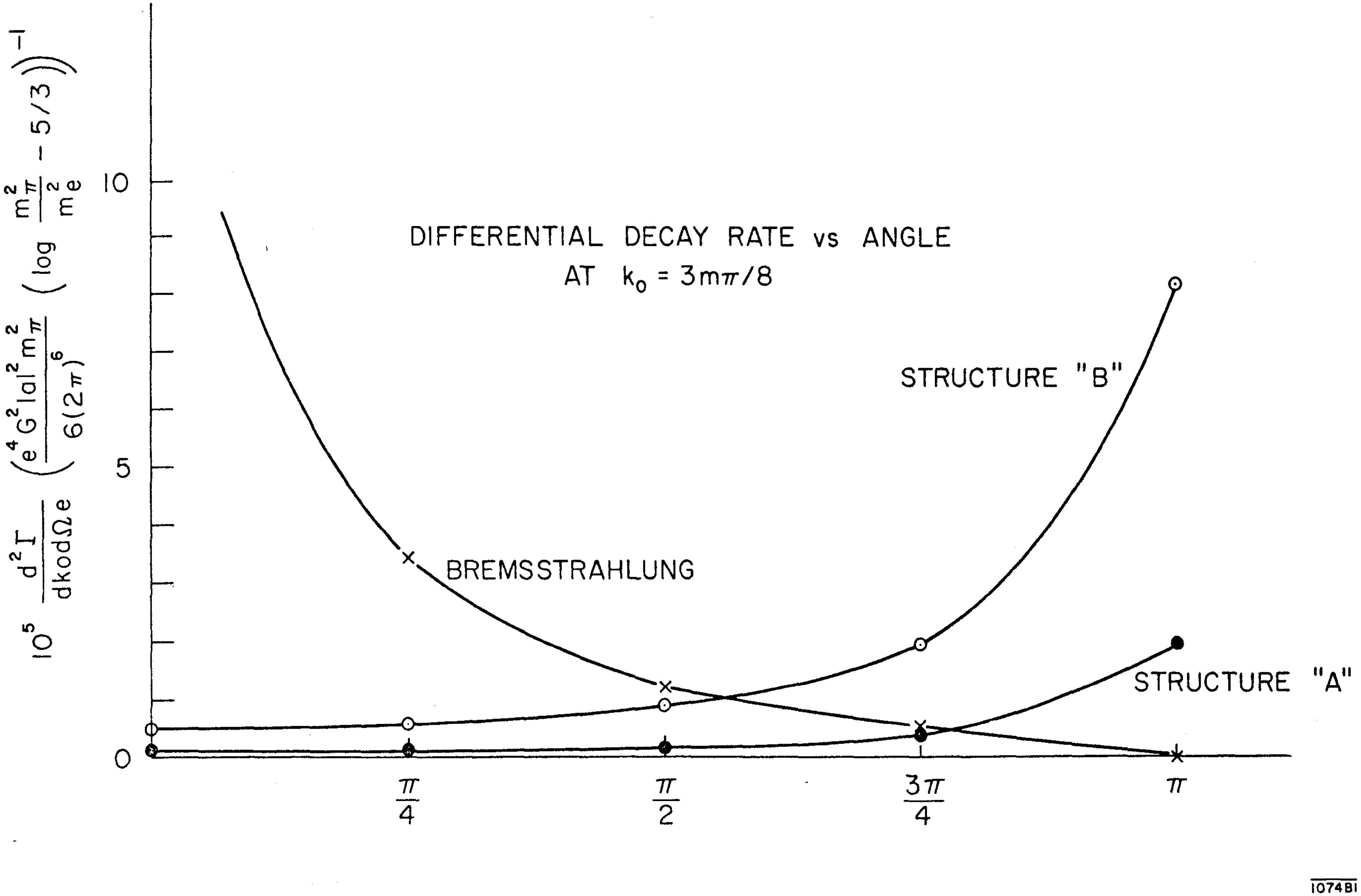
DIFFERENTIAL DECAY RATE vs ANGLE
AT $k_0 = 3m\pi/8$
STRUCTURE "B"
STRUCTURE "A"
BREMSSTRAHLUNG
$10^5 \frac{d^2\Gamma}{dk_o d\Omega_e} \left(\frac{e^4 G^2 |a|^2 m_\pi^2}{6(2\pi)^6}\left(\log \frac{m_\pi^2}{m_e^2} - 5/3\right)\right)^{-1}$
10
5
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
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Fig. 2