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PROPERTIES OF THE P AND P' REGGE TRAJECTORIES FROM LOW ENERGY πN AND KN SCATTERING AMPLITUDES †

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ABSTRACT

We show that the resonance-dominance approximation for the low energy part of finite energy sum rules for the $C = +1$, $I = 0$ t-channel πN and KN elastic amplitudes reproduces correctly properties of the P' trajectory and residue functions. The Pomeranchon is fully accounted for by the low energy background. The Gell-Mann ghost eliminating mechanism is favored for the P' trajectory.

Finite energy sum rules[1] (FESR) enable one to relate the phenomenological Regge description of high energy scattering amplitudes to the properties of low energy resonances or background amplitudes. The resonance-dominance approximation for the low energy region has been successful in computing various properties of trajectories other than the Pomeranchon[2], while in the case of $C = +1$, $I = 0$ t-channel amplitudes it is difficult to separate the contributions of the P and P' trajectories in a straightforward manner. It was recently proposed[3] that this difficulty can be removed if we assume that the Pomeranchon is mostly "built" (in the FESR sense) from the non-resonating "background" part of the low energy

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amplitude while all other "ordinary" trajectories, including P' , are mainly generated by the low energy resonances and can be appropriately analyzed in terms of the resonance-dominance approximation[4]. This hypothesis has already yielded a large number of interesting results[3] and, more significantly, in spite of its extremely restrictive nature it does not lead to any obvious inconsistency.

In this paper we present quantitative tests of this hypothesis. We consider the $C = +1$, $I = 0$ t-channel amplitudes for πN and KN scattering and find that the correspondence between the low energy resonances and the P' trajectory as well as the complementary association of the Pomeranchon with the low energy background are consistent with the experimental situation within the usual ambiguities of FESR and the experimental uncertainties. Assuming that this model for generating the P' trajectory is indeed correct, we study its behavior as a function of t and find that it probably follows the Gell-Mann ghost-eliminating mechanism.

We start by writing down FESR for the $C = +1$, $I = 0$ t-channel πN and KN scattering amplitudes. The non-spin-flip amplitudes $A'^{(+)}$ satisfy[5]:

$$S_{2n+1} \equiv \frac{1}{N^{2n}} \int_0^N \nu^{2n+1} \, \mathrm{Im}\, A'^{(+)}(\nu, t) d\nu = \sum_{i=P,P'} \beta_i^A(t) \, \frac{N^{\alpha_i(t)+2}}{\alpha_i(t)+2n+2} , \qquad (1)$$

while the spin flip amplitudes $B^{(+)}$ obey:

$$S_{2n} \equiv \frac{1}{N^{2n}} \int_0^N \nu^{2n} \, \mathrm{Im}\, B^{(+)}(\nu, t) d\nu = \sum_{i=P,P'} \alpha_i(t) \beta_i^B(t) \, \frac{N^{\alpha_i(t)}}{\alpha_i(t)+2n} \qquad (2)$$

At t=0 we can determine $\mathrm{Im}\, A'^{(+)}$ directly from the total cross section data. We can therefore explicitly separate $\mathrm{Im}\, A'^{(+)}(\nu, 0)$ into a resonance contribution, $\mathrm{Im}\, A'^{(+)}_{RES}(\nu, 0)$, and a background term, $\mathrm{Im}\, A'^{(+)}_{BG}(\nu, 0)$. The resonance part is computed by adding Breit-Wigner forms (including appropriate threshold corrections) for all known πN or KN resonances[6], while the background term is defined as:

$$\mathrm{Im}\,A'^{(+)}_{BG}(\nu, 0) = \mathrm{Im}\,A'^{(+)}(\nu, 0) - \mathrm{Im}\,A'^{(+)}_{RES}(\nu, 0). \tag{3}$$

The hypothesis that we are testing at t=0 can most simply be stated as:

$$S^{P}_{2n+1} \equiv \frac{1}{N^{2n}} \int_0^N \nu^{2n+1}\, \mathrm{Im}\,A'^{(+)}_{BG}(\nu, 0)d\nu = \beta^{A}_{P}(0)\, \frac{N^{\alpha_P(0)+2}}{\alpha_P(0)+2n+2}, \tag{4}$$

$$S^{P'}_{2n+1} \equiv \frac{1}{N^{2n}} \int_0^N \nu^{2n+1}\, \mathrm{Im}\,A'^{(+)}_{RES}(\nu, 0)d\nu = \beta^{A}_{P'}(0)\, \frac{N^{\alpha_{P'}(0)+2}}{\alpha_{P'}(0)+2n+2}. \tag{5}$$

While Eq. (5) is expected to hold only for averages over all resonances in a sufficiently large region in ν, Eq. (4) may be valid even "locally", for small ν-intervals. This follows from the expected smooth behavior of $A'^{(+)}_{BG}$ as a function of ν. If the equality:

$$\mathrm{Im}\,A'^{(+)}_{BG}(\nu, 0) = \beta^{A}_{P}(0)\nu \tag{6}$$

actually holds for any value of ν in the resonance region, we should be able to describe the total amplitude $\mathrm{Im}\,A'^{(+)}(\nu, 0)$ by a modified version[3] of the interference model[7], in which we add the s-channel resonances to the extrapolated contribution of the Pomeranchon alone. In this way we avoid the usual double counting committed by the ordinary interference model[7], when the P and P' contributions are both added to the resonances. Fig. 1 compares this modified model with the πN scattering data[8] indicating good agreement between the model and the experimental situation.

Another test of Eq. (4) can be performed by computing $\alpha_P(0)$ from S_1^P and S_3^P or by computing $\beta(0)$ from S_1^P, assuming $\alpha_P(0) = 1$. For cutoff values $1.1 \leq N \leq 1.8$ BeV (corresponding approximately to the upper ends of the "old" and "new" πN phase shift studies) we find an extremely sensitive dependence of $\alpha_P(0)$ on N and on the resonance parameters. The range of values obtained from the relation $\alpha_P = (2S_1^P - 4S_3^P)/(S_3^P - S_1^P)$ is consistent with the accepted value of $\alpha_P(0) = 1$ but it definitely does not predict it. In fact, any value of $\alpha_P(0)$ between -1 and $+2$ can be obtained in this way. The sensitivity follows here from the dominant contribution of the 1.6 - 1.9 BeV region to $\nu^3 \mathrm{Im}\, A'^{(+)}(\nu, 0)$ and therefore to S_3. Small ambiguities in the parameters of the resonances in this region are sufficient to prevent us from an accurate computation of α_P.[9] We may, however, assume that $\alpha_P(0) = 1$ in Eq. (4) and use S_1^P for computing $\beta_P(0)$. Choosing cutoffs $1.1 \leq N \leq 1.8$ we obtain for $\beta_P(0)$ numbers corresponding to an asymptotic πN total cross-section $\sigma_t^\infty(\pi N) = 14 \pm 4$ mb. This should be compared with typical high energy extrapolations such as[10]: 14.5(I); 18.4(II); 22.1(III).

We now proceed to compare Eq. (5) with the πN data. Figure 2 shows three typical curves for the extrapolated P' contribution[10] together with $\nu\, \mathrm{Im}\, A'^{(+)}_{RES}(\nu, 0)$. It is evident that the P' contribution is approximately accounted for by the resonances, in agreement with our basic conjecture. For cutoffs $1.1 \leq N \leq 1.8$ $S_1^{P'}$ and $S_3^{P'}$ give $\alpha_{P'}(0) = 0.65 \pm 0.25$ in reasonable agreement with the "high energy determinations"[10] $\alpha_{P'}(0) = 0.73$(I); 0.63(II); 0.31(III). Assuming $\alpha_{P'}(0) = 0.5$ we can compute $\beta_{P'}^A(0)$ from $S_1^{P'}$. For the same range of N-values we find[5] $\beta_{P'}^A(0) = 18 \pm 2$ to be compared with[10] 20.6(I), 16.8(II), 18.5(III). Our value for β is not very sensitive to modifications in $\alpha_{P'}(0)$.

We have performed similar calculations for the $C = +1$, $I = 0$ t-channel amplitude in KN scattering. The t=0 $A'^{(+)}$ amplitude is given by

$$\mathrm{Im}\, A'^{(+)}(\nu, 0) = \frac{1}{4}\sqrt{\nu^2 - m_K^2}\,[\sigma_t(K^+p) + \sigma_t(K^-p) + \sigma_t(K^+n) + \sigma_t(K^-n)] \qquad (7)$$

In addition to the points which we have already mentioned[6], the following ambiguities in handling the KN data should be noted: (i) The low energy total cross-section data ($\nu < 1$) have relatively large errors; (ii) The ΛKN and ΣKN coupling constants are not well determined. We have used the values given by Kim[11] but also computed the changes that would follow if Zovko's[11] results are correct. (iii) We used $g^2_{Y_o^*KN}/4\pi = 0.32$ as given by Warnock and Frye[12] for $Y_o^*(1405)$. For $Y_1^*(1385)$ we used[12] $g^2_{Y_1^*KN}/4\pi = 1.9$ but arbitrarily allowed an error of $\pm$ 25%.

In view of these errors we find that the only meaningful calculation that we can make is to compare $\mathrm{Im}\, A'^{(+)}_{RES}(\nu, 0)$ to the P' contribution, using Eq. (5). Assuming $\alpha_{P'}(0) = 0.5$ we obtain[5] from $S_1^{P'}$: $\beta^A_{P'}(0) = 5.7\ ^{+0.7}_{-1.7}$ where the errors indicate the combined effect of the above ambiguities and the cutoff dependence ($1.1 \leq N \leq 1.9$ BeV). Our value for $\beta^A_{P'}(0)$ should be compared with the "high energy value"[13] $\beta^A_{P'}(0) = 5.5 \pm 1.3$.

Encouraged by these successes of our assumption on the relation between the low energy resonances and the P' trajectory we now try to extend the analysis to $t \neq 0$. Here we do not have any reliable values of β_P or $\beta_{P'}$ since the separation of the A' and B contributions to $d\sigma/dt$ as well as the "popular" parametrizations of $\beta_{P'}$ suffer from many ambiguities. In particular, since the present high energy data do not explicitly <u>require</u> any zeros in $\beta_{P'}(t)$ in addition to those which occur at the point $\alpha_{P'} = 0$, most parametrizations <u>apriori assume</u> that no such additional zeros exist. This does not mean that other parametrizations of $\beta_{P'}(t)$ are inadequate, or that it is difficult to fit the data with a residue function that vanishes at t-values other than the $\alpha_{P'} = 0$ point. What we can do here is to <u>assume</u> that for $t \neq 0$ the P' trajectory is still "built" from resonances only and to use the FESR in order to compute the t-dependence of $\beta^A_{P'}(t)$ and $\beta^B_{P'}(t)$ for πN and KN scattering. Using the πN and KN resonances[6] and assuming $\alpha_{P'}(t) = 0.5 + t$ we find the t-dependence shown in Figure 3.

The behavior of the $A'^{(+)}$ amplitudes for both πN and KN scattering indicates very clearly that $\beta^A_{P'}$ has two (possibly coinciding) zeros in the region

$-1 \leq t \leq 0$. Since $\beta^{A}_{P'}(t)$ has to vanish at least once for $\alpha_{P'} = 0$, we are left here with two possibilities: (a) We have a double zero of $\beta^{A}_{P'}$ at $\alpha_{P'} = 0$, corresponding to the "no-compensation" ghost-eliminating mechanism[14]. (b) We have one zero at $\alpha_{P'} = 0$ and another "dynamical" zero elsewhere (probably around $t = -0.2$ or -0.3). In this case we would have either the Chew[15] or the Gell-Mann[16] mechanism. The accuracy of our analysis of the $A'^{(+)}$ amplitudes is certainly not sufficient to distinguish between possibilities (a) and (b). The ambiguity can be resolved, however, by looking at the t-behavior of $\beta^{B}_{P'}$. The Chew and "no-compensation" mechanisms demand that $\beta^{B}_{P'}$ (as defined in Eq. (2)) vanishes for $\alpha_{P'} = 0$, while in the Gell-Mann mechanism it does not. Figure 3(b) shows that at least in the πN case the Gell-Mann mechanism is definitely favored and $\beta^{B}_{P'}(t)$ does not vanish anywhere in the region of interest. In the KN case (Figure 3(d)) the situation is obscured by the large errors and it is hard to reach any conclusions. On the basis of the πN-analysis we propose, however, that the P' trajectory actually chooses to follow the Gell-Mann mechanism with an extra zero in the $\beta^{A}_{P'}(t)$ residue function. It is interesting to add that the same mechanism is also favored for the A_2 trajectory in processes such as $\pi N \rightarrow \eta N$ [17], $KN \rightarrow K\Delta(1236)$ [18] and $KN \rightarrow KN$ [19]. At least for $KN \rightarrow KN$, FESR predicts[19] an extra zero in the A' amplitude. In view of the SU(3) relation between the P' and A_2 trajectories, it would be embarrassing if they followed different ghost-eliminating mechanisms. Our conclusions with respect to P' are in accord with the requirement of a similar behavior for the two trajectories[20].

Our results can be summarized by the following points: (i) The low energy resonances in πN and KN scattering approximately account for the P' t-channel trajectory; (ii) The background amplitude which remains after subtracting the known resonances is sufficiently large to produce the Pomeranchon contribution via FESR; (iii) Our modified version of the interference model works very well at least in the πN case: (iv) The P' trajectory probably obeys the Gell-Mann ghost-eliminating mechanism.

Footnotes and References

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2. K. Igi and S. Matsuda, Phys. Rev. Letters 19, 928 (1967); M. Ademollo *et al.*, Phys. Rev. Letters 19, 1402 (1967) and Phys. Letters, to be published; C. Schmid, Phys. Rev. Letters 20, 628 (1968); A. Bietti *et al.*, Phys. Letters 26B, 530 (1968). See also R. Dolen *et al.*, Ref. 1.
3. H. Harari, Weizmann Institute preprint, submitted to Phys. Rev. Letters.
4. Throughout this paper we assume that the only relevant $C = +1$, $I = 0$ trajectories are P and P'. The possible existence of a P" will not affect our conclusions since (a) it probably couples weakly to the nucleon; (b) our calculated P' will be an "effective" P' - P" combination which presumably does not differ much from the P' itself.
5. We use the following notation and units: $\mathrm{Im}\, A'^{(+)}(\nu, 0) = \sqrt{\nu^2 - m^2}\ \sigma_t^{(+)}(\nu)$, ν is the laboratory energy (in BeV) of the incident meson with mass m, $\sigma_t^{(+)}$ is the average $(\pi^+ p, \pi^- p)$ or $(K^+ p, K^+ n, K^- p, K^- n)$ total cross section in mb.
6. For πN scattering we basically use the 18 N* states listed by A. Donnachie *et al.*, Phys. Letters 26B, 161 (1968). For KN scattering we use the Y_0^* and Y_1^* states listed by A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 40, 77 (1968). Three of these states, $Y_0^*(1670)$, $Y_1^*(1660)$ and $Y_1^*(1690)$, involve serious experimental ambiguities, but their total effect on our calculations is not very significant.

For resonances we introduce a threshold factor $(q/q_R)^{2\ell+1}$ where q_R is the C. M. momentum of the elastic decay products of the resonance. We modify only the $q < q_R$ part of the Breit-Wigner form. The sensitivity of our calculations with respect to (a) variations in resonance parameters, (b) possible other ways of making threshold corrections, (c) possible non-existence of some of the high-mass N* and Y* states, (d) different choices of KNY couplings, is reflected by the error bars in Figures 1 and 3.

7. V. Barger and M. Olsson, Phys. Rev. 151, 1125 (1966). See also V. Barger and L. Durand, Phys. Letters 26B, 588 (1968).

8. Im $A'^{(+)}(\nu, 0)$ is taken from G. Hohler *et al.*, Z. Physik 180, 430 (1964).

9. If $\alpha_P(0) = 1$, $S_3^P = 0.6\ S_1^P$. An error of $\pm 10\%$ in S_3^P changes $\alpha_P(0)$ between 0.3 and 1.9. The ambiguities in the resonance parameters above 1.5 BeV are at least of that order of magnitude.

10. Among the many high energy fits to $A'^{(+)}$ we have chosen three "typical" ones with which we compare our FESR results. These are - solution 1 of W. Rarita *et al.*, Phys. Rev. 165, 1615 (1968)(hereafter denoted as I); C. B. Chiu *et al.*, Phys. Rev. 161, 1563 (1967)(denoted as II) and K. Z. Foley *et al.*, Phys. Rev. Letters 19, 330 (1967)(denoted as III).

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13. R. J. N. Phillips and W. Rarita, Phys. Rev. 139B, 1336 (1965).

14. C. B. Chiu *et al.*, Ref. 10.

15. G. F. Chew, Phys. Rev. Letters 16, 60 (1966).

16. M. Gell-Mann, *Proceedings of the 1962 International Conference on High Energy Physics at CERN*, p. 539.

17. The absence of a dip in $\pi^- p \to \eta n$ hints that the correct mechanism is that of Gell-Mann.

18. M. Krammer and U. Maor, Nuovo Cimento 52A, 308 (1967).

19. Igi and Matsuda, Ref. 2.

20. Several recent papers dealing with high energy Regge fits have claimed that the no-compensation mechanism is preferred for P', and that a double zero at $\alpha_{P'} = 0$ probably exists in $\beta^A_{P'}$. Our result is numerically very close to this possibility since our two different zeros of $\beta^A_{P'}$ are not very far apart. It is only $\beta^B_{P'}$ (about which we know very little at high energies, experimentally) that tells us to prefer the Gell-Mann mechanism.

Figure Captions

Figure 1: Comparison of the modified interference model with the πN data. Dashed line: $\mathrm{Im}\, A'^{(+)}(\nu, 0)$ taken from Ref. 8. Straight line: Extrapolated Pomeranchon contribution corresponding to $\sigma_t^{\infty}(\pi N) = 18.4$ mb (II). Solid curved line: Sum of extrapolated P contribution and Breit-Wigner forms of all resonances of Donnachie et al., (Ref. 6). Error bars show the variation of the model's prediction when $\beta_P^A(0)$ is allowed to change between 14.5 (I) and 22.1 (III) (Ref. 10) and all πN resonances which are not well established are omitted.

Figure 2: Extrapolated P' contribution to $\nu\, \mathrm{Im}\, A'^{(+)}(\nu, 0)$ for πN scattering as determined by three different high energy fits (I-III, Ref. 10). and $\nu\, \mathrm{Im}\, A'^{(+)}_{RES}(\nu, 0)$ (line IV).

Figure 3: The t dependence of the P' residue functions for πN and KN amplitudes $\alpha_{P'}(t) = 0.5 + t$ is assumed. $N(t) = N(0) + t/4M$. $\beta_{P'}^{A,B}$ defined as in Eqs. (1), (2). (a) $\beta_{P'}^A(t)$ for πN. I - $N(0) = 1$; II - $N(0) = 1.4$; III - $N(0) = 1.8$. (b) $\beta_{P'}^B(t)$ for πN, I-III same as in (a). (c) $\beta_{P'}^A(t)$ for KN. Solid line for $N(0) = 1.2$ and for the coupling constants given in the text. Errors indicate the variation due to changes in N (up to 1.9 BeV), replacing Kim's couplings by Zovko's; 25% error in $g^2_{Y_1^* KN}$, and variation of the parameters of the poorly determined high mass Y* resonances (Ref. 6). (d) $\beta_{P'}^B(t)$ for KN. Notation as in (c).

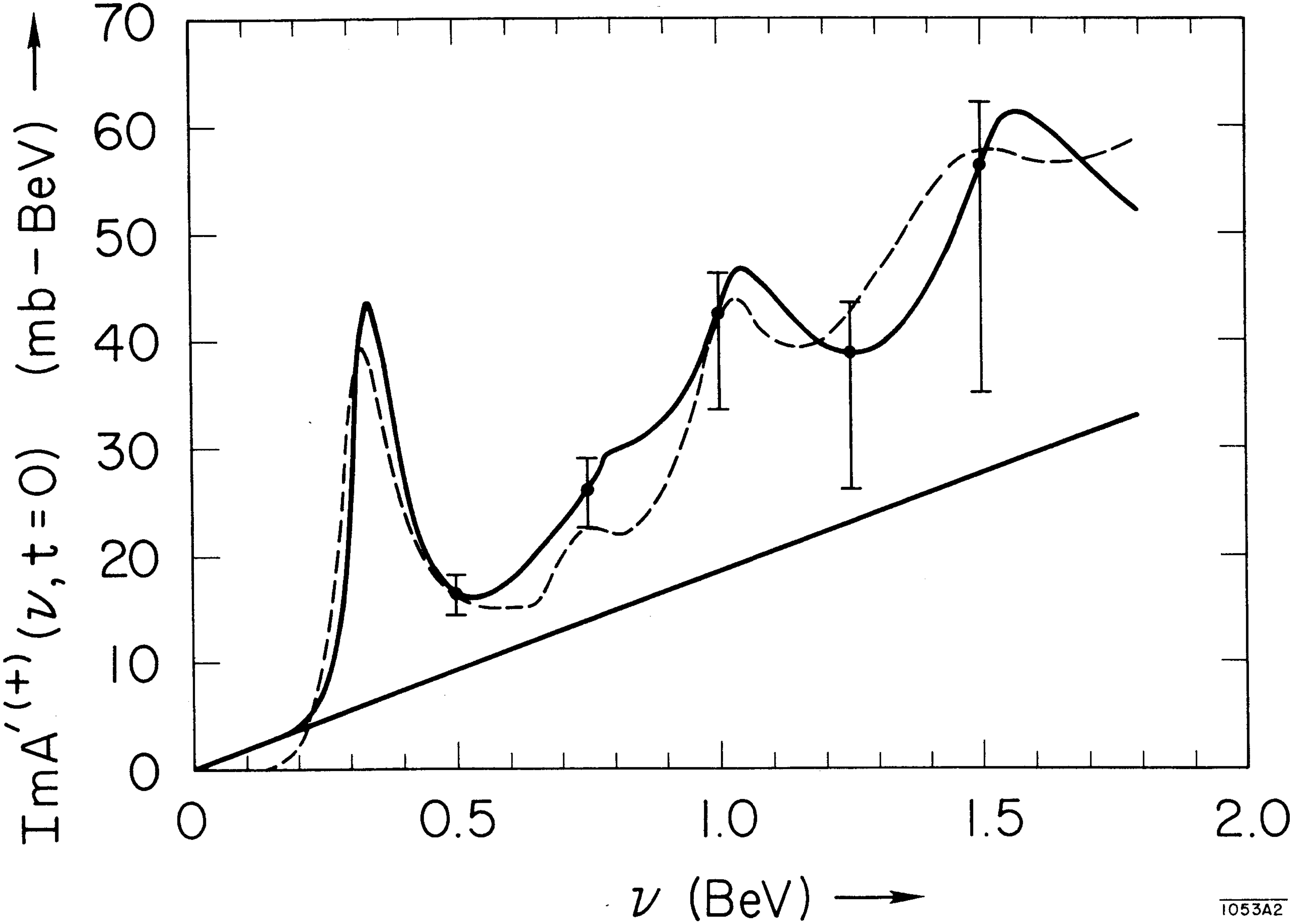
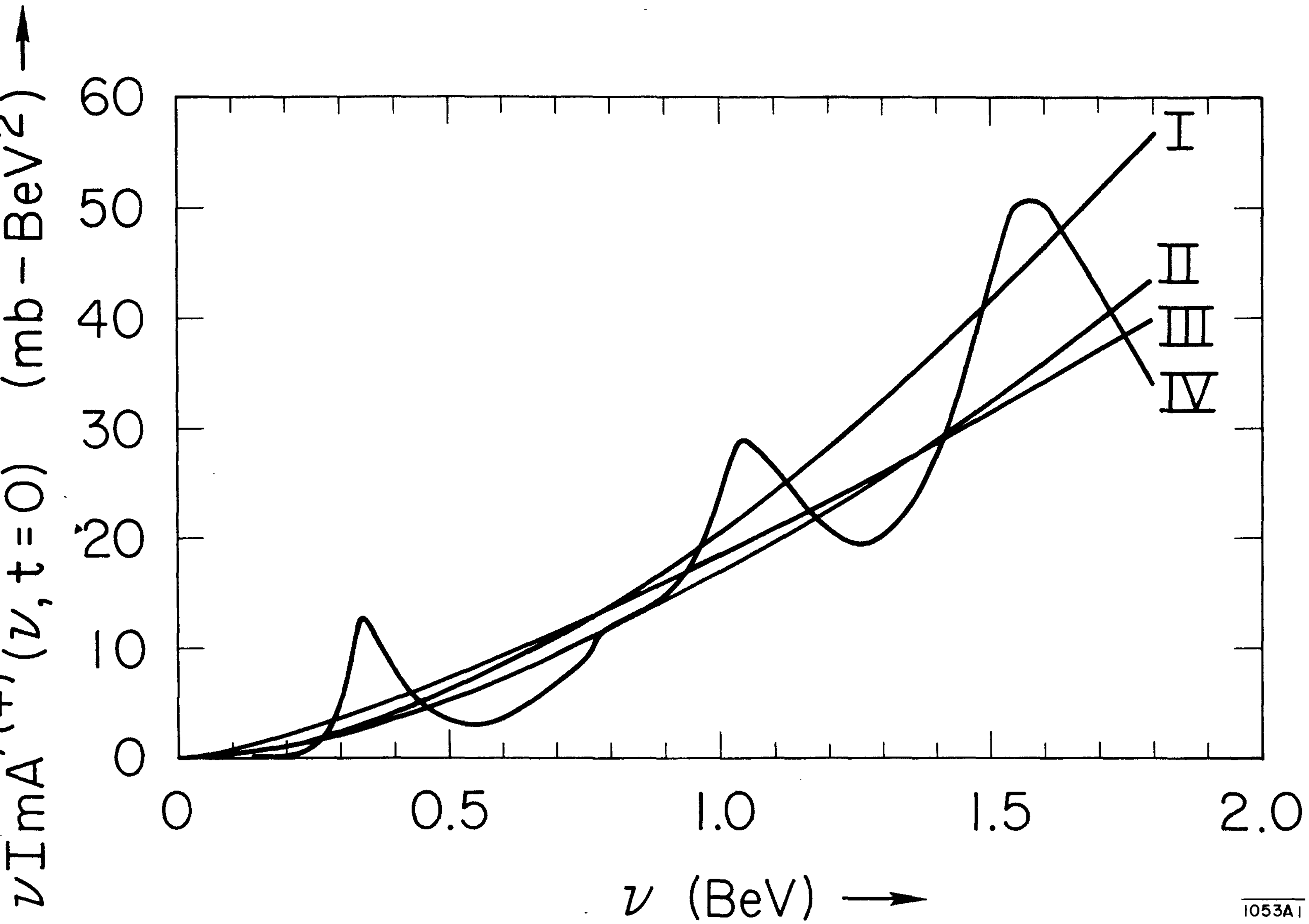


Fig. 1



60
50
40
30
20
10
0
0
0.5
1.0
1.5
2.0
I
II
III
IV
$\nu \mathrm{Im} A'^{(+)}(\nu, t=0)$ (mb - BeV^2)
ν (BeV)
1053A1

Fig. 2

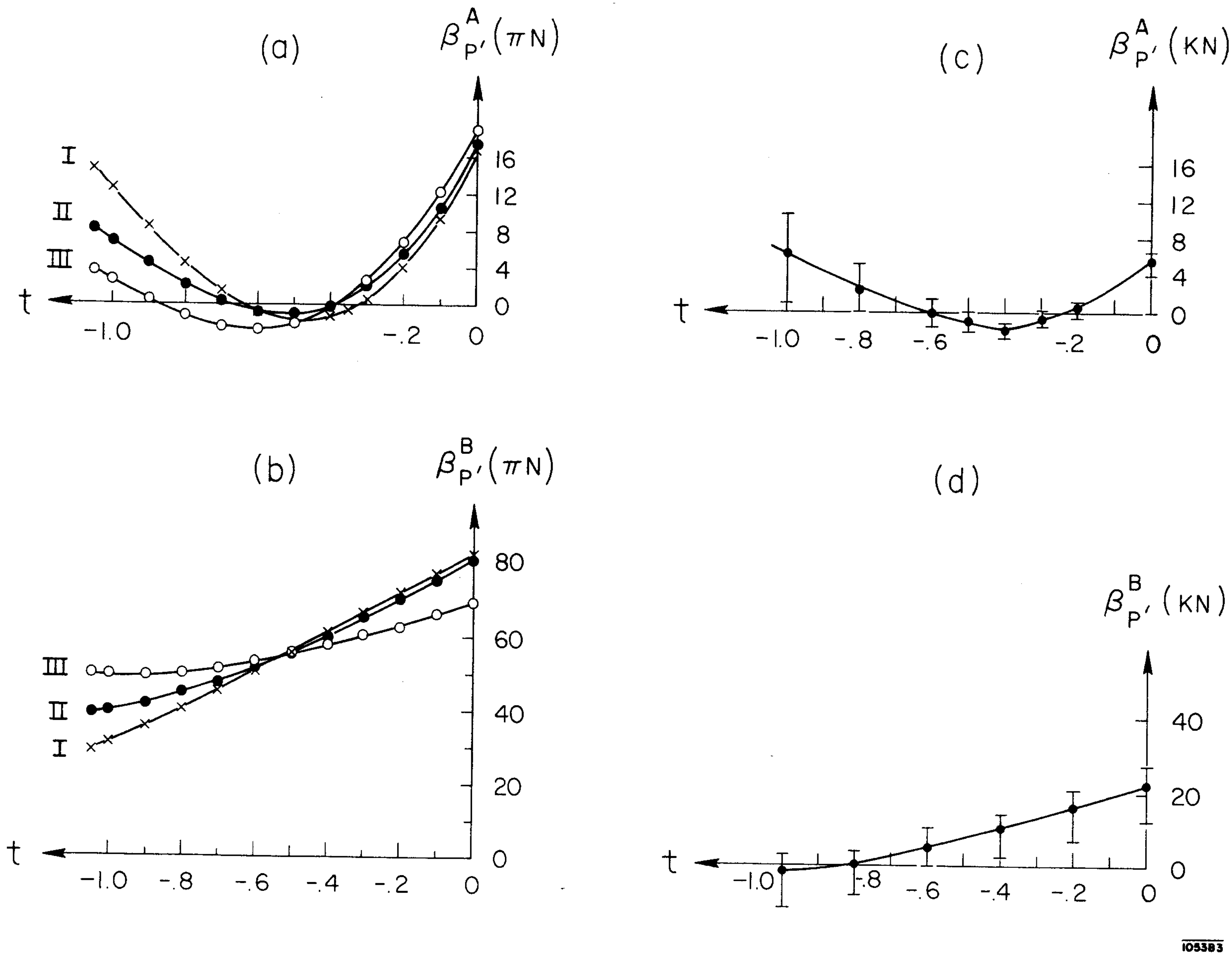

(a)
$\beta^A_{P'}(\pi N)$
I
II
III
t
-1.0
-.2
0
16
12
8
4
0
(b)
$\beta^B_{P'}(\pi N)$
III
II
I
t
-1.0
-.8
-.6
-.4
-.2
0
80
60
40
20
0
(c)
$\beta^A_{P'}(KN)$
t
-1.0
-.8
-.6
-.2
0
16
12
8
4
0
(d)
$\beta^B_{P'}(KN)$
t
-1.0
-.8
-.6
-.4
-.2
0
40
20
0
105383

Fig. 3