A SIMPLE MODEL OF π p ELASTIC SCATTERING NEAR 1 GeV/c*

W. A. Ross and D. W. G. S. Leith Stanford Linear Accelerator Center Stanford University, Stanford, California

ABSTRACT

Diffractive phenomena have been shown to play an important role in $\pi^- p$ elastic scattering even down to energies below 1 GeV/c where resonant effects are dominant. With this in mind, we attempt to fit the differential cross section with a model incorporating both resonant and diffractive effects. At these energies the diffractive term is taken from the experimental data to be exponential in the invariant momentum transfer, although at higher energies we plan to allow more structure. The resonant amplitudes were parameterized as Breit Wigner line shapes. A fit is made to the observed cross sections by varying, as parameters, the masses, widths, and elasticities of the resonances, as well as the parameters associated with the diffraction structure. In this way values are obtained for these parameters and a satisfactory fit to the cross section data is obtained.

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I. INTRODUCTION

In a recent paper by Levi-Setti <u>et al.</u>, ¹ a model incorporating both resonant and diffractive effects was used to fit $K^{-}P$ elastic scattering at energies near 1 GeV/c. We are using a similar approach to fit pion-nucleon scattering. The results given here are to be treated as a progress report on work which is now continuing.

In section II we present some motivation for our model. In section III we give our parameterization of the transition amplitudes. In sections IV and V we give the results and conclusions of the explicit application of this model to $\pi^- p$ elastic scattering near 1 GeV/c. This is the work which is presently being reported. We also point out in section V some other applications of this model which are now being made.

II. MOTIVATION FOR THE MODEL

In an attempt to construct a model of the pion-nucleon scattering amplitude we shall attempt to benefit from an examination of the differential cross section. First let us specify the energy region with which we shall deal. This is given by 925 MeV/c $\leq p_{LAB} \leq 1180$ MeV/c and is indicated on a plot of the total and total elastic $\pi^- p$ cross section in Fig. 1. The differential cross sections for this energy region can be seen in Fig. 3. The main two features observed in these cross sections are:

- 1. A large diffraction, or forward, peak;
- 2. Somenon-forward, energy-dependent structure.

The second of these features is believed to be due to resonance structure much of which is well established by previous work.²

-2-

The first of these observations (the diffraction peak) will be partly due to resonances (which are known to have a large forward peak), but it is clear from the higher energy data³ that there is more. In fact since we know that the pion-nucleon inelastic cross sections are rising rapidly above 600 MeV (725 MeV/c), we can anticipate from an understanding of the optical theorem that there should be a forward peak due to diffraction.

Another feature of diffraction scattering is that it is due to a predominantly imaginary amplitude. To see how true this is for our case we proceed somewhat indirectly. For the forward direction we can evaluate the imaginary part of the amplitude from the total cross sections via the optical theorem and the real part via dispersion relations. The result of these calculations is that the ratio of real to imaginary parts of the forward scattering amplitude is less than 25% throughout the energy region of interest to us.⁴

Suppose that we temporarily regard this to be ample evidence for the existence of some diffraction scattering. Then we proceed by choosing our scattering amplitude to be a sum of two terms:

$A = A_{RESONANCE} + A_{BACKGROUND}$

where we shall make the assumption that the background amplitude is purely diffractive, i.e.,

 $A = A_{RESONANCE} + A_{DIFFRACTION}$

One can go farther and claim that this sort of parameterization of the amplitude is very natural for the simple reason that it reflects the two dominant features seen in the differential cross section data. Thus, on this basis we could believe

-3-

this model for pion-nucleon scattering from 900 MeV/c up to 3 or 4 GeV/c and also for \overline{K} N scattering above 800 MeV/c. Indeed, this model was first presented by Levi-Setti, Predazzi, and Lasinski for \overline{K} P elastic scattering between .85 GeV/c and 1.2 GeV/c.¹

III. TRANSITION AMPLITUDES

A. General Conventions

First we shall define the transition amplitudes quite generally. For a given isospin state the scattering amplitude matrix is given by:

$$A^{I}(k,\theta) = f^{I}(k,\theta) + i \sigma \cdot \hat{n} g^{I}(k,\theta)$$

Here k and θ are the center-of-mass momentum and scattering angle. f^{I} and g^{I} are respectively the spin-non-flip and spin-flip amplitudes for isospin I. $\bigwedge_{n=1}^{n}$ is the normal to the scattering plane.

For a particular process the physical scattering amplitude is

A
$$(k,\theta) = \sum_{I} C_{I} A^{I} (k,\theta)$$

where C_{I} is the appropriate isospin coupling factor.

Next, writing

A
$$(k,\theta) = f(k,\theta) + i\sigma \cdot \hat{n} g(k,\theta)$$

one can easily get:

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\boldsymbol{\Omega}}(\mathbf{k},\boldsymbol{\theta}) = \left|\mathbf{f}(\mathbf{k},\boldsymbol{\theta})\right|^{2} + \left|\mathbf{g}(\mathbf{k},\boldsymbol{\theta})\right|^{2}$$

$$P(k,\theta) \frac{d\sigma}{d\Omega}(k,\theta) = 2 \operatorname{Im} \left[f(k,\theta) g^{*}(k,\theta) \right]$$

Here P is the polarization and $\frac{d\sigma}{d\Omega}$ the differential cross section. We can then perform the usual partial wave decomposition to get

$$f^{I}(\mathbf{k},\theta) = \frac{1}{\mathbf{k}} \sum_{\ell} \left\{ (\ell+1) \mathbf{a}_{\ell+}^{I} + \ell \mathbf{a}_{\ell-}^{I} \right\} \mathbf{P}_{\ell} (\cos\theta)$$
$$g^{I}(\mathbf{k},\theta) = \frac{1}{\mathbf{k}} \sum_{\ell} \left[\mathbf{a}_{\ell+}^{I} - \mathbf{a}_{\ell-}^{I} \right] \mathbf{P}_{\ell}^{1} (\cos\theta)$$

where

$$P_{\ell}^{1}(x) = \sqrt{1 - x^{2}} \frac{dP_{\ell}(x)}{dx}$$

B. Resonant Amplitude

The resonant contributions will of course appear only in the specific resonance channels whereas the diffraction term will spill over into all ℓ values.

The resonant term will be given by:

$$f_{RES}^{I}(k,\theta) = \frac{1}{k} \sum_{\ell=\ell_{RES}} \left\{ (\ell+1) a_{\ell+}^{IRes} + \ell a_{\ell-}^{IRes} \right\} P_{\ell} (\cos\theta)$$
$$g_{RES}^{I}(k,\theta) = \frac{1}{k} \sum_{\ell=\ell_{RES}} \left\{ a_{\ell+}^{IRes} - a_{\ell-}^{IRes} \right\} P_{\ell}^{1} (\cos\theta)$$

where the sum is over all resonant partial waves. Such resonant partial wave amplitudes can be taken to have a Breit-Wigner form:

$$a^{\text{Res}} = \frac{x}{\epsilon - i}$$

and

where $x = \Gamma_{elastic} (k) / \Gamma (k)$ is the elasticity of the resonance.

$$\epsilon = 2 \left[E_{RES} - E \right] / \Gamma(k)$$

 $E_{\rm RES}$ and Γ (k) are the energy and (energy-dependent) width of the resonance.

For our purposes we take⁵

$$\Gamma$$
 (k) = $\frac{k v_{\ell}$ (kR)}{k_{RES} v_{\ell} (k_{RES}R) Γ_{R}

Here $v_{\ell}(x)$ is the appropriate barrier penetration factor given by Blatt and Weisskopf.⁶ R is the interaction radius.

C. Diffraction Amplitude

This completes the specification of the resonant amplitude; next we must consider the diffraction amplitude. This we choose to determine phenomonologically. Thus, to obtain the form of the diffraction amplitude, we observe the near-forward differential cross section.

For our purposes we fit the near-forward cross sections emperically by the form $^{7} \,$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$
 (θ) = $\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\Big|_{\theta=0}$ e^{bt}

where t = $-2k^2(1 - \cos \theta)$ is the invariant momentum transfer.

While this dependence can be explained by various models, we shall take it as an emperical fit. Next, working specifically with π^-P elastic data we can examine the energy dependence of b. This is shown in Fig. 2. In doing so we observe:

1) b has some structure showing peaks at particular values of k.

A closer examination shows a distinct correlation between these peaks and the positions of πN resonances.

- 6 -

2) b may well level out approaching a constant at high energies far from where resonances are dominant.

Thus, one may hope that, ignoring resonant contributions, b = constant and we shall take our diffraction amplitude to give a cross section $a(k) e^{bt}$ with b constant. Next we examine $a(k) = \frac{d\sigma}{d\Omega}\Big|_{\theta=0} \cong \left(\frac{k\sigma_{\text{tot}}}{4\pi}\right)^2$. Note that we choose to use the optical cross section here since we want to treat a(k) as a diffraction term.

As a function of k this is observed to vary quite strongly with peaks at resonance values. The procedure we use to extract a diffraction term from this is as follows. First we ignore the resonance bumps and fit the smoothed result with a simple polynomial in k. Next we acknowledge that even this smooth result will be in part due to resonances, so we choose our diffractive amplitude to be

$$A_{\text{DIFF}} = i C \sqrt{a(k)} e^{1/2 bt}$$

where a(k) is fixed by the fit described above. C and b are two constant parameters which will be varied to fit the cross sections. Note that

$$|A_{\text{DIFF}}|^2 = C^2 a(k) e^{bt}$$

So we now take our diffraction amplitudes to be:

$$f_{DIFF}^{I} (k, \theta) = F^{I} (k) e^{1/2 b_{I}t}$$
$$g_{DIFF}^{I} (k, \theta) = \sin \theta G^{I} (k) e^{1/2 b_{I}t}$$

We have forced the spin-flip term to approach 0 like $\sin \theta$ as we know it must. For this reason we shall ignore this term and set $G^{I}(k) = 0$. This is mainly done in order to make a simple first approximation to the cross section. Such a term may later be found necessary, but for now we shall try to do without it. Note

-7-

that such a term will not make itself felt in the differential cross section (which we are now fitting) nearly as much as in the polarization data.

Before proceeding further, let us note the following important feature. We have explicitly used e^{bt} as the shape of the diffraction term. However, if one observes a different angular dependence on the forward peak, one can simply change e^{bt} to some new function which reflects the observed structure. Thus higher energies where secondary maxima are observed can be accomodated by altering the form of the diffraction term.

IV. APPLICATION TO π P ELASTIC SCATTERING

Now let us deal specifically with $\pi^- P \longrightarrow \pi^- P$ differential cross sections. We take our amplitudes to be:

$$f(k, \theta) = i C \sqrt{a(k)} e^{1/2bt} + 2/3 [I = 1/2 \text{ Resonance terms}]$$

+ 1/3 [I = 3/2 resonance terms]

$$g(k, \theta) = 2/3$$
 $[I = 1/2 \text{ Resonance terms}] + 1/3 [I = 3/2 \text{ Resonance terms}]$

Note that we have collapsed the two isospin diffraction terms into a single term. This makes the interpretation of a(k) more natural.

As an attempt to test our model we started to use data around 1 GeV/c where we believed that the resonance structure was known. By choosing only data in the region 900 MeV/c $\leq p \leq 1200$ MeV/c we could confine ourselves to a particular set of resonances as seen from the Rosenfeld tables of one year ago.⁸ Any attempt to fit the data using only those resonances was a failure. The reason for these failures is simply that the resonant structure was not then fully understood. In attempts to show that no further resonances were

- 8 -

needed we found that the opposite was true. Specifically a P_{11} resonance was the addition most necessary for improving the fit. We found soon after that the latest round of phase shift analyses² had indeed made such predictions.

Now let us present some of the characteristics of our best fit.

DATA USED.⁹ Differential cross section data of Duke <u>et al.</u>, Helland <u>et al.</u>, Grard <u>et al.</u>, and Wood <u>et al</u>. 925 MeV/c $\leq P \leq 1180$ MeV/c. NUMBER OF DATA POINTS 207

NUMBER OF FITTING PARAMETERS35NUMBER OF DEGREES OF FREEDOM172NUMBER OF RESONANCES USED (of these three were fixed)10 $\chi^2 = \Delta \sum_{\text{data points}} \left[\frac{\sigma_{\text{FIT}} - \sigma_{\text{EXP}}}{\Delta \sigma_{\text{EXP}}} \right]^2 = 385$

 χ^2 was minimized by the program MINFUN.¹⁰ NOTE: ϵ is a normalization factor.¹²

First we shall comment on the rather large value of χ^2 which, if taken at face value would correspond to a confidence level of about 10^{-3} or 10^{-4} . We are using data from four different experimental groups⁹ so that the relative compatibility may be slightly questionable. Probably a more important feature is the fact that specific points may be slightly questionable. To be specific, 7 of the 207 data points contribute 100 of the 385 units of χ^2 . In fact in terms of confidence levels, the removal of about 10 points could achieve a confidence level above 10 percent ($\chi^2 \cong 250$).

In any case the quality of the fit can be seen by observing the graphs in Fig. 3.

Next let us present the results in terms of the resonances. The low energy resonances used, $P_{11}(1470)$, $D_{13}(1518)$, and S_{11} (1550) had their resonance parameters fixed.¹¹Others used had their resonance parameters varied to fit the data; the results are shown in Fig. 4.

- 9 -

Note that the D_{15} and F_{15} resonances are well determined. Note also that these are the only resonances in their particular partial waves. Next observe that the S-wave resonance parameters are rather poorly determined; we believe that this is due to some mixing of the three resonances all with the same quantum numbers. Next we come to the P₁₁ resonance which we found necessary in order to obtain a good fit. The fact that its resonance energy as determined by our MINFUN best fit is so high is not to be regarded as a serious objection. We have used no data above 1770 MeV (c.m. energy) and only one experiment above 1720 MeV so that there are no constraints on the amplitude above this energy. Since we are only then seeing the lower energy part of this resonance, it is quite possible that a strong wide resonance at 2 GeV (as found by MINFUN) is equivalent to a weaker narrower resonance at 1750 MeV (as predicted by Lovelace). The difference between these two resonances is the energy dependence of the amplitude outside the range of our data so that we cannot distinguish between the two cases. Attempts were also made with the P_{11} artificially constrained to lie near 1751 MeV and finally ignoring it altogether. The results are:

with P₁₁ constrained $\chi^2 = 435$ E_{RES} = 1820 with no P₁₁ $\chi^2 = 560$.

The introduction of the P_{33} and D_{33} resonances causes a reduction of 20 or 30 units in χ^2 for each resonance.

Next let us see what results were obtained for the other parameters of out fit. These are shown in Fig. 5.

For the diffraction slope B we would expect the result to be $B \lesssim 7(GeV/c)^{-2}$ because this is the "asymptotic value" and some of this should be due to resonances. The result, $B = 2.54 (GeV/c)^2$ is consistent with this expectation.

- 10 -

The constant C corresponds to that part of the forward amplitude due to the diffraction term. Thus, we would expect C to be less than 1. The result, C = 0.293 is reasonable. An attempt was made to make C complex (equivalent to giving the diffraction term a real part). The results of this attempt were that C remains real with only a 2 percent imaginary part (corresponding to the diffraction amplitude remaining imaginary) and that the fit is not improved. (χ^2 drops by only 3 units.)

The interaction radius was found to be R = 0.95 FM which is a reasonable value.

The normalization of the experiments was also found to be quite acceptable.

V. CONCLUSIONS AND FUTURE APPLICATIONS

We regard these results as follows. We feel we have strong evidence for the existence of a P_{11} resonance with mass greater than 1750 MeV. Its elasticity is not small (\gtrsim 30 percent) and it is probably a fairly wide resonance but until we extend the energy range of our analysis, we can say no more.

We also claim that this constitutes weak evidence for P_{33} and D_{33} resonances in the energy range suggested by phase shift analyses.² We claim that these resonances are quite inelastic but beyond that we do not determine their resonant parameters very well.

We also have very strong evidence for the other well-established resonances as would be expected.

We have also tried adding extra resonances in every partial wave. This resulted in no improvement in any case and all such resonances "went away" i.e., their resonant parameters varied in such a way that the extra resonances made no contribution to the amplitude in the region of study.

- 11 -

With respect to future applications we feel that this can be a fruitful approach to obtaining some knowledge about resonant amplitudes. With this in mind we are, at present, making attempts to apply this model to energies above 2 GeV/c where the resonant structure is quite poorly determined.

We are also making attempts to fit π^+ p elastic scattering in the 1 GeV/c energy range and will shortly include charge exchange and polarization data as well.

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- 11. The fixed resonance parameters used were:

S ₁₁ (1550)	X =	.30	M = 1565 MeV	$\Gamma = 125 \text{ MeV}$
D ₁₃ (1518)	<i>X</i> =	.66	M = 1526 MeV	Γ = 119 MeV
P ₁₁ (1470)	<i>X</i> =	.60	M = 1400 MeV*	Γ = 205 MeV

*Note: The same results were obtained with M = 1470 MeV.

12. ϵ is a scaling factor which was allowed to vary around 1 to account for normalization errors in a given experiment. ϵ could vary from one experiment to another but was fixed for all data from a given experiment at a given energy. For the forward points (dispersion points) ϵ was set equal to 1.

- 14 -

FIGURE CAPTIONS

- 1. π P total and total elastic cross sections as a function of energy. The hatched region indicates the energy range with which this analysis deals.
- 2. Behaviour of the diffraction slope B as a function of energy. B is obtained from fits to the near forward differential cross section of the form

$$\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\boldsymbol{\Omega}}(\boldsymbol{\theta}) = \frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\boldsymbol{\Omega}}(\boldsymbol{\theta} = 0) \ \mathrm{e}^{\mathrm{Bt}}$$

- 3. π P differential cross sections from 0.925 GeV/c to 1.180 GeV/c. The experimental points are taken from reference 10; the solid curves represent the best fit of our model.
 - (a) p = 925, 975, 1000 MeV/c
 - (b) p = 1003, 1016, 1030 MeV/c
 - (c) p = 1055, 1080, 1120 MeV/c
 - (d) p = 1151, 1180 MeV/c
- 4. Resonant parameters corresponding to our best fit. These are labelled
 "Ross, Leith". Also included for a comparison are the resonance parameters
 listed in Reference 3 and/or Reference 9. These are labelled "Other Reference".
- 5. A list of the non-resonance parameters of our best fit; for an explanation of the symbols see text.



 π p Total and Total Elastic Cross Sections

Fig. 1



Fig. 2



 $\pi^- P$ ELASTIC DIFFERENTIAL CROSS SECTION

Fig. 3a



 π^-P ELASTIC DIFFERENTIAL CROSS SECTION

Fig. 3**b**



 π -P ELASTIC DIFFERENTIAL CROSS SECTION

Fig. 3c



 π -P ELASTIC DIFFERENTIAL CROSS SECTION

Fig. 3 **d**

RESONANCE		x	E (MeV)	$\Gamma(\text{MeV})$
D	OTHER REFERENCE	0.40	1675	155
15	ROSS, LEITH	0.41	1682	118
s_{31}	OTHER REFERENCE	0.35	1655	180
	ROSS, LEITH	0.84	1679	76
F ₁₅	OTHER REFERENCE	0.65	1689	120
	ROSS, LEITH	0.61	1685	174
s ₁₁	OTHER REFERENCE	0.85	1705	270
	ROSS, LEITH	0.35	1696	213
, P ₁₁	OTHER REFERENCE	0.32	1751	330
	ROSS, LEITH	0.81	2021	413
P ₃₃	OTHER REFERENCE	0.10	1688	280
	ROSS, LEITH	0.26	1729	72
Daa	OTHER REFERENCE	0.15	1691	280
-33	ROSS, LEITH	0.24	1803	246
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Fig. 4

 $B = 2.54 (GeV/c)^{-2}$ C = 0.293 R = 0.95 Fm. $\epsilon = 1.00, 1.01, 1.04,$ 1.01, 1.02, 1.06, 1.06, 1.06, 0.91, 0.97, 0.99

Fig. 5