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PROTON FORM FACTORS

Over the past decade there have been a progression of beautiful experiments on elastic electron proton scattering starting with the work of Hofstadter and collaborators and extending to broad new ranges of higher energies and momentum transfers in the recently reported results from the Stanford Linear Accelerator Center. From these the proton electromagnetic form factors have been deduced up to momentum transfers of $t = q^2 \sim 25\ (\mathrm{GeV}/c)^2$.

The original measurements in the very beginning showed that the proton was a rather fat, diffuse charge and current distribution with a root mean square radius of ~ 0.8f. Nambu first recognized the need for the existence of an isoscalar vector meson resonance, the ω^o, and Frazer and Fulco subsequently showed in detail the case for the isovector ρ^o in order to provide a theoretical basis for the form factor behavior. Indeed the early form factor work led to predictions that there existed vector mesons of sub-nucleonic mass, and from this grew the generally very successful vector dominance model.

However, as discussed by Wilson in a recent Comments (Vol. 1, No. 3, May 1967), a ρ^o dominant model for the isovector form factor is inadequate to fit the data accurately--and in particular it fails for the large t behavior which shows the form factor $G_M^{\,p}$ (see Wilson) falling at least as fast as $\frac{1}{t^2} \sim \frac{1}{q^4}$. To see this we turn to the canonical

(Comments on Nuclear and Particle Physics. 2, 36-40 (March-April 1968))

starting point of dispersion theory which expresses form factors as a sum of Yukawa-like terms [$q^2 > 0$ for scattering measurements]

$$G(q^2) = \int_{4m_\pi^2}^{\infty} \frac{s(\sigma^2)d\sigma^2}{\sigma^2 + q^2} \tag{1}$$

The spectral amplitude $s(\sigma^2)$ describes the exchange of one or of a few of the neutral vector mesons or resonant enhancements from the electromagnetic current to the proton line in Fig. 1. Each resonance contributes a bump to $s(\sigma^2)$ at its mass $\sigma^2 = M_r^2$, and evidently a cancellation between several resonances must be contrived in order to give

$$\int d\sigma^2 s(\sigma^2) \approx \sum_r s_r(M_r^2) = 0 \tag{2}$$

and lead to an asymptotic behavior decreasing as $1/q^4$ or faster. Thus ρ^o dominance alone in the isovector channel will not suffice. Also, such cancellations artificially arranged to accommodate data but without an underlying theoretical principle or dynamical raison d'etre are but faint beacons toward deeper understanding.

One very direct way to achieve a $1/q^4$ fall off is to construct a theoretical argument which orders you to multiply a Yukawa form for the vector meson propagator by a similar form vanishing as $q^2 \rightarrow \infty$ for the form factor describing the vector coupling to the nucleon line (see Fig. 2). Toward this end we can appeal to dispersion theory language

or to a Lagrangian formalism. The assumption that the entire hadronic electromagnetic current operator is identical to the vector meson field amplitude (the current-field identity) provides an explicit statement of what vector dominance means in a Lagrangian field theory. From either of these approaches we can derive the relation

$$G^{\gamma}(q^2) = \frac{m_V^{\,2}}{m_V^{\,2} + q^2} \; G^V(q^2) \tag{3}$$

where G^V is the vector meson-nucleon form factor and when subjected to a dispersion analysis would be treated as in Eq. (2). $G^V(q^2)$ contains contributions from all but the vector meson pole itself at $-q^2 = m_V^{\,2}$ and a priori there is no reason for its value at large q^2 to decrease as $1/q^2$.

In fact, the general dispersion approach has a very severe limitation when applied to a study of the behavior at large q^2. This is because the dispersion integral converges only very slowly and all contributions are essentially equally weighted in Eq. (1) up to large masses $\sigma^2 \sim q^2$. In contrast, a mean square radius calculation

$$< R^2 > \; \equiv \; 6 \int \frac{s(\sigma^2) d\sigma^2}{\sigma^4} \tag{4}$$

has a $1/\sigma^4$ convergence factor to enhance the low lying resonance terms. It may be reasonable to assume that only a few low lying resonance contributions dominate in Eq. (4) for $< R^2 >$ but it is certainly an extravagant optimism to extend that same assumption to calculating Eq. (1) for large values of $q^2 \gtrsim 10$ BeV^2. In other words, the ρ^0 dominant model may be fine when applied to processes involving real photons or virtual electromagnetic currents transferring $q^2 \lesssim 1(\text{GeV/c})^2$. However, for extrapolations to distant q^2 regions it is (not surprisingly) inadequate.

Experience has proved that theorists often find a very useful guiding light when most desperately needed by returning to what Gell-Mann has referred to as our theoretical laboratory of the Schrodinger equation. In this spirit we may compose an apologue of how it might have been had our experimental colleagues constructed the GeV electron accelerator twenty years earlier in the 1930's. Theorists would have been much less sophisticated back then and without the big apparatus of local quantum field theory and dispersion relations at our disposal, we would have fallen back quite naturally upon the Schrodinger equation to provide the formal basis of our conjectures. Being familiar then with the model of a nuclear atom and accepting the concept that a nuclear force field responsible for binding neutrons and protons to each other in a nucleus could give rise to a spatial distribution for the nucleon's electromagnetic structure, a bright graduate student might have argued as follows. The proton's electromagnetic form factor is the fourier transform of the charge distribution (neglecting spin and magnetic effects),

$$F(q^2) = \int \rho(r)\, e^{i\underline{q}\cdot\underline{r}}\, d^3r \tag{5}$$

For a point proton the charge density is a delta function, $\rho(r) = \delta^3(\underline{r})$ and $F(q^2) = F(0) \equiv 1$ is a constant independent of momentum transfer. Any structure in $\rho(r)$ introduces a q dependence and $F(q^2)$ generally decreases with increasing q^2 since the scattered waves do not all add coherently upon scattering from different points. Evidently as $q \rightarrow \infty$ in

Eq. (5), the behavior of $\rho(r)$ at the origin $r \to 0$ controls the behavior of $F(q^2)$. In fact, upon integrating over all momentum transfers,

$$\int^{\infty} d^3q \; F(q^2) = \int d^3r \rho(r) \int d^3q e^{i\underline{q}\cdot\underline{r}} = (2\pi)^3 \rho(0)$$

or

$$\int_0^{\infty} q^2 F(q^2) dq \propto \rho(0) \tag{6}$$

Now this student would have known, or perhaps quickly recalled upon opening the Pauli or Kramers treatise on quantum mechanics (or whichever text he had been nourished on in the 1930's) that $\rho(0)$ is finite or zero for any but pathological potentials--which means for any binding potentials less strongly attractive than $1/r^2$ as $r \to 0$. For example, $\rho(0) = |\psi(0)|^2 \propto r^{2\ell}$, $\ell = 0, 1, \ldots\ldots$ for solutions of a bound state in a $1/r$ potential as $r \to 0$. This says that it is "natural" for the integral in Eq. (6) to exist so that $F(q^2)$ decreases more rapidly than $1/q^3$, and loosely $F(q^2) \propto 1/q^4$ as $q \to \infty$.

There is a moral to this tale: There is something very simple about this particular feature of the rapid decrease of $F(q^2)$ with increasing q^2 that is not apparent and which does not shine to the fore in a dispersion approach. Let us return to the relativistic form of Eq. (1) and compute in the center of mass frame so that the invariant momentum transfer q^2 is purely a spatial momentum transfer. The condition of a finite charge density at the origin again is seen to lead to the superconvergence condition of Eq. (2)

$$\text{viz.} \quad \int e^{i\vec{q}\cdot\vec{r}}\, G(q^2)\, d^3q \;\equiv\; \rho(r) \;=\; 2\pi^2 \int_{4m_\pi^2}^{\infty} s(\sigma^2)\, d\sigma^2\, \frac{e^{-\sigma r}}{r}$$

or

$$\rho\,(r \to 0) \;\to\; 1/r \left[\int s(\sigma^2)\, d\sigma^2 \right]$$

Does this result mean we must accept the bootstrap idea of the democratic proton built as a compound system as opposed to its being knighted as an elementary aristocrat? Does it mean that the proton is more appropriately viewed as a bound structure of quarks?

GENERAL REFERENCES

1. Murray Gell-Mann and Fredrik Zachariasen, Phys. Rev. 124, 953 (1961).
2. Norman M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. 157, 1376 (1967).
3. Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies, Stanford Linear Accelerator Center, 1967.

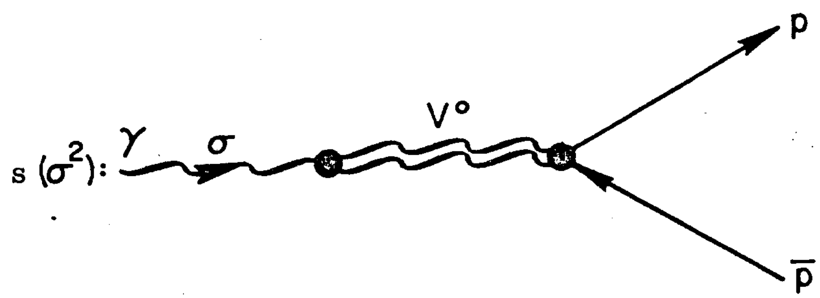

Fig. 1

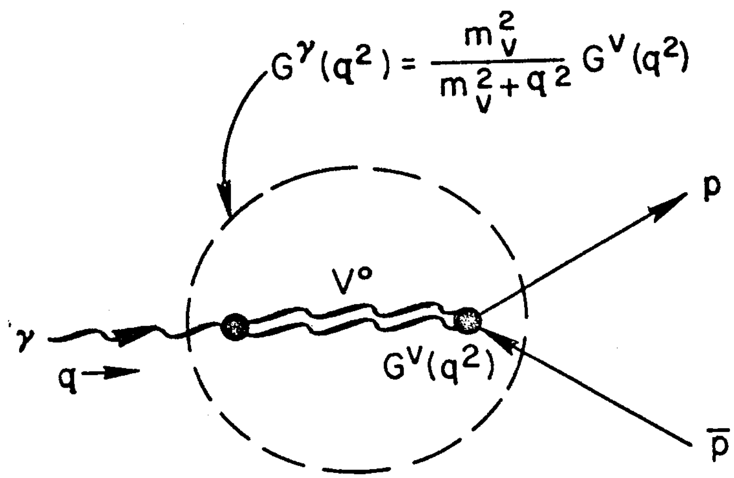

Fig. 2