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REGGE POLE HYPOTHESIS IN AMPLITUDES WITH PHOTONS

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Photopion production and Compton scattering cross sections from hydrogen have been studied with electron accelerators since the early 1950's and have provided crucial evidence for determining pion quantum numbers as well as establishing conclusively the existence of the 3-3 resonance. With the successful operation of the Cambridge Electron Accelerator and of the Deutsches Elektronen-Synchrotron at Hamburg as well as the initiation of experiments at SLAC during 1966, more recent interest has focused on the multi-GeV region and on the questions:

What clues do we see which may be joined with the full body of ideas and concepts that have proved of value in analyzing hadron reactions at high energies in order to provide some basis for an understanding of these photo initiated reactions? In particular, what experimental evidence is there indicating that the Regge pole, or moving pole, hypothesis for the scattering amplitude as a function of angular momentum j is applicable.

There are several well-known and major differences between the amplitudes for electromagnetic interactions with protons or nuclei and the amplitudes for purely hadronic interactions. This suggests that it would be wise to tread cautiously before we embrace all that pure and simple Reggeism offers and apply it indiscriminately to electromagnetic processes. The first of these differences is in the unitarity relation. Unitarity, or the requirement of probability conservation is expressed as a nonlinear equation of the following form: Let S denote the S-matrix and $\mathcal{J}$ the transition amplitude defined by

$$S = 1 + i\mathcal{J} .$$

Unitarity states that $S^{\dagger}S = 1$ or

$$i(\mathcal{J}^{\dagger} - \mathcal{J}) = \mathcal{J}^{\dagger}\mathcal{J} . \tag{1}$$

As a nonlinear relation, Eq. (1) limits how large a transition amplitude can grow. We are familiar with the meaning of this in potential scattering: $|\sin\delta\, e^{i\delta}| \leq 1$ no matter how large the phase shift becomes. When coupled with general conditions of analyticity and relativistic invariance from field theory, unitarity leads to severe constraints on the growth of two body transition amplitudes in the high energy limit $s \to \infty$. One of these constraints in fact sparked the application of the Regge pole ideas for studying high energy behavior: If "elementary particles" or quanta with fixed poles in the angular momentum plane at j = integer (≥ 0) are exchanged in the t (momentum transfer) channel, they lead to contributions to the transition amplitude $\propto s^j$. For $j > 1$ this brings us in conflict with unitarity, and there are even embarrassments in some circumstances for $j = 1$. According to the moving pole hypothesis, the angular momentum varies with t, $j = \alpha(t)$, and $\alpha(t) < 1$ for scattering processes with $t < 0$, in accord with unitarity.

However, for photon processes there is a small coupling parameter, the fine structure constant $e^2 = \frac{1}{137} \ll 1$, and generally we work to lowest order in e using perturbation theory. Unitarity, approximated to lowest order in e in Eq. (1), poses no threat or offers no aid to the high energy growth since in the perturbation approximation it is an identity in e which can be simply scaled out of Eq. (1). Perhaps then there are fixed poles in j in electromagnetic amplitudes, or contributions from Feynman graphs in which "elementary particles" of fixed $j > 0$ are exchanged, and perhaps the real part of the amplitude is much larger than the imaginary part at high energies--i.e., for $\delta \ll 1$, $\sin\delta\, e^{i\delta} \simeq \delta$ is real.

A second difference between electromagnetic and hadron cross sections is that we are dealing with a conserved current interaction when photons are involved. All the ramifications of current conservation are not clear, but in some examples t channel exchanges must be coupled with s-channel ones, or contact interactions. In the conventional Regge-peripheral view applied to high s low t reactions we assume that the cumulative effect of many high partial waves (large impact parameters) as described by the t channel exchanges plays the dominant role with the low angular momentum contributions to the s channel amplitude playing a minor one. It remains to be established that this view will still be applicable in photon processes when s and t channel contributions are related by current conservation, and we shall explore this question in a subsequent installment.

After these general preliminaries, let us see where we stand with the data and its implications for Regge behavior. Consider first two examples of the successful application of Regge ideas to electromagnetic amplitudes, one involving asymptotic behavior and the other the appearance of "non-sense" zeros in the residues.

As described in the May issue of Comments, Harari has used Regge pole asymptotics for t channel exchanges in virtual forward Compton scattering and standard assumptions as to the positions of the intercepts of the trajectories on which they lie to give a natural and simple explanation of the failure of calculations of $\Delta I = 1$ mass splittings such as the neutron-proton mass difference and of the success of the $\Delta I = 2$ ones such as the $\pi^+ - \pi^0$ mass splitting. In analogy to the discussions by Chew and Goldberger in the March issue of Comments about the pion charge exchange scattering, there is a dip in the π^0 photoproduction angular distribution at a momentum transfer of $t = -0.6\ \mathrm{GeV}^2$. This persists for photon energies ranging from 2 to 5 GeV and suggests an origin associated with a non-sense point in the presumably dominant ω-exchange trajectory. In the crossed, or t, channel the $\gamma + \pi^0$ form a state of unit helicity since the photon is transverse. However, at a value of t at which the ω trajectory crosses $j = 0$ it acts like a spin 0 particle under the three dimensional (Euclidean) rotations and cannot support a unit of helicity--thus the non-sense zero. By the way, inelastic μ or e scattering can always be analyzed in terms of known proportions of longitudinal photons for which this non-sense zero does not occur. An experiment is hereby advertized! An appreciable fraction of longitudinal photons can be achieved with large energy losses and their filling in of this non-sense zero studied.

Next let us turn to the example of forward Compton scattering in which simple Reggeism must be supplemented by a requirement of a fixed pole or a singular residue as first discussed by Mur. We expect, in complete analogy with πp and pp elastic scattering, and as also observed for forward photoproduction of the ρ^0, that we will find a forward diffraction peak corresponding to Pomeron exchange. However, a Pomeron leading to a constant total cross section, σ_t, at high energies must have a Regge trajectory intersecting at $\alpha_p(0) = 1$ and thus behave under three dimensional rotations as a vector. It can then not couple to 2γ's any more than a vector π^0 could have decayed to 2γ's. More precisely in the forward direction the photon cannot flip helicity and an incident right circularly polarized γ (rhγ) must emerge as a rhγ simply by angular momentum conservation. Upon crossing to the t channel and the process $\gamma + \gamma \rightarrow p + \bar{p}$, the emerging rh$\gamma$ crosses to a ℓhγ incident and the two incoming γ's form a system with two units of helicity. This cannot, however, be deposited upon a Pomeron of unit spin when $\alpha_p(0) = 1$.

If the Pomeron does not couple or if we must contrive to make $\alpha_p(0) < 1$ in order to restore its coupling, we do not predict a constant σ_t at high energies and we lose in an instant the motivating charm of the Pomeranchuk trajectory in Reggeism. Originally it was designed to reproduce in hadron physics the classical diffraction picture in the classical problem of light scattering. Thus we must give the Pomeron a singular residue to cancel its non-sense zero at t = 0 for forward Compton scattering, or we must abandon the classical diffraction analogy of $\alpha_p(0) = 1$.

Once this Pandora's box is opened, we have a new ball game and several experiments acquire enhanced interest. One is a study of forward Compton scattering at low energies ($< \mu = 140$ MeV) which can reveal whether we are led to the requirement of a subtraction constant at infinite energy for the real part of the forward non-spin flip Compton amplitude from a proton. To amplify this observation we write the amplitude for forward Compton scattering from a proton as given by Gell-Mann, Goldberger, and Thirring

$$f(\nu) = f_1(\nu)\, \underline{e}'^{*} \cdot \underline{e} + i\underline{\sigma} \cdot \underline{e}'^{*} \times \underline{e}\, \nu f_2(\nu) \qquad (2)$$

where ν is the photon energy and $\underline{e}$ and $\underline{e}'$ are the transverse polarization vectors of the incident and outgoing photon. The dispersion relation for $f_1(\nu)$ usually appears as

$$f_1(\nu) = -e^2/M + \frac{\nu^2}{2\pi^2} \int_\mu^\infty \frac{d\nu' \sigma_t(\nu')}{\nu'^2 - \nu^2} \tag{3}$$

where the exact classical Thomson limit is introduced as a subtraction constant at zero energy and $\sigma_t(\nu')$ is the total photoabsorption cross section by the proton. We are working to lowest order in $e^2 = \frac{1}{137}$ but to all orders in the strong interactions and the threshold in the dispersion integral is $\mu \approx 140$ MeV, the threshold for photopion production. In the present context, total cross sections means the photoabsorption to form hadron final states and the large but well understood Bethe-Heitler processes are excluded. Whether or not the spin dependent amplitude $f_2(\nu)$ requires a subtraction in its dispersion relation, its zero energy limit is exactly known in terms of the proton charge, mass, and anomalous moment k_p to be

$$f_2(0) = -\frac{\alpha}{M^2} k_p^{\,2} \; ; \quad k_p = 1.79 \tag{4}$$

Combining equations (2), (3), and (4), we have an exact result for the forward angle differential elastic Compton cross section

$$\lim_{\nu^2 \to 0} \left(\frac{d\sigma}{d\Omega}\right)_{0^\circ} = \left|f_1(\nu)\right|^2 + \nu^2 \left|f_2(\nu)\right|^2$$

$$= \frac{e^4}{M^2} \left[1 - \left(\frac{\nu}{\mu}\right)^2 \left\{\frac{\mu^2 M}{e^2 \pi^2} \int_\mu^\infty \frac{d\nu' \sigma_t(\nu')}{\nu'^2} - \frac{\mu^2}{M^2} k_p^{\,4}\right\} + 0\left[\left(\frac{\nu}{\mu}\right)^4\right]\right] \tag{5}$$

The coefficient of the low energy slope is already known very accurately from measured photoabsorption cross sections up to 6 GeV since the integral converges rapidly and to one significant figure

$$\frac{\mu^2 M}{e^2\pi^2}\int_{\mu}^{6\ \mathrm{GeV}} \frac{d\nu'\sigma_t(\nu')}{\nu'^2} - \frac{\mu^2}{M^2}k_p^{\ 4} = +0.7\ . \tag{6}$$

Further refinement in this number will result from measurements at higher energies but in any case the changes will be small ($\lesssim$ 10%). Evidently there is a sizable and measurable slope with $(\text{energy})^2$ to be measured and checked against the very general assumptions that are the input into the forward dispersion relations for scattering of light (relativity, macroscopic causality, and unitarity). The only possible source of disagreement between the predictions of Eqs. (5) and (6) and experiment, short of a theoretical catastrophe of the highest order, could come about as follows: Due to the contribution from a t channel exchange of an "elementary particle" of fixed spin 2 contributing to the real part of the forward spin independent amplitude, we must add a term $\lambda\nu^2$ on the right hand side of Eq. (3)--or more generally a real polynomial in ν^2 without disturbing the low energy Thomson limit. We may not welcome such a contribution, and we may not understand whence it originates, but evidently it would not be the first appearance of corrections to simple Reggeism in processes with photons. On general principles it cannot be ruled out--in particular, we cannot fall back on the usual unitarity arguments that are invoked at this point in hadron amplitudes since we are working only to lowest order in e^2. I view an experimental confrontation of Eqs. (5) and (6) as a problem of very high urgency in "medium energy" photon physics.

GENERAL REFERENCES

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