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SUPERCONVERGENCE AND THE CALCULATION OF ELECTROMAGNETIC MASS DIFFERENCES

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The assumption that the high energy behavior of the forward amplitude for Compton scattering on hadrons is controlled by the leading trajectory in a Regge pole description has led recently to several new and significant results.

The first of these is the work of Harari who has given important clarification to long-standing puzzles in the electromagnetic mass differences between particles (hadrons) within a given isospin multiplet. The essential idea is as follows:

1) To lowest order in $\alpha = \frac{1}{137}$ the electromagnetic self energy of a hadron is given by the amplitude corresponding to the following graph which may be viewed as the amplitude for the forward Compton scattering of a virtual photon of "mass" q^2 and energy $q_0 = \nu$ from a hadron. In forming the self energy bubble the photon is tied back onto itself, and we integrate over all photon four momenta in the manner of a Feynman graph so that the formal expression for the self energy takes the form

$$\Delta M = \frac{1}{8\pi^2} \int_{-\infty}^{\infty} \frac{d^4q}{q^2 + i\epsilon} T_\beta{}^\beta(q^2,\nu)$$

where $\epsilon^\alpha \epsilon^\beta T_{\alpha\beta}(q^2,\nu)$ is the forward non-spin-flip Compton amplitude for a virtual photon with polarization ϵ, mass q^2, and energy ν. The blob

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in the picture represents all possible states that are formed by the hadron plus virtual photon system. A perturbation expansion of the strong interactions operating within this blob will be useless in general since such expansions fail to converge rapidly, if at all.

2) Cottingham in 1963 made the important observation that the contour of integration in Eq. (1) can be rotated to the imaginary axis in the complex ν-plane. This is because the singularities of the Feynman propagator lie in the second and fourth quadrants, viz

$$q^2 + i\epsilon = [\nu - |q| + i\epsilon][\nu + |q| - i\epsilon]$$

and so do those of the scattering amplitude as a function of energy ν. This rotation allows us to express ΔM completely in terms of scattering amplitudes for space-like photons, viz

$$q^2 = \nu^2 - |q|^2 \rightarrow (i\nu)^2 - |q|^2 < 0$$

In principle, therefore, it is possible to use experimental electron scattering data to compute the integral for ΔM, since for elastic and inelastic electron scattering the invariant four momentum transfer to the target hadron is space-like, i.e., $q^2 = (\Delta E)^2 - |q|^2 < 0$.

3) In practice what one does to disperse the Compton amplitude in the energy variable for fixed photon mass $q^2 < 0$ and to compute it in terms of its absorptive parts, i.e., in terms of the cross sections for inelastic electron scattering. Since data on inelastic electron scattering exists only for nucleon targets and over a limited range of energy transfers ν and invariant momentum transfers

$q^2 < 0$ to the nucleon, success will be achieved with this program in calculating the neutron-proton mass difference as well as the ΔM for other hadron multiplets only if the dispersion integrals are dominated by the contributions of the first few low-lying states. If many high-lying ones play a dominant role, the convergence of the dispersion integrals will be determined by the high energy behavior of the absorptive parts--i.e., by the inelastic scatterings to form massive hadron states in the blob of Fig. 1. Their contributions may require subtractions in the dispersion relations, in which case there would be no hope of calculating ΔM exclusively in terms of the first few low-lying states of the hadron.

4) In the simple approximation of retaining only a few low-lying states in the calculation of the forward Compton amplitude, one obtains the correct sign and magnitude for the $\pi^+ - \pi^0$ mass difference as well as for a particular combination of the Σ mass difference, viz

$$\begin{aligned} m(\pi^+) - m(\pi^0) &\sim (5\pm1)\ \text{MeV} = 4.61\ \text{MeV} \quad \text{(observed)} \\ m(\Sigma^+) + m(\Sigma^-) - 2m(\Sigma^0) &\sim (1.5\pm0.5)\ \text{MeV} = 1.76\pm0.23\ \text{MeV} \quad \text{(observed)} \end{aligned} \tag{1}$$

In the same approximation the calculations of the neutron-proton, the $K^0 - K^+$, the $\Sigma^- - \Sigma^+$ and $\Xi^- - \Xi^0$ mass differences all fail, in some cases even leading to the wrong sign.

The outstanding problem is the sign of the neutron-proton mass difference:

$$m(n) - m(p) = 1.3\ \text{MeV} .$$

Naively, one expects the proton to be more massive because of its electrostatic self-energy. Back in 1954[4], Feynman and Speisman first pointed out that the currents producing the nucleon magnetic moments could reverse the sign of their mass difference. Retaining only the nucleon pole contribution to the blob of Fig. 1, as illustrated in Fig. 2, they required that contributions of very massive photons with $q^2 \gtrsim (\text{few BeV})^2$ play a prominant role to achieve this sign reversal. Since this notion is at variance with the picture of the nucleon structure emerging from the electron scattering studies according to which the nucleon electromagnetic form factors severely suppress contributions for $q^2 \gtrsim \frac{1}{2}(\text{BeV})^2$ the problem remained somewhat of a mystery, not resolved by retaining just the low-lying resonance excitation of the nucleon in the intermediate state in addition to the nucleon pole of Fig. 2.

5) The central observation of Harari was that the successful ΔM calculations of Eq. (1) had this essential difference with the unsuccessful ones mentioned above: The forward Compton amplitudes may be broken down into their different contributions according to whether there is an isospin exchange $\Delta I = 1$ or 2 from the photon line to the hadron line through the blob in Fig. 1. Since the two photons (initial and final) in the Compton scattering each carry isospin of 0 and 1 unit, they can transfer up to 2 units of isospin to the hadron-anti-hadron pair as we view the Compton scattering in the crossed channel $\gamma + \gamma \rightarrow H + \overline{H}$. For I = 0 exchange, all members of a particular isospin multiplet are shifted by the same amount and no mass splitting is produced. The reactions

of Eq. (1), the successful ones, all correspond to an exchange of $\Delta I = 2$. The unsuccessful ones all correspond to $\Delta I = 1$. In particular, this includes the n-p mass difference which admits no $\Delta I = 2$ since the neutron and proton form an $I = \frac{1}{2}$ doublet. Evidently, the dispersion amplitudes must converge much better for $\Delta I = 2$ than for $\Delta I = 1$ exchanges in the crossed (or t-channel) reactions. Why? How can one understand such a difference in their asymptotic behaviors?

6) Harari appeals at this point to the most basic and successful prediction of Regge pole theory: Forward non-spin-flip amplitudes with a given set of crossed-(or t-)channel quantum numbers behave at high energies ν as $\nu^{\alpha(0)}$ where $\alpha(0)$ is the intercept at zero momentum transfer $t = 0$ of the leading Regge trajectory with appropriate quantum numbers being exchanged from photon to hadron line in the t-channel. For $\Delta I = 1$ exchange, the energy dependence is determined by the intercept of the leading trajectory with quantum numbers $I = 1$, charge conjugation $C = 1$ (forming 2 photons) and parity $P = (-)^J$. This is the trajectory on which the A_2 meson lies and which has intercept $\alpha_{A_2}(0) \sim 0.4 > 0$. On the other hand, for $\Delta I = 2$ exchange, one searches in vain for low-lying $I = 2$ mesons and in their absence assumes, following deAlfaro, Fubini, Furlan, and Rossetti that all $I = 2$ trajectories have $\alpha_{I=2}(0) < 0$; i.e., we have superconvergent dispersion relations for $I = 2$ channel exchanges.

The consequence of these assumptions is that the $\Delta I = 2$ amplitudes obey unsubtracted dispersion relations, and it is reasonable to find their absorptive parts dominated by low-lying states. In contrast, the dispersion integral for the $\Delta I=1$ amplitudes requires subtractions. It is due to the presence of this additional and essentially unknown subtraction term which results from high-energy contributions that one cannot reduce the n-p and all $\Delta I=1$ mass difference calculations to contributions of simply the low-lying mass states. On the basis of simple estimates, Harari concludes that the subtraction term may well have the proper sign and magnitude to fix up what is missing from the naive low-mass dominant calculations in order to reproduce the observed neutron-proton mass difference.

Independent of this estimate, the central point remains that the success of the $\Delta I=2$ calculations as well as the failure of the $\Delta I=1$ ones on the basis of a simple approximation of low-mass state dominance now has a straightforward explanation in terms of the same basic ideas of Regge-pole theory that have achieved wide success in high-energy scattering analyses. Both the Feynman-Speisman result and the "tadpoles" with $\Delta I=1$ that Coleman and Glashow introduced earlier in their calculation of the mass-splittings among isospin multiplets are now interpretable in terms of standard dispersion theory ideas and high-energy behaviors of forward scattering amplitudes.

The superconvergence assumption on the high-energy behavior of amplitudes involving isospin exchanges in the t channel—i.e., the assumption that $\alpha_{I=2}(0)<0$ introduced by de Alfaro et al.—leads to a number of additional sum rules and coupling-constant and mass relations of considerable interest and impressive success. It has also been noted that they provide alternative derivations for relations otherwise derived from current commutation relations or specific quark models.

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will be found full bibliographies)

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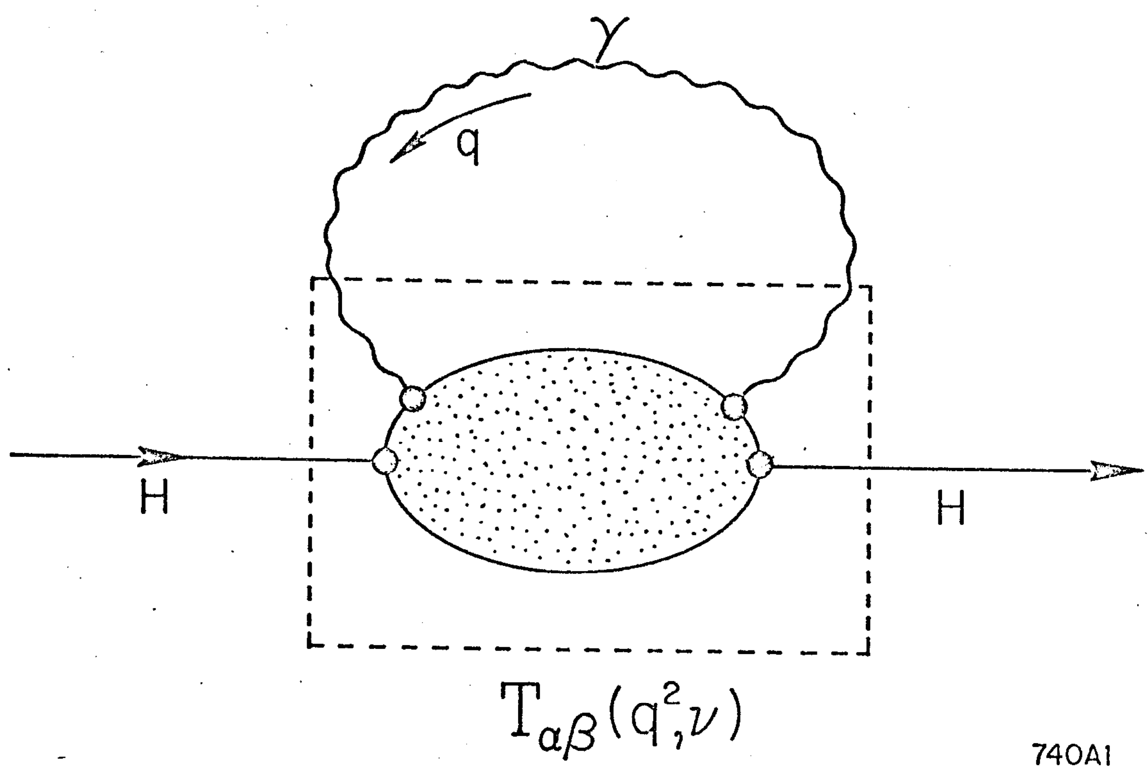

Fig. 1

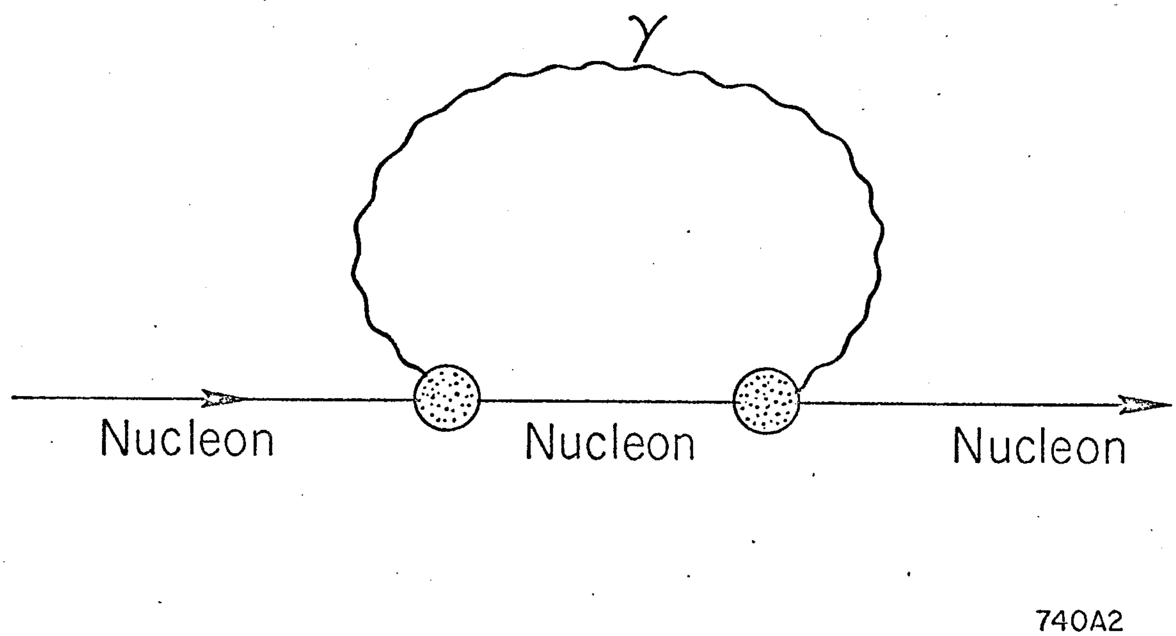

Fig. 2