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A STATISTICAL MODEL FOR HIGH ENERGY REACTIONS*

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ABSTRACT

A statistical model for high energy reactions is constructed using the internal symmetry invariance properties of the scattering amplitudes which yields simple predictions for average branching ratios. The model is so constructed as to be valid no matter what invariance group or reaction one considers and also to be independent of the channel through which one views the reaction. Branching ratio predictions are made for reactions of the type baryon + meson → baryon + meson and photon + baryon → meson + baryon.

The statistical model for nuclear reactions which arose from Bohr's[1] original suggestion of a compound nucleus picture has been extensively studied both experimentally and theoretically and its validity in certain experimental regimes well established[2]. In recent years further confirmation of its applicability in these situations has been found in the observation of the statistical fluctuations, predicted by Ericson[3], which are an essential concomitant of any statistical phenomenon. The use of statistical concepts in the analysis of high energy collisions between elementary particles seems to have originated with Fermi[4], but in spite of a long period of development, the applicability of such concepts to reactions at accelerator energies has always been in some doubt. Particular interest was recently stimulated in such models by the observations of Cocconi et al.[5] on large angle elastic proton-proton scattering which seemed to show an energy dependence such as would be expected on the basis of a statistical model. Valid questions concerning such an interpretation have been raised[6], however, and a further shadow over this interpretation results from the failure to observe the Ericson fluctuations[7].

The role which might be played by internal symmetries in a statistical model of reactions does not appear to have been raised in a general way until very recently. Drell, Speiser, and Weyers[8] have pointed out that internal symmetries such as charge-independence (isospin or SU_2 invariance) or charge symmetry in nuclear phenomena and SU_2 (isospin), SU_3 (eightfold way) or possible higher symmetries in elementary particle phenomena, could play a decisive role in the determination of average branching ratios for reactions. The statistical assumptions which they chose to introduce for the purpose of implementing this observation can

be subjected to some criticism, and it is this question which is examined in the present paper; it is a question with considerable intrinsic interest even apart from any pragmatic questions as to the applicability of the statistical model to any particular situation.

The approach of DSW consists in observing that in the presence of an internal symmetry group G, the ingoing and outgoing particles in a reaction are each a member of a multiplet (or supermultiplet) of particles which is associated with some irreducible representation of the symmetry group. The amplitude for going from the ingoing to the outgoing state received contributions from many possible "paths" where each such "path" can be considered to be associated with or correspond to a "compound state" which carries among its labels those appropriate to an irreducible representation of the symmetry group G. The sum of the amplitudes arising from compound states belonging to any one irreducible representation of G is the "reduced amplitude" associated with the particular irreducible representation. The desired amplitude is then given as a linear combination of these reduced amplitudes, one associated with each irreducible representation which is common to the reduction of the product representations formed of the ingoing and outgoing particle representations, respectively. The coefficients in this linear combination are just products of Clebsch-Gordan coefficients associated with these reductions. To illustrate, consider a reaction in which two particles go into two particles. We write the reaction as

$$a(A) + b(B) \rightarrow c(C) + d(D) \qquad (1)$$

where A and B represent the irreducible representations (multiplets) to which the incoming particles belong and a and b designate the particular

member of the multiplet. Similarly C and D represent the multiplets and c and d the members corresponding to the outgoing particles. The kinematic variables as, for example, the momenta and helicities of each of the particles, are presumed fixed. The amplitude for the reaction (1) which we represent as $\langle CcDd|M_s|AaBb\rangle$, can then be written as[9]

$$\langle CcDd|M_s|AaBb\rangle = \sum_{X\tilde{\xi}\xi x} \langle CcDd|X(\tilde{\xi})x\rangle \; S_X(\tilde{\xi}\xi) \; \langle X(\xi)x|AaBb\rangle. \qquad (2)$$

Here $\langle X(\xi)x|AaBb\rangle$ is the Clebsch-Gordan coefficient associated with the reduction of the product of the representations A and B into irreducible representations X, with x representing the member of the multiplet X and ξ is a symbol used to designate and distinguish the same irreducible representation X if it occurs more than once in the reduction. Similarly $\langle CcDd|X(\xi)x\rangle$ is the (conjugate) Clebsch-Gordan coefficient associated with the reduction of the product of the representations C and D into irreducible representations X distinguished, if occurring multiply, by the symbol $\tilde{\xi}$. The quantities $S_X(\tilde{\xi}\xi)$ are the reduced amplitudes referred to above and are associated with those irreducible representations which are common to the reduction of the products of representations associated with the ingoing and outgoing particles, respectively. The Clebsch-Gordan coefficients are in principle completely determined by purely mathematical considerations once the invariance group G and its irreducible representations are specified. All considerations of dynamics are contained in the reduced amplitudes $S_X(\tilde{\xi}\xi)$.

Now, apart from purely kinematical coefficients, the cross section for the reaction (1) is determined by the absolute square of the amplitude (2) and is therefore a linear combination of products of the reduced

amplitudes and their complex conjugates:

$$\sigma(ab\to cd) \propto \sum_{X\tilde{\xi}\xi} \sum_{X'\tilde{\xi}'\xi'} F_s[X'(\tilde{\xi}'\xi')X(\tilde{\xi}\xi):CcDdAaBb] \left[S_{X'}^{*}(\tilde{\xi}'\xi')\, S_X(\tilde{\xi}\xi)\right] \tag{3a}$$

where the functions F_s are sums of products of four Clebsch-Gordan coefficients:

$$F_s[X'(\tilde{\xi}'\xi')X(\tilde{\xi}\xi):CcDdAaBb] = \sum_{x'x} \langle CcDd|X'(\tilde{\xi}')x'\rangle^{*}\langle CcDd|X(\tilde{\xi})x\rangle \langle X'(\xi')x'|AaBb\rangle^{*}\langle X(\xi)x|AaBb\rangle \tag{3b}$$

These results are completely general and depend only on the validity of the internal symmetry group as an invariance group; no statistical considerations have entered so far. If the reduced amplitudes $S_X(\tilde{\xi}\xi)$ are known, the branching ratios corresponding to decays into different members c,d of the final multiplets C,D are fixed. It is in the absence of such information that one might hope to employ statistical assumptions about these amplitudes to gain statistical information about branching ratios.

One manner in which statistical features may enter into these considerations is in the instance that each of the reduced amplitudes $S_X(\tilde{\xi}\xi)$ can be considered to arise as the sum of contributions corresponding to many compound states each belonging to the same irreducible representation X of the symmetry group. If the statistical properties of these contributions could be correctly characterized, it would be possible to calculate not only relative average cross sections for different branchings of a reaction but also the fluctuations about these averages. A complete characterization of these statistical

properties would in general require an appropriate statistical ensemble to be specified. The calculation of average branching ratios, however, requires only that one be able to assign statistically averaged values to products of the form $S^*_{X'}(\tilde{\xi}'\xi')S_X(\tilde{\xi}\xi)$, which may be an easier task.

Let us examine first what kind of information must be known in order to make any reasonable guess as to an "average" value like $\langle S^*_{X'}(\tilde{\xi}'\xi')S_X(\tilde{\xi}\xi)\rangle$. Now the Clebsch-Gordan coefficients such as $\langle X(\xi)x|AaBb\rangle$ which occur in the "definition" of the reduced amplitudes $S_X(\tilde{\xi}\xi) \equiv \langle X(\tilde{\xi})x|M_s|X(\xi)x\rangle$ depend in general on a choice of the overall phase of the set of states $|X(\xi)x\rangle$; that is, on an overall phase factor in the selection of a basis in that invariant subspace of the product space $A\otimes B$ associated with the irreducible representation X. Since the choice of this overall phase is completely arbitrary, it would appear that one would have to have complete information about this choice before one could hazard a guess as to the value of an average of the kind required above. One could argue, however, that there exists a statistical assumption which is consistent with any choice of these phases, namely the assumption[10]

$$\langle S^*_{X'}(\tilde{\xi}'\xi')S_X(\tilde{\xi}\xi)\rangle = \mathcal{S}_X\delta_{X,X'}\delta_{\tilde{\xi}',\,\tilde{\xi}}\,\delta_{\xi',\,\xi}\quad , \tag{4}$$

where $\mathcal{S}_X$ depends (for given A, B, C, D)[11] on the irreducible representation X but not on X', ξ, ξ', or $\tilde{\xi}'$. Indeed, there seems to be no reasonable alternative to this statistical assumption.[12] On the basis of it, the statistical average of Eq. (3) yields

$$\langle\sigma(ab \to cd)\rangle \propto \sum_X \mathcal{F}_s(X)\mathcal{S}_X \tag{5a}$$

where

$$\mathcal{F}_s(X) = \sum_{x'x} \sum_{\tilde{\xi}\xi} [\langle X(\xi)x'|AaBb\rangle^* \langle CcDd|X(\tilde{\xi})x'\rangle^* \langle X(\xi)x|AaBb\rangle \langle CcDd|X(\tilde{\xi})\, x\rangle] \tag{5b}$$

and

$$\mathcal{S}_X = \langle S_X^*(\tilde{\xi}\xi) S_X(\tilde{\xi}\xi)\rangle \quad . \tag{5c}$$

The complete solution of the average branching ratio problem requires then the determination of the dependence of the weights $\mathcal{S}_X$ on the irreducible representations X; or more specifically, the dependence of the relative weights $\mathcal{S}_X : \mathcal{S}_{X'}$, where X and X' are both irreducible representations of the same group G. This is the more difficult part of the problem.

Before attempting a solution, one can ask what form an answer might take. If we ask that we give an alogarithm for the rates $\mathcal{S}_X : \mathcal{S}_{X'}$, valid for every irreducible representation of every compact group G, it is clear that the result must be a function of such numbers as can be associated with every irreducible representation of every compact group. We have been able to think of only one such universal number which is of any reasonable value to us - the dimensionality of the irreducible representation. This suggests that $\mathcal{S}_X : \mathcal{S}_{X'} = f(\Delta_X) : f(\Delta_{X'})$ where Δ_X is the dimensionality of the irreducible representation X. In fact, the result which we shall obtain has this form with f a universal function independent of the group G, namely: $f(\Delta_X) = \frac{C}{\Delta_X}$ where C is a constant independent of X.

We have sought a basis on which to find or define reasonable statistical ensembles that in some way determine the relative weights to be given to different irreducible representations of the same group. The

only approach which provided a definite result was one based on an idea which we now describe.

The decomposition of reaction amplitudes into contributions associated with compound states belonging to particular irreducible representations of the invariance group G; that is, the reduced amplitudes, is not the only decomposition along group theoretical lines which is possible. It is also possible to decompose such amplitudes into contributions which can be considered to arise from the exchange of systems between the two interacting systems, and the exchanged systems can also be classified according to irreducible representations of the invariance group to which they are associated. The procedure has been outlined in previous papers[13] and will be discussed at length in a forthcoming publication[14]. We remark simply that the reduced amplitudes which are designated $S_X(\tilde{\xi}\xi)$ in the case already discussed are replaced by reduced amplitudes $T_Y(\tilde{\eta}\eta)$ and $U_Z(\tilde{\zeta}\zeta)$ in the case of the two types of exchange (t-exchange and u-exchange). If again we assume that $\langle T_{Y'}^{*}(\tilde{\eta}'\eta')\, T_Y(\tilde{\eta}\eta)\rangle$ and $\langle U_{Z'}^{*}(\tilde{\zeta}'\zeta')\, U_Z(\tilde{\zeta}\zeta)\rangle$ shall be independent of the phases involved in the definition of the Clebsch-Gordan coefficients then the analog of Eq. (5a) would be

$$\langle\sigma(ab \to cd)\rangle \propto \sum_Y \mathcal{F}_t \mathcal{J}_Y \tag{6a}$$

$$\propto \sum_Z \mathcal{F}_u \mathcal{U}_Z \tag{6b}$$

where

$$\mathcal{F}_t \equiv \sum_{y'y} \sum_{\tilde{\eta}\eta} [\langle Y(\eta)y'|Aa\bar{C}\bar{c}\rangle^{*} \langle \bar{B}\bar{b}Dd|Y(\tilde{\eta})y'\rangle^{*} \langle Y(\eta)y|Aa\bar{C}\bar{c}\rangle \langle \bar{B}\bar{b}Dd|Y(\tilde{\eta})y\rangle]$$

and

$$\mathcal{F}_u \equiv \sum_{z'z} \sum_{\tilde{\zeta}\zeta} [\langle Z(\zeta)z'|Aa\bar{D}\bar{d}\rangle^* \langle Cc\bar{B}\bar{b}|Z(\tilde{\zeta})z'\rangle^* \langle Z(\zeta)z|Aa\bar{D}\bar{d}\rangle \langle Cc\bar{B}\bar{b}|Z(\tilde{\zeta})z\rangle] .$$

Here $|\bar{C}\bar{c}\rangle$, for example, represents the conjugate state to $|Cc\rangle$ and describes the antiparticle $\bar{c}$.

The weights $\mathcal{S}_X$, $\mathcal{T}_Y$ and $\mathcal{U}_Z$ are not independent. Consider specifically the u-channel representation for the cross section in Eq. (6b). The s-and u-channel reduced amplitudes are related by[14]

$$S_X(\tilde{\xi}\xi) = \sum_{Z\tilde{\zeta}\zeta} Q_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta)\, U_Z(\tilde{\zeta}\zeta) \tag{7}$$

where the s- to u-channel crossing matrix $Q_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta)$ is defined by

$$Q_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta) \equiv (1/\Delta_X) \sum_{xz} \sum_{\substack{ab \\ cd}} [\langle X(\tilde{\xi})x|cd\rangle \langle c\bar{b}|Z(\tilde{\zeta})z\rangle \langle Z(\zeta)z|a\bar{d}\rangle \langle ab|X(\xi)x\rangle] . \tag{8}$$

Thus, multiplying Eq. (7) for $S_X(\tilde{\xi}\xi)$ by its complex conjugate and then taking the ensemble average (as defined earlier) of the product, we find that

$$\mathcal{S}_{X^\delta X';X^\delta \tilde{\xi}';\tilde{\xi}^\delta \xi';\xi} \equiv \langle S^*_{X'}(\tilde{\xi}'\xi')\, S_X(\tilde{\xi}\xi)\rangle = \sum_{Z\tilde{\zeta}\zeta} Q^*_{X'Z}(\tilde{\xi}'\xi'|\tilde{\zeta}\zeta)\, Q_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta)\mathcal{U}_Z \tag{9}$$

where, by assumption, $\mathcal{U}_Z$ depends only on the dimensionality, Δ_Z. From the orthogonality properties of the Clebsch-Gordan coefficients in terms of which the crossing matrices $\underline{Q}$ are defined, it is easy to verify that these matrices must satisfy the property;

$$\Delta_X\, Q^*_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta) = \Delta_Z\, Q^{-1}_{ZX}(\tilde{\zeta}\zeta|\tilde{\xi}\xi) , \tag{10}$$

Using Eq. (10), Eq. (9) becomes

$$[\Delta_X \mathcal{S}_X(\Delta_X)]\delta_{X',X}\,\delta_{\tilde{\xi}',\tilde{\xi}}\,\delta_{\xi',\xi} = \sum_{Z\tilde{\zeta}\zeta} Q_{XZ}(\tilde{\xi}\xi|\tilde{\zeta}\zeta)\, Q^{-1}_{ZX'}(\tilde{\zeta}\zeta|\tilde{\xi}'\xi')\,[\Delta_Z \mathcal{U}_Z(\Delta_Z)] \quad . \tag{11}$$

As it stands, Eq. (11) is not sufficient to determine the relative dependence of either $\mathcal{S}_X$ or $\mathcal{U}_Z$ on their respective dimensionalities. We need to know at least how the dependences of $\mathcal{S}_X$ and $\mathcal{U}_Z$ are related. We shall assume that the dependence of $\mathcal{S}_X$ on Δ_X is the same as the dependence of $\mathcal{U}_Z$ on Δ_Z; that is,

$$\mathcal{S}_X(\Delta_X) = \mathcal{U}_X(\Delta_X) \quad . \tag{12}$$

From a purely pragmatic point of view, this is the simplest assumption one can make for it treats the s-and u-channels equally. But also it seems appropriate that a statistical assumption should not depend on the channel through which one views a reaction because in many situations ($\pi^0\ \pi^0$ elastic scattering, for example) the various channel reactions are indistinguishable. With this assumption Eq. (11) yields, for a given invariance group and set of the representations A, B, C and D, a linear set of equations for the weights $\mathcal{S}_X$. In general, these equations do not have a unique solution. It is clear, however, that $\mathcal{S}_X = C/\Delta_X$, where C is a constant independent of X, is always a possible solution of this set of equations. Also, in some particular situations, it is possible to prove that this is the only solution[15]. Thus, although Eqs. (11) do not generally admit a unique solution, $\mathcal{S}_X = C/\Delta_X$ is the only solution which is <u>universally valid no matter what invariance group or reaction one considers and is independent of through which channel one views the reaction.</u> The

DSW assumption which in our notation amounts to taking $\mathcal{L}_X = \mathcal{U}_X = \text{const.}$, is not channel independent.

Taking $\mathcal{L}_X = C/\Delta_X$, the average branching ratios become

$$\frac{\langle\sigma(ab \to cd)\rangle}{\langle\sigma(ab \to c'd')\rangle} = \frac{\sum_X \mathcal{F}^X_{s(ab\to cd)} \; [1/\Delta_X]}{\sum_{X'} \mathcal{F}^{X'}_{s(ab\to c'd')} \; [1/\Delta_{X'}]} \tag{13}$$

where the functions $\mathcal{F}^X_s$ are sums of products of Clebsch-Gordan coefficients which can be calculated once an invariance group is chosen.

Thus far we have concentrated on calculating average branching ratios. But the same arguments can be applied to any ratio of average cross sections for which corresponding particles in the two reactions belong to the same internal symmetry multiplet. That is, under the same assumptions as above, we also have

$$\frac{\langle\sigma(ab \to cd)}{\langle\sigma(a'b' \to c'd')\rangle} = \frac{\sum_X \mathcal{F}^X_{s(ab\to cd)} [1/\Delta_X]}{\sum_{X'} \mathcal{F}^{X'}_{s(a'b'\to c'd')} \; [1/\Delta_{X'}]} \tag{14}$$

so long as a and a' belong to the same multiplet of the invariance group, b and b' to the same multiplet, c and c' to the same multiplet and d and d' to the same multiplet.

In order to calculate average branching ratios using Eq. (13) we have to choose an invariance group. At present there are several choices which one can make. Do the calculated average branching ratios depend on this choice? The answer is that they do. To see this, let us use Eq. (13) to calculate the average branching ratio for the processes

$$\pi^- p \to \pi^- p \tag{15a}$$

and

$$\pi^- p \to \pi^0 n \tag{15b}$$

under both SU_2 and SU_3. Assuming the interaction is invariant under SU_2, two reduced amplitudes result in the s- and u-channels labeled by the total isospins I = 1/2 and I = 3/2. The average branching ratio obtained from Eq. (13) for these reactions is,

$$\frac{\langle\sigma(\pi^- p \to \pi^- p)\rangle}{\langle\sigma(\pi^- p \to \pi^0 n)\rangle} = 3/2 \tag{16}$$

If, on the other hand, we had assumed SU_3 invariance where the pion and nucleon both belong to octets, seven reduced amplitudes would result in the s- and u-channels; $S_8(ss)$, $S_8(sa)$, $S_8(as)$, $S_8(aa)$, S_{10}, $S_{\overline{10}}$ and S_{27} in the s-channel with a similar set of u-channel amplitudes[16]. In this case the average branching ratio calculated from Eq. (13) is,

$$\frac{\langle\sigma(\pi^- p \to \pi^- p)\rangle}{\langle\sigma(\pi^- p \to \pi^0 n)\rangle} = 37/24 \tag{17}$$

At first sight the fact that the SU_2 and SU_3 results differ (even slightly) is a bit surprising because SU_2 is a subgroup of SU_3 and one usually expects any results which hold for SU_2 to also hold for SU_3. The reason for the difference can be traced back to Eq. (4) which expresses the statistical independence of the reduced amplitudes S_X. This independence was basic to the argument that followed. But because of the restrictions placed on the reduced amplitudes by group invariance, assuming invariance under a particular group G (SU_3 in this case) requires correlations between certain reduced amplitudes which correspond to irreducible representations of invariance groups which are subgroups of G. As an example of this type

of behavior, consider again our calculation of the SU_2 and SU_3 average branching ratios for the processes in Eqs. (15). Since SU_2 is a subgroup of SU_3, each SU_3 multiplet will generally contain several SU_2 submultiplets. The two 10-dimensional representations in the $8 \otimes 8$ decomposition, for example, contain $I = 3/2$, $I = 1$, $I = 1/2$ and $I = 0$ SU_2 submultiplets. Under SU_3 invariance, amplitudes connecting different states in the 10-dimensional representation must be equal. In particular, amplitudes connecting states within the $I = 1/2$ submultiplet of the 10-dimension SU_3 multiplet must equal amplitudes connecting states within the $I = 3/2$ submultiplet. Thus, if SU_3 is the assumed invariance group and if the SU_3 reduced amplitudes are assumed to be independent, the corresponding SU_2 reduced amplitudes ($S_{I = 1/2}$ and $S_{I = 3/2}$ in the above example) cannot all be statistically independent; in fact, some of them must be equal. It is because of this incompatibility of statistical independence for SU_2 and SU_3 that the branching ratios in Eqs. (16) and (17) differ.

These arguments can easily be generalized to any compact group and its subgroups. Thus, <u>if one assumes statistical independence of all the reduced amplitudes corresponding to irreducible representations of a compact group G, then no similar assumption can be made for any group which is a subgroup of G</u>.

In a sense, this means that we cannot test our statistical sssumption unless we know the "true" internal symmetry group. However, there are "approximate" internal symmetry groups which appear to be fairly well satisfied. At present, the internal symmetry group which has met with the most success in high energy physics is the Gell-Mann - Ne'eman form[17] of SU_3. This is not an exact symmetry group because of the mass splitting

within the multiplets. However, at the high energies we are considering [$s >> (\text{mass})^2$], we should hope this splitting to represent only a small correction to the exact SU^3 predictions.

Following DSW, we shall calculate average branching ratios for reactions of the form,

$$\text{meson} + \text{baryon} \longrightarrow \text{meson} + \text{baryon} \qquad (18)$$

and

$$\text{photon} + \text{baryon} \longrightarrow \text{meson} + \text{baryon}. \qquad (19)$$

We shall assume that the interactions are invariant under SU_3 and that the mesons and baryons belong to octet representations of SU_3. We shall further assume that within SU_3, the electromagnetic current transforms like a pure octet and U-spin scalar[17] (which conserves charge). For computational purposes, this is equivalent to assuming that the photon belongs to a U=0 state of an octet[18]. Labeling the states in SU_3 by $|YII_Z\rangle$ (Y = hypercharge, I = total isospin and I_Z = projection of I along the axis of quantization), a U-spin scalar is given by[19],

$$|U = 0\rangle \equiv |\gamma\rangle = \sqrt{3}/2\ |010\rangle - 1/2\ |000\rangle \quad . \qquad (20)$$

In carrying out the calculations of the average branching ratios we have used the tables of Clebsch-Gordan coefficients for SU_3 compiled by McNamee and Chilton[20]. The results of these calculations are summarized in Tables 1 through 6. For comparison we have also listed the results obtained with the DSW assumption[21]. One notices that although the predicted branching ratios for our assumption differ from those for the DSW assumption in each table except Table 1; this difference is for the most part small. The principal differences appear in Tables 4 and 6. Whether or not one can use these

differences to distinguish experimentally (when experimental data is available) between the two assumptions is an open question. Note that the average cross sections in each table is normalized to unity.

In order to compare the average branching ratios in Tables 1 through 6 with experiment, some experimental interpretation must be given to the average in Eqs. (13) and (14). The only reasonable experimental interpretation is an average over some range of energies and scattering angles[22]. DSW argue that if such a statistical behavior occurs at all in the high energy realm, it is most likely to occur in central collisions where there is the possibility of forming a large number of compound states. Thus one can hope to test the validity of the assumed statistical behavior of the amplitudes for high energy reactions by comparing the calculated average branching ratios in Tables 1 through 6 with the corresponding experimental branching ratios averaged over energy and momentum transfer intervals large compared with the mass splittings within the individual multiplets. Unfortunately, there is not yet enough high energy, large angle data available to make such a comparison.

TABLE 1

Branching Ratios for $\pi^+(8) + p(8) \to M(8) + B(8)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\pi^+ p \to \pi^+ p$	0.500	0.500
$\to K^+ \Sigma^+$	0.500	0.500

TABLE 2

Branching Ratios $\pi^-(8) + p(8) \to M(8) + B(8)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\pi^- p \to \pi^- p$	0.214	0.218
$\to K^+ \Sigma^-$	0.214	0.218
$\to \pi^o N$	0.152	0.141
$\to \Sigma^o K^o$	0.152	0.141
$\to K^o \Lambda$	0.134	0.141
$\to \eta N$	0.134	0.141

TABLE 3

Branching Ratios for $\pi^+(8) + p(8) \rightarrow M(8) + B^*(10)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\pi^+ p \rightarrow \pi^+ N^{*+}$	0.188	0.216
$\rightarrow K^+ Y^{*+}$	0.188	0.216
$\rightarrow \pi^o N^{*++}$	0.281	0.324
$\rightarrow \eta N^{*++}$	0.344	0.243

TABLE 4

Branching Ratios for $\pi^-(8) + p(8) \to M(8) + B^*(10)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\pi^- p \to \pi^+ N^{*-}$	0.210	0.337
$\to \pi^o N^{*o}$	0.153	0.202
$\to \pi^- N^{*+}$	0.221	0.180
$\to K^+ Y^{*-}$	0.110	0.112
$\to K^o Y^{*o}$	0.145	0.101
$\to \eta N^{*o}$	0.160	0.067

TABLE 5

Branching Ratios for $\gamma + p(8) \rightarrow M(8) + B(8)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\gamma p \rightarrow \pi^+ N$	0.246	0.250
$\rightarrow \pi^o p$	0.227	0.211
$\rightarrow K^+ \Sigma o$	0.227	0.211
$K^+ \Lambda$	0.151	0.164
ηp	0.151	0.164

TABLE 6

Branching Ratios for $\gamma + p(8) \rightarrow M(8) + B^{*}(10)$ Processes Under SU_3

Process	Branching Ratio	
	DSW	Ours
$\gamma p \rightarrow \pi^{+} N^{*o}$	0.231	0.219
$\rightarrow \pi^{o} N^{*+}$	0.089	0.131
$\rightarrow \pi^{-} N^{*++}$	0.209	0.263
$\rightarrow K^{+} Y^{*o}$	0.167	0.131
$\rightarrow K^{o} Y^{*+}$	0.070	0.088
$\rightarrow \eta N^{*+}$	0.234	0.168

FOOTNOTES AND REFERENCES

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9. The notation used in this paper will follow for the most part the notation in Kottler and Foldy, reference 13.
10. The same result would have been obtained if we had argued that we should average over the overall phases associated with the irreducible subspaces in the products $A \otimes B$ and $C \otimes D$.

11. The weights δ_X depend not only on the multiplets A, B, C and D, but also on which set of particles belongs to each of these multiplets. In the SU_3 case, for instance, the weights δ_X would in general depend on whether one of the representations A, B, C or D was a baryon octet or a meson octet.

12. Any other would require that one be provided with a table of phase conventions for Clebsch-Gordan coefficients before one could even approach the question of considering the relative values of the averages $\langle S_X^* S_X \rangle$.

13. L. L. Foldy and R. F. Peierls, Phys. Rev. 130, 1585 (1963).
D. Amati, L. L. Foldy, A. Stanghellini and L. Van Hove, Nuovo Cimento 32, 1685 (1964).
H. Kottler and L. L. Foldy, Phys. Rev. 155, 1736 (1967).

14. L. L. Foldy and H. Kottler, University of Pittsburgh Report #NYO-3829-19, soon to be published.

15. In SU_2, for example, $\delta_X = C/\Delta_X$ is the only solution of Eqs. (11) for the processes $A + 1/2 \to A + 1/2$ where A and 1/2 label the total isospins of the incoming and outgoing particles.

16. The s and a here refer to the symmetric and antisymmetric octet representations contained in the decomposition of the direct product $8 \otimes 8$.

17. See, for instance, M. Gell-Mann and Y. Ne'eman, The Eightfold Way (W. A. Benjamin, Inc., New York, 1964). See also D. R. Speiser and J. Tarski, J. Math. Phys. 4, 588 (1963) [T. D. Lee and C. N. Yang, Phys. Rev. 122, 1954 (1961)].

18. H. J. Lipkin, Lie Groups for Pedestrians (North Holland Publishing Co., Amsterdam, 1965) p. 161.

19. H. J. Lipkin, loc. cit., p. 141.

20. P. McNamee and F. Chilton, Rev. Mod. Phys. 36, 1005 (1964).

21. Many of the results in DSW are expressed in turns of a parameter α, which specifies the transformation from the symmetric-anti-symmetric set of octets to the baryon octet plus another octet. Therefore, α is explicitly basis dependent and, in fact, represents the transformation from one set of Clebsch-Gordan coefficients to another. This factor has not been included here because we have specifically required that our statistical model results not depend on our choice of Clebsch-Gordan coefficients.

22. It should be pointed out that if we interpret the averages in Eqs. (13) and (14) as representing averages over a range of energies and momentum transfers which are physical in the s-channel then the corresponding u-channel averages will be over an unphysical range of energies and momentum transfers. This, however, does not alter the argument because all we have used is the fact that one can define a set of weights $\mathcal{U}_Z$ for each set of weights $\mathcal{S}_X$ (and this is guaranteed by crossing symmetry) independent of whether or not the set $\mathcal{U}_Z$ is physical.