$$
\begin{gathered}
K_{2}^{\circ}+p>K_{1}^{\circ}+p \text { AND THE REGGE TRAJECTORY } \\
\text { OF THE } \omega-\text { MESON }
\end{gathered}
$$

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## ABSTRACT

The behavior of the process $K_{2}^{0}+p \rightarrow K_{1}^{0}+p$ at high energy is shown to be dominatcd near the forward direction by Reggeized $\omega$ exchange with an intercopt $\alpha_{\omega}(0) \simeq 0.5$. It is shown how a study of the energy dependence and angular distribution in this process will determine the important characteristics of the $\omega$ Regge trajectory and residue functions.

[^0]
## I. INTRODUCTION

Perhaps the most impressive success of the Regge pole theory of high energy scattering has been the fit of $\rho$ Regge pole exchange to the $\pi N$ charge exchange data. In this reaction Reggeized $\rho$ exchange explains three critical features of the data ${ }^{l}$ : (1) the energy dependence of the forward amplitude; (2) the shrinkage of the diffraction peak with increasing energy; and (3) the dip at $t \simeq-0.6 \mathrm{BeV}^{2}$ due to the vanishing of the spin flip amplitude. In this and other cases, the Regge pole model has been most successful in explaining the data when only one, or at most, a few, Regge poles can be exchanged ${ }^{2}$. Working in the other direction, processes where only one or two Regge poles can be exchanged enable us to determine in detail the trajectories and residues of the leading Regge poles.

We propose in this note that the process $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$ is dominated by Reggeized $\omega$-exchange and that a study of the energy dependence and angular distribution in this process will allow us to "pick-off" the important characteristics of the $\omega$ Regge trajectory in much the same way that study of $\pi^{-}+p \rightarrow \pi^{0}+n$ determined the important characteristics of the $p$ Regge trajectory. In fact, it appears that the only other important Regge pole contributing to $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$ is the $\rho$ pole, and we know all its important characteristics precisely from $\pi N$ charge exchange.

A detailed knowledge of the $\omega$-trajectory will also permit us to predict the ratio of the real to imaginary parts of the forward amplitude for $K_{2}^{\circ}+p \rightarrow K_{1}^{O}+p$ at high energies. A knowledge of this phase is crucial. in experiments to determine the relative phase (of $\eta_{+_{+}}$) of the CP violating
ampiitude for the decay of neutral $K$ mesons into two pions ${ }^{3,4}$. Conversely, weak interaction experiments can be used to determine the regeneration phase and thus provide strong restrictions on the possible exchanged Regge trajectories. There is then a rather beautiful overlap of strong and weak interactions in the determination of the energy dependence and phase of the forward amplitude for $K_{2}^{\circ}+p \rightarrow K_{7}^{\circ}+p$. We shall return to this point later after we have investigated the evidence for the dominance of $\omega$ exchange and the energy dependence of the forward amplitude.

$$
\text { II. REGGEIZED } \omega \text { EXCHANGE AND } K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p
$$

In the process $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$ only the exchange of $C=-1$, natural spin-parity, neutral meson trajectories is allowed in the t-channel (see Figure 1). The only known mesons satisfying these criteria are the $p$, $\omega$, and $\varphi$ mesons. In the $q \bar{q}$ model of mesons with orbital angular momentum $I=0, I$, only the neutrai ${ }^{3} S_{I}(\rho, \omega, \varphi)$ and $I_{P_{I}}(B ?, H$ ? ) states have $C=-1$, but the ${ }^{I} P_{I}$ states have $J^{P}=I^{+}$and cannot couple to $K_{I}^{\circ} K_{2}^{0}$.

Using isospin rotations, the amplitude $A\left(K_{2}^{\circ}+p \rightarrow K_{l}^{\circ}+p\right)$ for $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$ can be related to other $K N$ elastic scattering amplitudes as follows ${ }^{5}$ :

$$
\begin{align*}
2 A\left(K_{2}^{\circ}+p \rightarrow K_{l}^{O}+p\right) & =A\left(K^{\circ}+p \rightarrow K^{\circ}+p\right)-A\left(\bar{K}^{\circ}+p \rightarrow \bar{K}^{-}+p\right)  \tag{I}\\
& =A\left(K^{+}+n \rightarrow K^{+}+n\right)-A\left(K^{-}+n \rightarrow K^{-}+n\right)
\end{align*}
$$

In the forward direction, $t=0$, the imaginary part of the amplitude $A\left(K_{2}^{\circ}+p \rightarrow K_{l}^{\circ}+p\right)$ is then related, using the optical theorem, to the
total cross section differcncc, $\sigma_{\mathrm{T}}\left(\mathrm{K}^{+} \mathrm{n}\right)-\sigma_{\mathrm{T}}\left(\mathrm{K}^{-} \mathrm{n}\right)$. In the Regge pole theory of high energy scattering, such total cross section differences are expected to have the behavior ${ }^{6}$

$$
\begin{equation*}
\Delta \sigma_{T} \propto s^{\alpha(0)-1} \propto p_{I}^{\alpha(0)-1} \tag{2}
\end{equation*}
$$

where $\alpha(0)$ is the intercept at $t=0$ of the leading Regge trajectory in the crossed channel. For example, in Figure 2 the total cross section difference ${ }^{7,8} \sigma_{T}\left(\pi^{-} p\right)-\sigma_{T}\left(\pi^{+} p\right)$ is plotted vs. $p_{L}$, and it can be seen that $\alpha(0) \simeq 0.5$ for the leading $t$-channel trajectory, that of the p-meson.

Turning back to $\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)$, whose energy dependence is presumed governed by $\rho, \omega$ and $\varphi$ exchange, the high energy data ${ }^{7}$ are plotted vs. $p_{I}$ in Figure 3. Although it is probably possible to at least conclude that $\alpha(0)<l$ for the leading trajectory, it is difficult, given the errors, to conclude much more about the energy dependence of $\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)$, and hence of $A\left(K_{2}^{0}+p \rightarrow K_{1}^{0}+p\right)$.

However, since we know the leading $I=I, C=-I$ ( $\rho$ ) trajectory rather well from $\pi N$ charge exchange, and we cen calculate its contribution to KN processes, let us concentrate on the $I \neq 0, C=-1 \omega$ and $\varphi$ trajectories. Because the deuteron has isospin zero, $\sigma_{T}(\bar{p} d)-\sigma_{T}(p d)$ and $\sigma_{T}\left(K^{-} d\right)-$ $\sigma_{T}\left(K^{+} d\right)$ only involve $\omega$ and $\varphi$ exchange. Furthermore, these cross sections are free of the screening corrections needed to extract $\sigma_{T}\left(K^{-} n\right)$ and $\sigma_{T}\left(K^{\dagger} n\right)$, and the resulting errors are smaller. The data in Figure 4 , especially those for $\sigma_{\mathrm{T}}(\overline{\mathrm{p}} \mathrm{d})-\sigma_{\mathrm{T}}(\mathrm{pd})$, rather clearly indicate that $\alpha(0) \simeq 0.5$.

We thus find that the leading $I=0, C=-1$ trajectory has $\alpha(0) \simeq 0.5$, just as the leading $I=1, C=-1(0)$ trajectory does. Furthermore, the
leading trajectory with $C=-1, I=0$ should be that of the $\omega$ meson since
(1) With the usual slope of $\sim 1 / \mathrm{BeV}^{2}$,

$$
\alpha_{\rho}(0) \simeq \alpha_{\omega}(0) \simeq I / 2, \text { while } \alpha_{\varphi}(0) \simeq 0
$$

(2) Experimentally, it appears that the $\varphi$ is not coupled to nucleons or nuclei. This is also a consequence of the usual $\varphi$-山 mixing theory with universal vector meson couplings (equivalently, the $\varphi$ only couples to particles with strange quarks in the quark model). The $\varphi$ then shouldn't contribute to scattering off nucleons or nuclei.
(3) In the usual $\omega-\varphi$ mixing model with universal vector meson couplings (or quark model) the $\omega$ couplings are proportional to $Y+2 B$, where $Y$ is the hypercharge current and $B$ the baryon number current (in the quark model this amounts to a coupling proportional to the number of non-strange quarks). We immediately have the relation ${ }^{10}$

$$
\begin{equation*}
3\left[\sigma_{\mathrm{T}}\left(\mathrm{~K}^{-} \mathrm{d}\right)-\sigma_{\mathrm{T}}\left(\mathrm{~K}^{+} \mathrm{d}\right)\right]=\sigma_{\mathrm{T}}(\overline{\mathrm{p}} \mathrm{~d})-\sigma(\mathrm{pd}) \tag{3}
\end{equation*}
$$

which is well satisfied (see Figure 4). Also, since we have vector exchange and $K^{-}$or $\bar{p}$ and $d$ have opposite values of $Y+2 B$, (1) exchange is attractive for $K^{-} d$ or $\overline{p d}$ and predicts $\sigma_{\Gamma}\left(K^{-} d\right)>\sigma\left(K^{+} d\right)$ and $\sigma_{\Gamma_{1}}(\bar{p} d)>\sigma(p d)$, again in agreement with experiment. Thus we find strong experimental and theoretical reasons for believing that the leading $C=-1, I=0$ and 1 Regge trajectories are the $\omega(\underline{n o t} \varphi)$ and $\rho$ and that both $\alpha_{\omega}(0)$ and $\alpha_{\rho}(0)$ are approximately 0.5 . These are then the leading Regge trajectories to be considered for $K_{2}^{0}+p \rightarrow K_{1}^{0}+p$.
III. PREDICTION OF $\frac{d \sigma}{d t}\left(K_{2}^{0}+p \rightarrow K_{1}^{0}+p\right)$ AT $t=0$

From Eq. (1) and the optical theorem it is simple to show that the part of do/dt at $t=0$ for $K_{2}^{\circ}+p \rightarrow K_{1}^{0}+p$ which comes from the imaginary part of the forward amplitude is

$$
\begin{equation*}
\left(\frac{d \sigma}{d t}\right)_{\text {optical }}=\frac{1}{64 \pi}\left[\sigma_{T}\left(K^{+} n\right)-\sigma_{T}\left(K^{-} n\right)\right]^{2} \tag{4}
\end{equation*}
$$

To dctcrminc the real part of the forward amplitude we use the Regge pole model and the specific results for the $\omega$ and $\rho$ trajectories at $t=0$ determined in the previous section. In the Regge pole model the phase of the forward amplitude is given by the signature factor $11,1-e^{-i \pi \alpha(0)}$. This gives a ratio of real to imaginary part of the forward amplitude of $\tan \frac{\pi \alpha(0)}{2}$. We therefore have for $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$ :

$$
\begin{equation*}
\left.\frac{d \sigma}{d t}\right|_{t=0}=\frac{1}{64 \pi \cos ^{2} \frac{\pi \alpha(0)}{2}}\left[\sigma_{T}\left(K^{+} n\right)-\sigma_{T}\left(K^{-} n\right)\right]^{2} \tag{5}
\end{equation*}
$$

where $\alpha(0)$ is the intercept at $t=0$ of the leading Regge trajectory in the t-channel.

Since we found in the previous section that both the leading trajectories $\left(\rho\right.$ and $\omega$ ) have $\alpha(0) \simeq I / 2, \tan \frac{\pi \alpha(0)}{c} \simeq 1$ and we predict roughly equal real and imaginary parts of the forward regeneration amplitude ${ }^{l 2}$. Eq. (5) then predicts $\left.\frac{d \sigma}{d t}\right|_{t=0}$ is roughly twice the optical theorem value in Eq. (4).

If we choose $p_{L}=6 \mathrm{BeV}$ as an arhitrary point to normalize with respect to, we can now summarize the predictions of our Regge pole model
for the forward regeneration amplitude as ${ }^{13}$

$$
\begin{equation*}
\left.\frac{d \sigma}{d t}\right|_{t=0}=\frac{\left(\left[\sigma_{T}\left(K^{+} n\right)-\sigma_{T}\left(K^{-} n\right)\right]_{p_{L}}=6 \mathrm{BeV}\right)^{2}}{64 \pi} \cdot \frac{\left(\left.\frac{p_{I}}{6 \mathrm{BeV}}\right|^{2 \alpha(0)-2}\right.}{\cos ^{2} \frac{\pi \alpha(0)}{2}} \tag{6}
\end{equation*}
$$

Taking ${ }^{14}\left[\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{\dagger} n\right)\right]_{p_{L}}=6 \mathrm{BeV}=4.5 \mathrm{mb}$, we find

$$
\begin{equation*}
\left.\frac{d \sigma}{d t}\right|_{t=0}=\left(260 \frac{\mu b}{B e V^{2}}\right) \frac{\left(\frac{p_{L}}{6 \overline{B e V}}\right)^{2 \alpha(0)-2}}{\cos ^{2} \frac{\pi \alpha(0)}{2}} \tag{7}
\end{equation*}
$$

'lhe values of $\left.\frac{d \sigma}{d t}\right|_{t=0}$ are shown in Figure $\rangle$ for the most probable value of $\alpha(0)=1 / 2$, as well as for $\alpha(0)=1 / 3$ and $2 / 3$. It is important to note that the Regge pole model connects the energy dependence of the forward amplitude with the ratio of its real to imaginary parts, and that both of these enter Eqs. (6) and (7). Measurement of both the energy dependence of $\left.\frac{d \sigma}{d t}\right|_{t=0}$ and of its magnitude in comparison with ( $\frac{d \sigma}{d t}$ ) optical is then a critical test of the Regge model of the forward amplitude at high energies and of the value of $\alpha(0)$.

## IV. DETERMINATION OF THE $\omega$ REGGE TRAJECTORY'S

 PARAMETERS FROM $K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p$In Section II we saw that the high energy behavior of the forward $\mathrm{K}_{2}^{0}+\mathrm{p} \rightarrow \mathrm{K}_{\mathrm{l}}^{0}+\mathrm{p}$ amplitude is governed by the leading $\mathrm{C}=-I, \quad \mathrm{I}=0$ and I Regge trajectories, the $\omega$ and $\rho$ trajectories. If we assume that at $t=0$ the Reggeized $\rho$ meson is coupled at the meson-meson- $\rho$ vertex universally through the isospin current ${ }^{15}$ (or that the $\rho$-meson-meson Regge residues obey $\operatorname{SU}(3)$ ), then we find that the relative contributions of the $\omega$ and $\rho$ trajectories to $A\left(K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p\right)$ at $t=0$ are proportional to $\left[\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)+\frac{1}{2}\left[\sigma_{T}\left(\pi^{-} p\right)-\sigma_{T}\left(\pi^{+} p\right)\right]\right.$ and $\frac{1}{2}\left[\sigma_{T}\left(\pi^{-} p\right)-\sigma_{T}\left(\pi^{+} p\right)\right]$. Over the range of $p_{L}$ from 6 to 20 BeV this gives a ratio of $\omega$ to $p$ contributions of four or five to one ${ }^{16}$. The forward regeneration amplitude is thus dominately due to $\omega$ exchange and the measurement of $\left.\frac{d \sigma}{d t}\right|_{t=0}$ as a function of energy (see the previous section) gives $\alpha(0)$ for the a meson.

Both the $\rho$ and $\omega$ have spin non-flip and spin-flip residues at the nucleon vertex and as we move away from t=O there are contributions to $\frac{d \sigma}{d t}$ from both types of residue functions (at $t=0$, only non-flip contributes). In most Regge fits ${ }^{17}$ to high energy $\pi^{ \pm} p, K^{ \pm} p, p p$ and $\bar{p} p$ scattering the $\omega$ is assumed to contribute predominately through the non-flip residue function, while the $\rho$ gives a large spin-flip contribution ${ }^{18}$. Since we have seen that the $\omega$ is much more important at $t=0$ than the $\rho$, we expect no rise in $\frac{d \sigma}{d t}$ as we move away from $t=0$, as occurs in $\frac{d \sigma}{d t}$ for $\pi^{-}+p \rightarrow \pi^{\circ}+n$ where the large $\rho$ meson spin-flip residue dominates.

In fact, the usual Regge pole models would predict a dip in $\frac{d \sigma}{d t}\left(K_{2}^{\circ}+p \rightarrow K_{2}^{\circ}+p\right)$ at $t \simeq-.15 \mathrm{BeV}^{2}$. This is because the usual Regge
pole models explain the cross-over phenomenon ${ }^{17}\left(\frac{d \sigma}{d t}(\overline{p p})-\frac{d \sigma}{d t}(p p)\right.$, $\frac{d \sigma}{\partial t}\left(\pi^{-} p\right)-\frac{d \sigma}{d t}\left(\pi^{+} p\right)$, and $\frac{\partial \sigma}{\partial t}\left(K^{-} p\right)-\frac{d \sigma}{d t}\left(K^{+} p\right)$ all change $\operatorname{sign}$ at $\left.t \simeq-.15 \mathrm{BeV}{ }^{2}\right)$ by assuming that there are dynamical zeroes in the $\rho$ and $\omega$ non-flip residue functions at $t \simeq-.15 \mathrm{BeV}^{2}$. Since the a trajectory is expected to have a large non-flip residue and to dominate $A\left(K_{2}^{0}+p \rightarrow K_{1}^{0}+p\right)$, we expect a large dip in $\frac{d \sigma}{d t}\left(K_{2}^{0}+p \rightarrow K_{1}^{0}+p\right)$ at $t \simeq-.15 \operatorname{BeV}^{2}$ if the usual Regge pole models are correct ${ }^{19}$. This is again a critical test of part of the Regge pole model. We note in passing that the presence of such a dip at $t \simeq-.15 \mathrm{BeV}^{2}$ requires experiments with a resolution in $t$ of $\lesssim .05 \mathrm{BeV}{ }^{2}$ if we are to determine the shape of $\frac{d \sigma}{d t}$ with any accuracy. A dip in $\frac{d \sigma}{d t}$ at $t \simeq-.15 \mathrm{BeV}^{2}$ together with the fact that the bins in $t$ are $0.2 \mathrm{BeV}^{2}$ wide in the experiment of Firestone, et al. 4 would help explain why their extrapolated value for $\left.\frac{d \sigma}{d t}\right|_{t=0}$ is more than a factor of two less than $\left(\frac{d \sigma}{\partial t}\right)$
optical.
At values of $t \simeq-0.6 \mathrm{BeV}^{2}$ we expect another aip in $\frac{\partial \sigma}{d t}$ from the vanishing of the $\rho$ and $\omega$ spin-flip residues when $\alpha_{\rho}$ or $\alpha_{\omega}$ equals zero. We do not expect the dramatic dip observed in $\pi N$ charge exchange, since here we expect that the $\omega$ non-flip residue is the largest contributor and it has no known zero in the region of $t \simeq-0.6 \mathrm{BeV}^{2}$.

Finally, lct us return to the overlap of strong and weak interactions in this problem. Given the cnergy dependence of the forward amplitude, $A\left(K_{2}^{0}+p \rightarrow K_{I}^{0}+p\right)$, we have scen that the Regge model gives us its phase, which can then be used in analyzing experiments on the interference between the CP violating amplitude for long lived neutral $K$ mesons to decay into $\pi^{+}+\pi^{-}$and the amplitude for regenerated $K_{l}^{0}$ 's to decay into $\pi^{+}+\pi^{-}$.

Conversely, knowledge of the phase of $\eta_{+-}$can be used to get the phase of the forward regeneration amplitude. Even away from $t=0$, the phase of the regeneration amplitude, $A\left(K_{2}^{\circ}+p \rightarrow K_{1}^{O}+p\right)$, can be measured relative to that for elastic scattering, $A\left(K_{2}^{\circ}+p \rightarrow K_{2}^{\circ}+p\right)$, by measuring the charge asymmetry between the $\pi^{-"} e^{+} \nu$ and $\pi^{-} e^{-\bar{\nu}}$ modes of the scattered neutral $K$ mesons ${ }^{20}$. This is a very much more difficult experiment, however, than those proposed here to simply measure the shape and energy dependence of $\frac{d \sigma}{d t}$ (with present beam intensities such a charge asymmetry measurement is a "theorist's experiment" away from $t=0$ ).

In summary, the following key aspects of the $\omega$ trajectory can be determined from measuring $\frac{d \sigma}{d t}\left(K_{2}^{O}+p \rightarrow K_{1}^{\circ}+p\right)$ :

1. Measurement of the magnitude and energy dependence of $\left.\frac{d \sigma}{d t}\right|_{t=0}$ gives $\alpha(0)$.
2. The absence of a rise in $\frac{d \sigma}{d t}$ for very small increasing values of -t would show the dominance of the non-flip $\omega$ residue function (in contrast to the $\rho$ ).
3. The energy dependence of $\frac{d \sigma}{d t}$ for fixed $t \neq 0$ determines $\alpha(t)$.
4. The presence of a dip at $t \simeq-.15 \mathrm{BeV}^{2}$ would demonstrate the presence of a dynamical zero in the non-flip residue functions which is used to explain the cross-over phenomenon in Regge pole theory.
5. The size of the dip at $t \simeq-0.6 \mathrm{BeV}^{2}$ indicates the magnitude of the $\omega$ and $\rho$ spin flip residues.
6. At least in principle, weak interaction experiments can determine the phase of the regeneration amplitude for all tor comparison with the Regge pole model prediction of the phase, 1 - $e^{-i \pi \alpha(t)}$.

## ACKNOWLEDGEMENTS

The author thanks Professor David Leith for many useful discussions and for putting him on the trail of the $\omega$ in $K_{2}^{0}+p \rightarrow K_{1}^{0}+p$ to start with.

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5. We write $K_{1}^{0}=\left(K^{0}+\overline{K^{0}}\right) \sqrt{2}$ and $K_{2}^{0}=\left(K^{0}-\overline{K^{0}}\right) / \sqrt{2}$, neglecting for the moment the fact that the actual short and long lived neutral $K$ states observed in nature differ slightly from this because of $C P$ noninvariance.
6. Actually $p_{I}$ and $s$ are not exactly proportional, but we neglect terms of order $M_{N}^{2} / s$ which are only a few percent at the energies we consider here.
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10. C. A. Ievinson, H. J. Lipkin, and N. S. Wall, Phys. Rev. Letters 17, 1122 (1966).
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12. A similar discussion of the phase of the forward regeneration amplitude in Regge theory has been given by $N$. Cabibbo, Phys. Letters 22 , 212 (1966). It was not shown there, however, that the with $\alpha_{\omega}(0) \simeq I / 2$ is the dominant $I=0, C=-1$ trajectory, and not the $\varphi$. With the usual slope of $\sim I / \mathrm{BeV}^{2}, \alpha_{\varphi}(0) \simeq 0$, and the regeneration amplitude would be almost purely imaginary.
13. We actually should write the contribution of the $\rho$ and $\omega$ Regge poles separately, but since they have the same signature and $\alpha_{p}(0) \simeq \alpha_{\omega}(0)$, we have written Eq. (6) as if only one Regge pole contributes to the forward amplitude. We will in fact show in the next section that the $\omega$ exchange contribution to the regeneration amplitude is much larger than that due to $\rho$ exchange.
14. W. Galbraith, et al., (Ref. 7) give $\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)=4.4 \pm 0.6 \mathrm{mb}$. The weak Johnson-Treiman relation, $\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)=\sigma_{T}\left(K^{-} p\right)$ $-\sigma_{T}\left(K^{+} p\right)-\sigma_{T}\left(\pi^{-} p\right)+\sigma_{T}\left(\pi^{+} p\right)$, which is well satisfied (see Ref. 2), gives $\sigma_{T}\left(K^{-} n\right)-\sigma_{T}\left(K^{+} n\right)=4.7 \pm 0.4 \mathrm{mb}$ from the better measured $K^{ \pm} p$ and $\pi^{ \pm} p$ total cross sections.
15. This leads to the weak Johnson-Treiman relation which is well satisfied (see Ref. 2).
16. A ratio of three to one is predicted assuming universality (pure F-type) for all the vector meson couplings and the usual $\varphi$ - $\omega$ mixing. The ratio of four or five to one given here corresponds to the statement in Ref. 10 that the $\omega$ contribution to KN scattering is about 40 percent larger than the value predicted by $\mathrm{SU}(6)$ from the $\rho$ coupling.
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18. One expects this in a model for the vector meson residues based on vector meson dominance of the electromagnetic form factors where the ratio of spin flip to spin non-flip residues are proportional to $G_{M}^{V} / G_{E}^{V}$ and $G_{M}^{S} / G_{E}^{S}$ for the $\rho$ and $\omega$ mesons respectively.
19. Arguments against the usual assumption of a zero in the $\omega$ residues at $t \simeq-.15 \mathrm{BeV}^{2}$ are found in $V$. Barger and L. Durand, Phys. Rev. Letters 19, 1295 (1967). In processes such as NN scattering and $\gamma+p \rightarrow \pi^{0}+p$ the $B$ meson can be exchanged, so $K_{2}^{0}+p \rightarrow K_{1}^{0}+p$ may provide the cleanest test of the vanishing of the 0 residue function alone.
20. The author thanks Dr. D. Dorfan for a discussion on this subject.

| Figure 1 | Diagram representing the Regge pole contributions to the regeneration amplitude $A\left(K_{2}^{\circ}+p \rightarrow K_{1}^{\circ}+p\right)$. |
| :---: | :---: |
| Figure 2 | The total cross section difference $\sigma_{T}\left(\pi^{-} p\right)-\sigma_{T}\left(\pi^{+} p\right)$. All data are taken from Ref. 8, except for the point at $p_{\mathrm{I}}=6 \mathrm{BeV}$, which is from Ref. 7. The dashed Iines represent the behavior $p_{L}{ }^{\alpha(0)-1}$ for the cross section difference. For purposes of normalization, the dashed lines were drawn through the (arbitrarily chosen) point at $p_{I}=6 \mathrm{BeV}$. |

Figure 3

Figure 4

Figure $\left.5 \quad \frac{d \sigma}{\partial t}\right|_{t=0}$ for the process $K_{2}^{0}+p \rightarrow K_{7}^{0}+p$ for $\alpha(0)=1 / 3$, $1 / 2$, and $2 / 3$.

$\overline{100145}$

Fig. 1


Fig. 2


Fig. 3


Fig. 4


Fig. 5


[^0]:    * Work supported by the U. S. Atomic Energy Commission.

