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ION LOSS BY ELECTRON SCATTERING IN COMPRESSED ELECTRON RINGS†

R. H. Helm

Stanford Linear Accelerator Center
Stanford University, Stanford, California

INTRODUCTION AND SUMMARY

The heating by electron scattering on the ions trapped in a compressed electron ring can in principle lead to ion energies sufficient to permit escape from the potential well. A correct treatment of this question presumably would include the effects of random and systematic fluctuations in electron density, in which many particles might act collectively in exciting the ions.

The purpose of the present note is to investigate the much simpler problem in which the ring is assumed to be uniform and quiescent, and the scattering events are assumed to be random and uncorrelated.

Three separate processes may be thought of: (1) Single scattering, in which the recoil from one large-angle scattering event imparts escape momentum to the ion; (2) Azimuthal drag, in which the recoils from many small-angle events result in a cumulative tangential acceleration of the ions; and (3) Random walk, or statistical accumulation of impulses normal to the electron velocity.

As had been expected, under typical conditions for the ERA* these processes all lead to insignificant losses of protons (lifetimes much greater than one second). In the case of highly ionized heavy ions, the random walk and azimuthal drag effects could have lifetimes on the order of the ionization time.**

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*See e.g., A. M. Sessler, ERAN-1

**R. H. Levy, WC/ERA-2

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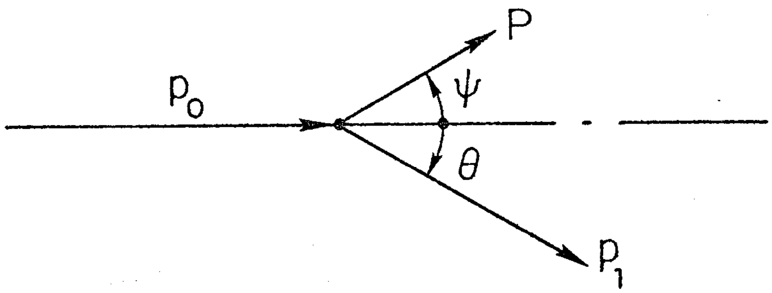
KINEMATICS, ETC.

The following inequalities are assumed to hold:

$$P \ll p \ll Mc \quad \text{and} \quad p \gg mc$$

where P is the ion momentum, p is the electron azimuthal momentum, M is the ion mass, and m is the electron rest mass. The collision kinematics (see sketch)



then are extremely simple. To first order in small quantities, it may be shown that the ion recoil energy is

$$E = \frac{1}{2} P^2/M \cong \frac{2}{M} p_0^2 \sin^2 \theta/2 \tag{1}$$

and the recoil angle is

$$\sin \psi \cong \cos \theta/2 \tag{2}$$

The coulomb scattering cross section is

$$\sigma = \frac{Z^2 r_0^2}{4\gamma^2\beta^4} \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \tag{3}*$$

where $r_0 = e^2/mc^2 = 2.8 \times 10^{-13}$ cm, Z = ion charge/e, β = (electrons azimuthal velocity)/c $\cong$ 1 and $mc\beta\gamma = p_0$. We neglect nuclear form factor and center-of-mass corrections to the cross section.

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*In the present note, γ will always refer to the azimuthal energy. It will be assumed throughout that $\beta = 1$ for the electrons.

SINGLE SCATTERING

The rate of loss of ions by single large-angle scattering is

$$\dot{N}_i/N_i = \frac{J}{e} \int_{\theta_1}^{\theta_2} \sigma \quad 2\pi \sin\theta \, d\theta \tag{4}$$

where J is the current density in the ring; θ_1 and θ_2 are the minimum and maximum angles which lead to escape from the well.

To estimate the angles $\theta_{1,2}$ we consider an ion starting at the major ring radius and moving in a harmonic oscillator potential with initial momentum $P \sin\psi$, related to the maximum amplitude by

$$P \sin\psi = M\,\omega_1\,\Delta r_{max} \tag{5}$$

$$\text{where } \omega_1 \cong \frac{c}{a}\sqrt{\frac{N_e Z r_0 m}{\pi \rho M}} \tag{6}$$

$$= \text{the ion frequency.}$$

Here a and ρ are the minor and major ring radii, respectively.

Roughly defining the escape momentum as $\sqrt{2}\,M\omega_1 a$ and using Eq. (1), we find

$$\sin^2\left(\frac{1}{2}\,\theta_{1,2}\right) \approx \frac{1}{2}\left(1 \mp \sqrt{1 - \frac{2N_e M Z r_0}{\pi m \gamma^2 \rho}}\right) \tag{7}$$

Integration of (4), using (2), (3) and (7) gives

$$\dot{N}_i/N_i \approx \frac{N_e c Z^2 r_0^2}{\pi a^2 \rho \gamma^2}\sqrt{1 - \frac{2N_e M Z r_0}{\pi m \gamma^2 \rho}} \tag{8}$$

Typical numbers from ERA-1 are:

γ	$= 47$	azimuthal energy
ρ	$= 3.7$ cm	major radius of ring
a	$= 0.1$ cm	minor radius of ring
N_e	$= 10^{13}$	number of electrons

Equation (8) then gives (for hydrogen ions)

$$\dot{N}_i/N_i \approx 0.7 \times 10^{-4} \text{ sec}^{-1}$$

as the rate of ion loss by single scattering. For slightly larger values of N_e, Eq. (7) cannot be satisfied; i.e., no single scatter can knock out an ion if the well is deep enough.

Inclusion of nuclear form factor would make the result even smaller.

Since the loss by single scattering is negligible, it will be assumed that plural scattering, involving a few large-angle events, may be ignored.

AZIMUTHAL DRAG

The rate of accumulation of azimuthal momentum by forward recoil is

$$\dot{P} = \frac{J}{e} \int_{\theta_{min}}^{\pi} \sigma P \cos \psi \; 2\pi \sin\theta \, d\theta \tag{9}$$

where θ_{min} is the minimum angle at which any scattering can occur. It will be assumed here that θ_{min} is determined by the maximum impact parameter, that is the maximum distance at which an electron can pass by an ion. In this case, the relation for an ion near the center of the ring is

$$\theta_{min} \approx \frac{2Z r_0}{a\gamma}$$

Integration of (9) then gives

$$\dot{P}/mc \cong \frac{2N_e cZ^2 r_0^2}{\pi a^2 \rho \gamma} \left(\ln \frac{a\gamma}{Zr_0} - \frac{1}{2}\right) \tag{10}$$

To estimate the rate of ion loss, we assume that the azimuthal acceleration is adiabatic on the time scale of the ion oscillation frequency. Then the centrifugal force of the azimuthal motion may be equated to the electrostatic restoring force

$$M\omega_1^2 \Delta r \approx Mv^2/\rho \approx \dot{P}^2 t^2/(M\rho) \tag{11}$$

Using (6), (10) and (11) and assuming that the ion is effectively lost if $\Delta r \approx a\sqrt{2}$, we find

$$1/t = \frac{2^{3/4}}{\gamma} \frac{c}{\rho} \left(\frac{Nm}{\pi M}\right)^{1/2} \left(\frac{Zr_0}{a}\right)^{3/2} \left(\ln \frac{a\gamma}{Zr_0} - \frac{1}{2}\right) \tag{12}$$

The above numerical values then give

$$1/t \approx 1.7 \times 10^{-3} \text{ sec}^{-1}$$

for the rate of loss of ions by the azimuthal drag effect.

RANDOM WALK

The impulse normal to the electron velocity, due to the passage of the i-th electron, is

$$\delta P_i = P_i \sin \psi_i = 2p_0 \sin \frac{\theta_i}{2} \cos \frac{\theta_i}{2}$$

It is assumed here that the cyclic and betatron motions are irrelevant; this may be justified either by saying that the coherence of a given electron on successive passages is destroyed by the scattering processes, or simply by assuming that no resonance conditions $\left[\text{such as } (1 - Q)\omega_0 \approx \omega_1\right]$ are satisfied.* Then the

*Instabilities associated with resonances of this type are treated by T. K. Fowler, WC/ERA-3.

cumulative impulse is

$$P = \sum_{t_i \le t} \delta P_i \cos \omega_1 (t - t_i)$$

(ω_1 = ion frequency) in which we assume that the same electron on successive passages is a "different" particle. To find the expectation value $\langle P^2 \rangle$, we note that $\langle \delta P_i \ \delta P_j \cos \omega_1 (t-t_i) \cos \omega_1 (t-t_j) \rangle = \frac{1}{2} \langle (\delta P_i)^2 \rangle \delta_{ij}$, because the events have been assumed to be uncorrelated; and that the weighting factor as a function of scattering angle is $\sigma(\theta_i) \Delta \Omega_i$. Then replacing the sum by an integral and using previous results, we find

$$\langle P^2 \rangle \cong \frac{Jt}{e} \int_{\theta_{min}}^{\pi} \frac{1}{2} \left(2p_0 \sin \frac{\theta}{2} \cos \frac{\theta}{2} \right)^2 \sigma(\theta) \cdot 2\pi \sin \theta \ d\theta$$

$$\cong \frac{2N_e ct \, Z^2 r_0^2 m^2 c^2}{\pi \rho a^2} \left\{ \ell n \frac{a\gamma}{Zr_0} - \frac{3}{4} \right\} \tag{13}$$

Suppose that we define the escape energy as

$$E^* = \frac{N_e Z e^2}{\pi \rho} = \frac{N_e Z r_0 m c^2}{\pi \rho}$$

which is twice the energy required to lift the ion from the center of the well to $\Delta r = a$. (This is consistent with the definition of escape momentum used previously.) Then using $\langle E \rangle = \frac{1}{2M} \langle P^2 \rangle$, we have

$$\langle \dot{E} \rangle / E^* \cong \frac{Zmr_0 c}{Ma^2} \left\{ \ell n \frac{a\gamma}{Zr_0} - \frac{3}{4} \right\} \tag{14}$$

for the rate of ion loss by the random walk process.

The numerical values used previously then give

$$\langle \dot{E} \rangle / E^* \approx 1.4 \times 10^{-2} \ \sec^{-1} .$$