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RENORMALIZATION OF WEAK FORM FACTORS, AND THE CABIBBO ANGLE*

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In order to test the Cabibbo[1] postulate of universality of the weak interactions one compares the Cabibbo angle as measured in K_{e_3} decay with that measured in O_{14} β-decay. Since accurate measurements of both these processes exist, the chief problem in such a comparison is the evaluation of small corrections to the theoretical rates. With this in mind, we consider the K_{e_3} decay rate. The purpose of this note is to argue that SU(3) breaking corrections to the K_{e_3} form factor $f_+(q^2)$ are such as to decrease its magnitude, for space-like q^2, from that expected in the limit of exact SU(3). The basis for this argument is an Ademollo-Gatto[2] relation obtained by considering the appropriate current commutator between kaon states, and assuming the contribution from states of zero strangeness is dominant over states of $|S| = 2$. The derivation of this relation and generalizations of it are discussed below; for the present we return to the question of the Cabibbo angle.

If we assume that a reasonable representation of f_+ over a range of q^2 is

$$f_+(q^2) = \frac{f_+(o)}{1 - \lambda q^2} \quad , \tag{1}$$

where the form factor f_+ is defined by

$$\langle \pi^+ (P+q) \left| V_\mu \ (o) \right| \overline{K}_0 (P) \rangle = \left[(2P+q)_\mu f_+(q^2) + q_\mu f_-(q^2) \right] \frac{1}{\sqrt{4P_0(P_0+q_0)}}$$

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and $q^2 > 0$ is time-like in our metric, then our relation is

$$\left|f_+(q^2)\right| \leq 1\,, \quad q^2 \leq 0 \tag{2}$$

which implies $\left|f_+(o)\right| \leq 1$ and $\lambda \geq 0$ in Eq. (1). Previous estimates of $\left|f_+(o)\right|$ have given values both greater and less than 1.[3] Unfortunately the inequality [Eq. (2)] does not provide a simple parameter-free estimate of $f_+(o)$ but it may help to choose between previous estimates. If we believe the form of Eq. (1) holds also for small time-like q^2 without any significant change in λ, the condition $\lambda \geq 0$ is in agreement with the experimental evidence[4] which gives

$$\lambda\, m_\pi^2 = 0.016 \pm .016$$

As remarked by Sirlin,[5] if $\left|f_+(o)\right| \leq 1$ and $\lambda \geq 0$, the effects of the $q^2 = 0$ renormalization and the q^2 dependence of f_+ tend to cancel one another in calculating θ from the K_{e_3} decay rate. The value of λ quoted above gives $-(6 \pm 6)\%$ correction to $\sin^2 \theta$.[6] Estimates of $\left|f_+(o)\right|$ vary, with a deviation from 1 of 5 - 15%. Thus one would expect a similar correction to the value of $\sin\,\theta$ obtained from K_{e_3} decay using $f_+(q^2) = 1$, which means less than 1% correction to $\cos\,\theta$. This, however, is an interesting quantity, as it is of the same order of magnitude as the possible discrepancy between θ as measured in O_{14} β-decay or in K_{e_3} decay, and also comparable to the uncertainties in the O_{14} β-decay value of θ due to the model dependence of the radiative corrections.[7] In particular, as the estimate of $f_+(o)$ decreases, the cutoff in the radiative correction needed to maintain agreement with Cabibbo theory increases.

We now discuss the derivation of Eq. (2). For all space-like q we can write a sum rule by the $P \rightarrow \infty$ technique, using the commutator

$$\left[V_0(\vec{x}, o),\, V_0^\dagger(\vec{x}', o)\right] = \delta^3(\overrightarrow{x - x'})\; V_0^3(\vec{x}, o)$$

For $\vec{q} = 0$ we need only the charge commutator, so the relation is more general in that case. We find

$$\left|f_+(q^2)\right|^2 = 1 - \lim_{P \to \infty} \sum_{n \neq \pi^+} \delta^3(\overrightarrow{P+q-P_n}) \left\{ \left|\langle \bar{K}_0 \right| V_0^\dagger(o) \left| n \rangle \right|^2 - \left| \langle \bar{K}_0 \right| V_0(o) \left| n \rangle \right|^2 \right\} \tag{3}$$

where $\vec{q} \cdot \vec{P} = 0$ and $q^2 = -\vec{q}^2$. In the exact SU(3) limit the sum $n \neq \pi^+$ vanishes and $\left|f_+(q^2)\right|^2 = 1$. Without having to make the usual arguments about interchanging the limit $P \to \infty$ and the infinite sum $\sum_{n \neq \pi^+}$ in Eq. (3), we can argue plausibly that the correction term in Eq. (3) reduces $\left|f_+(q^2)\right|$ from its exact symmetry value. The states contributing positively to the sum in Eq. (3) have the same quantum numbers as the Born term, while the negative terms differ by two units of charge and, since these are strangeness-changing currents, two units of strangeness. (We assume the validity of the $\Delta Q = \Delta S$ rule for such processes.) For K_{e_3} decay the $S = 0$ contribution includes many channels and has a contribution from the A_1 and any other abnormal parity multipion resonances or Regge recurrences, while the $S = -2$ bosonic states include no known resonances. Thus we argue that the former terms can be expected to dominate over the latter, giving a net negative contribution to Eq. (3) from the sum, $\sum_{n \neq \pi^+}$.

To summarize, we obtain Eq. (2) from an Ademollo-Gatto theorem for which we can argue with some plausibility that the dispersion integral contributes with a particular sign. This argument is based on the fact that we can relate the correction term to a difference of squared terms, where the terms of strangeness zero, which include many channels and any abnormal parity, $S = 0$, multipion resonances such as the A_1, contribute with one sign while the terms of the opposite sign have the quantum numbers $B = 0$, $Y = -2$ and include no known resonances. In such an integral it is very likely that the former terms dominate over the latter, and thus we infer the sign of the dispersion contribution. Apart from K_{e_3} decay the only interesting case where such a condition applies appears to be that of $\Sigma^- \longrightarrow N e \bar{\nu}$ where a condition similar to Eq. (2) for both the vector and axial form factors may be obtained.

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