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A CALCULATION OF POLARIZED ELECTRON MULTIPLE SCATTERING ASYMMETRIES*

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ABSTRACT

The first order polarization dependent electron multiple scattering theory of Toptygin has been evaluated for realistic experimental conditions. The result is a prediction of essentially no beam depolarization. Recent experiments are discussed, and calculations are made to illustrate the influence of multiple scattering on Mott double scattering asymmetries.

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INTRODUCTION

In recent years much labor has been devoted to the measurement of the longitudinal polarization of electrons from radioactive nuclei. One of the classical methods of making this measurement has been the use of a Mott double scattering apparatus[1]. This technique uses a first scattering process to change the initial longitudinal polarization component (directed along the electron momentum direction) to a transverse component (in a plane perpendicular to the momentum of the electron after scattering). This transverse polarization then produces an asymmetry in the second scattering process which occurs in a plane perpendicular to the plane of the first scattering (cf. Section II). If both of these processes are single scattering events, the theoretical predictions of the asymmetry are straightforward and well-known. However, in scattering foils thick enough to facilitate counting statistics, multiple scattering processes must be taken into account, especially in the first scattering foil. Mulhschlegel and Koppe[2] have developed a polarization dependent multiple scattering theory valid for small angles[3]. Toptygin[4] has obtained analytical results which are in principle valid for all angles.

In this paper a single scattering polarization dependent theory is briefly sketched (Section I), which is consistent with the Toptygin multiple scattering theory, and formulas used for asymmetries in the Mott double scattering apparatus are given (Section II). The pertinent conclusions of Toptygin's theory are indicated (Section III), and these results are then used in the asymmetry formulas previously developed to give comparisons with the predictions of single scattering theory (Section IV). Curves are given for a possible check with experiment.

I. SINGLE SCATTERING THEORY

To develop a polarization dependent, electron single scattering theory consistent with the Toptygin multiple scattering analysis, one may fruitfully use the density matrix approach.

Since we are following the general development of Mulhschlegel and Koppe[2], we need only outline the basic procedure, for notational reasons, and give the most general results not previously published.

A "transition amplitude" $A = A(\vec{k},\vec{k}',Z,\vec{\sigma})$ may be defined by the equation:

$$A\rho(\vec{P})A^{+} = w(\vec{k}';\vec{k},\vec{P})\rho(\vec{P}') \ , \tag{1}$$

where $\vec{\sigma}$ is the vector of Pauli spin matrices, ρ and ρ' are the density matrices for the initial and final beam polarizations $\vec{P}$ and $\vec{P}'$, and w is the probability of scattering into momentum $\vec{k}'$ and arbitrary polarization given initial momentum $\vec{k}$ and polarization $\vec{P}$. An explicit form for the transition amplitude can be constructed in the usual way as[5]:

$$A = f - ig\vec{u}\cdot\vec{\sigma} \ , \tag{2}$$

where f and g are the ordinary Mott functions of the scattering angle θ, atomic number Z of the scattering nucleus, and the electron energy. The vector $\vec{u} = (\vec{k} \times \vec{k}')/ \vec{k} \times \vec{k}'$ is a unit vector perpendicular to the scattering plane defined by $\vec{k}$ and $\vec{k}'$.

Rewriting the matrix produce $A\rho A^{+}$ in terms of the more convenient functions J, G, F, and D, we find that:

$$A\rho A^{+} = (\tfrac{1}{2})(J + D\vec{u}\cdot\vec{P} + (J\vec{P} + D\vec{u} + F\vec{u} \times \vec{P} + G\vec{u}\vec{u}\cdot\vec{P} - G\vec{P})\cdot\vec{\sigma}) \tag{3}$$

where

$$J = |f|^2 + |g|^2 \qquad D = -i(\bar{f}g - \bar{g}f)$$
$$G = 2|g|^2 \qquad F = \bar{f}g + \bar{g}f \ . \tag{4}$$

Factoring out $(J + D\vec{u}\cdot\vec{P})$ from Eq. (3) and independently expanding the final beam polarization density matrix in terms of the unit matrix (I) and the vector of Pauli spin matrices $(\vec{\sigma})$ in the form:

$$w\rho' = \frac{w}{2}\,(I + \vec{P}'\cdot\vec{\sigma}) \ , \tag{5}$$

we find by comparison of these two equations that the spin dependent cross section is:

$$w(\vec{k}';\vec{k},\vec{P}) = J + D\vec{u}\cdot\vec{P}) \ , \tag{6}$$

and that

$$\vec{P}' = \frac{(J - G)\vec{P} + (D + G\vec{u}\cdot\vec{P})\vec{u} + F\vec{u} \times \vec{P}}{J + D\vec{u}\cdot\vec{P}} \tag{7}$$

This polarization vector after single scattering may be more usefully resolved along axes which are always orthogonal as follows:

$$\vec{P}' = \frac{(D + J\vec{P}\cdot\vec{u})\vec{u} - F(\vec{P} \times \vec{u}) + (J - G)\vec{u} \times (\vec{P} \times \vec{u})}{J + D(\vec{P}\cdot\vec{u})} \tag{8}$$

or

$$\vec{P}' = (J + D(\vec{P}\cdot\vec{u}))^{-1} \times$$
$$\Big\{[-F(\vec{P}\times\vec{u})\cdot(\vec{n}'\times\vec{u}) + (J - G)\vec{u}\times(\vec{P}\times\vec{u})\cdot(\vec{n}'\times\vec{u})]\,\vec{n}'\times\vec{u}$$
$$+ [-F(\vec{P}\times\vec{u})\cdot\vec{n}' + (J - G)\vec{u}\times(\vec{P}\times\vec{u})\cdot\vec{n}']\,\vec{n}' \tag{9}$$
$$+ [D + J\vec{P}\cdot\vec{u}]\,\vec{u}\Big\}$$

where $\vec{n}'$ is a unit vector in the direction of $\vec{k}'$.

The scalar product of initial and final polarization vectors yields:

$$\cos\eta = \frac{(JP + D\cos\alpha - GP\sin^2\alpha)}{(J^2P^2 + 2JPD\cos\alpha + D^2P^2\sin^2\alpha + D^2)^{\frac{1}{2}}} \tag{10}$$

where η, the angle between $\vec{P}$ and $\vec{P}'$ is the polarization precession angle, and α is the angle between $\vec{P}$ and $\vec{u}$. For initial longitudinal polarization $\vec{P} = \vec{P}_L$. This reduces to:

$$\cos\eta = \frac{(JP_L - GP_L)}{(J^2P_L^2 + D^2P_L^2 + D^2)^{\frac{1}{2}}} \tag{11}$$

II. DOUBLE SCATTERING ASYMMETRIES

Eq. (6) is a form of the polarization dependent single scattering cross section which is applicable to the second scattering in a Mott double scattering experiment. For this second scattering the vector $\vec{P}_M$ is defined by writing Eq. (6) in the form:

$$w = J(1 + \frac{D}{J}\,\vec{u}\cdot\vec{P}) \equiv J(1 + \vec{P}_M\cdot\vec{P}) \ . \tag{12}$$

We see that the ratio of two events which differ only in azimuthal angle (and therefore have J functions equal) can be expressed as

$$\frac{w_1}{w_2} = \frac{1 + P_M P \cos \alpha_1}{1 + P_M P \cos \alpha_2} . \tag{13}$$

This w_1/w_2 asymmetry can be maximized by choosing the three vectors $\vec{k}$, $\vec{k}_1'$ and $\vec{k}_2'$ to lie in a plane perpendicular to initial polarization $\vec{P}$ (subscripted P_T, for transverse, in this geometry), as in Fig. 1.

We may therefore define for this geometry, a left-right asymmetry as:

$$\frac{w_L}{w_R} = \frac{1 + P_M P_T}{1 - P_M P_T} , \tag{14}$$

which is better written in the more sensitive form:

$$\delta = P_M P_T = \frac{w_L - w_R}{w_L + w_R} . \tag{15}$$

P_M is commonly known as the Mott asymmetry function, and has been extensively calculated in exact form by Sherman and Lin[6]. It is a function of scattering angle θ, scattering material Z, and electron energy, and except for relatively small scattering angles is a negative quantity. P_T in a Mott apparatus has been prepared by a scattering process in the first foil. This first process may be considered for theoretical simplicity from the point of view of either single or multiple scattering.

For a polarization prepared by a first foil <u>single</u> scattering we may use Eq. (9), written for initial beta longitudinal polarization $\vec{P} = \vec{P}_L$:

$$\vec{P}' = \frac{D}{J}\vec{u} + \frac{P_L}{J}((J - G)\cos\theta + F\sin\theta)\,\vec{n}' + \frac{P_L}{J}((J - G)\sin\theta - F\cos\theta)\,\vec{n}' \times \vec{u}. \tag{16}$$

Choosing the largest transverse component for the second scattering process gives, from this equation:

$$P_T = \frac{P_L}{J}((J - G)\sin\theta - F\cos\theta). \tag{17}$$

A function analogous to P_T may be derived if the polarization vector incident on the second scattering foil has been prepared through first foil <u>multiple</u> scattering. This is discussed in the following section.

III. MULTIPLE SCATTERING ANALYSIS

In a manner similar in principle to that of Mulhschlegel and Koppe, I. N. Toptygin has developed a multiple scattering theory in terms of the solutions to two transport equations[4]. One solution yields a scalar beam intensity I, and the other gives a polarization transport intensity vector $\vec{G} = \vec{P}I$. Thus for our consideration here we are interested in the polarization vector after multiple scattering, which is the ratio of these two solutions; i.e., $\vec{P} = \vec{G}/I$. In order to solve the transport equations, Toptygin separates the two equations into four, by the substitution of $I = I_0 + I_1$, and $\vec{G} = \vec{G}_0 + \vec{G}_1$; where the zero subscript indicates the contribution from scattering events which are made up of small angle scatterings, such that the true path length $dz/\cos\theta$ can be

replaced by its component dz in the direction of the beam incident on the dz lamina. The subscripts (1) then account for all other processes. The separated transport equations for I_0 and $\vec{G}_0$ are then solvable exactly, in a form amenable to computer evaluation as follows, where we give the result which presumes total initial longitudinal polarization $\vec{P}_0 = \vec{P}_L$:

$$I_0(\vec{n},T) = \frac{I^o}{4\pi} \sum \frac{(2m+1)}{1-k_m^2} \left(e^{-\alpha_1 T} - k_m^2 e^{-\alpha_2 T}\right) P_m(\cos\theta) \tag{18}$$

and the components of $\vec{G}_0$ are:

$$G_\chi = \frac{I^o}{4\pi} \sum \frac{k_m (2m+1)}{(1-k_m^2)\sqrt{m(m+1)}} \left(e^{-\alpha_1 T} - e^{-\alpha_2 T}\right) P_m^1(\theta) \tag{19a}$$

$$G_\eta = -\frac{I^o P^o}{4\pi} \sum \frac{1-s_m^2 - s/\sqrt{m(m+1)}}{1+s_m^2} \left(e^{-\beta_1 T} - e^{-\beta_2 T}\right) P_m^1(\theta) \tag{19b}$$

$$\begin{aligned} G_\xi = \frac{I^o P^o}{4\pi} \sum (1-s_m^2)^{-1} \Big\{ & (m+1)\left(e^{-\beta_1 T} + s_m^2 e^{-\beta_2 T}\right) \\ & + m\left(s_m^2 e^{-\beta_1 T} + e^{-\beta_2 T}\right) \\ & + 2s_m \sqrt{m(m+1)}\left(e^{-\beta_1 T} - e^{-\beta_2 T}\right) \Big\} P_m(\cos\theta) \end{aligned} \tag{19c}$$

where χ, η, and ξ denote the direction of $\vec{k}' \times \vec{k}$, $\vec{k} \times (\vec{k}' \times \vec{k})$ and $\vec{k}$ respectively. I^0 and P^o are the magnitudes of the initial beam intensity and polarization; k_m, $\alpha_{1,2}$, $\beta_{1,2}$, and s_m are functions of the summation index, the electron energy, the Z of the scattering nucleus

and the screening angle χ_0; T is foil thickness, and θ is the total momentum scattering angle. The constituent functions k, α, β and s are derived analytically by Toptygin in the second Born approximation (up to terms in $(\alpha Z)^3$).

The contributions to the polarization vector from large angle scatterings, which are contained in terms I_1 and G_1, are calculable by a method of successive approximations. However, up through the zeroth and first order approximations, the polarization is representable by the equation $\vec{P} = \vec{G}_0/I_0$ to a high order of accuracy, and it is the predictions of this solution which we investigate here.

IV. RESULTS OF TOPTYGIN'S THEORY

A computer evaluation of the ratios of Eqs. (18) and (19) has been made, and values tabulated for the zeroth order polarization vector after multiple scattering. The theory as it stands has two major restrictions relating to scattering foil characteristics and electron energy range.

The first is the limitation of the Born approximation, which requires that $\alpha Z/\beta \ll 1$, limiting validity to low Z foils and high energies.

The second involves the screening angle χ_0 used in the cross section integral. The Thomas-Fermi model has been used to evaluate this angle, and gives $\chi_0 = \lambda/a$; there λ is the de Broglie wave length of the electron, the atomic radius $a = 0.8853 \times a_0 \times Z^{-1/3}$, and a_0 is the Bohr radius. This calculation imposes the restriction that $m\chi_0 \ll 1$, where

m is the series summation index in Eqs. (18) and (19). For Z = 13 and $v/c = 0.95$, χ_0 has a value of 6×10^{-3}, and for Z = 79 and $v/c = 0.65$, $\chi_0 = 4 \times 10^{-2}$. The use of a more refined screening angle[3] is expected to improve convergence, but will not substantially alter the results.

For practical experimental considerations, valid series convergence was obtained for aluminum foils thinner than 10^{-1} cm. and thicker than 5×10^{-4} cm. for all $v/c \geq 0.65$; and for gold foils in the vicinity of 10^{-4} cm. in thickness and for $v/c > 0.85$.

The major conclusion from this analysis can be stated quite simply: to within less than 1%, for all electron energies, total scattering angles and foil materials, there is no beam depolarization in multiple scattering under the Toptygin theory.

The small angle theory of Mulhschlegel and Koppe was first thought to be in disagreement with this result, but an inconsistency in their calculation was discovered by Scott[3], and the corrected version now agrees with the above conclusion. Neither of these results accounts for the high degree of depolarization observed by van Klinken, et al.[7] in forward diffusion scattering. In these experiments, however, energy degradation of the beam is significant. Iddings, et al.[8] have shown that depolarization can be appreciable for large momentum transfers in single scattering, but that the cross section decreases so rapidly for increasing q^2 that this process is an unlikely candidate for the production of the large depolarizations observed in multiple scattering. The more highly probable low q^2 events seem unable to account for depolarizations of more than a few percent. Only in the case of experimental discrimination in favor of high q^2 events could these momentum transfer

processes be responsible for the observed depolarizations of up to 50%. This seems not to have been the case in the van Klinken apparatus. Higher order solutions of the Toptygin equations are also unlikely sources for such large effects, especially since in the forward direction small angle scattering is expected to dominate, and the lowest order solutions analyzed above should apply. Several other theoretical approaches to the problem have been attempted[9], but they either do not directly apply to the above experiment or else yield predictions significantly at variance with the data[7]. In particular, no theory predicts the large Z dependence observed in the experiment. A similar discrepancy seems to prevail in positron scattering experiments[10,8,9].

V. ASYMMETRY UNDER MULTIPLE SCATTERING

The multiple scattering polarization vector of Eq. (19) must be transformed onto the axes of Eq. (9) for use in the second scattering asymmetry expression, Eq. (14). Using this multiple scattering value for P_T and the value of P_M for single scattering in the second foil we have calculated the asymmetry function δ. Figure 2 compares the asymmetry results for single scattering in both foils to those from a process made up of a multiple scattering in the first foil and single scattering in the second foil. The angles and energy were chosen to maximize the single scattering asymmetry. The same type of comparison is made in Figure 3, for 90° first scattering, chosen to maximize the multiple scattering asymmetry. The thickness of the first foil was arbitrarily selected to be 10^{-3} cm.

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FOOTNOTES

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FIGURE CAPTIONS

Figure 1 Geometry for the second scattering in a Mott double scattering apparatus.

Figure 2 Double scattering asymmetry function δ vs second scattering angle θ_2. Single scattering is presumed in the second scattering foil for all curves. The subscripts 1 and 2 refer to the first and second scatterings respectively. Angles were chosen for maximum single scattering asymmetry.

Figure 3 Notation is as in Fig. 2, except that the angles were chosen for maximum multiple scattering asymmetry.

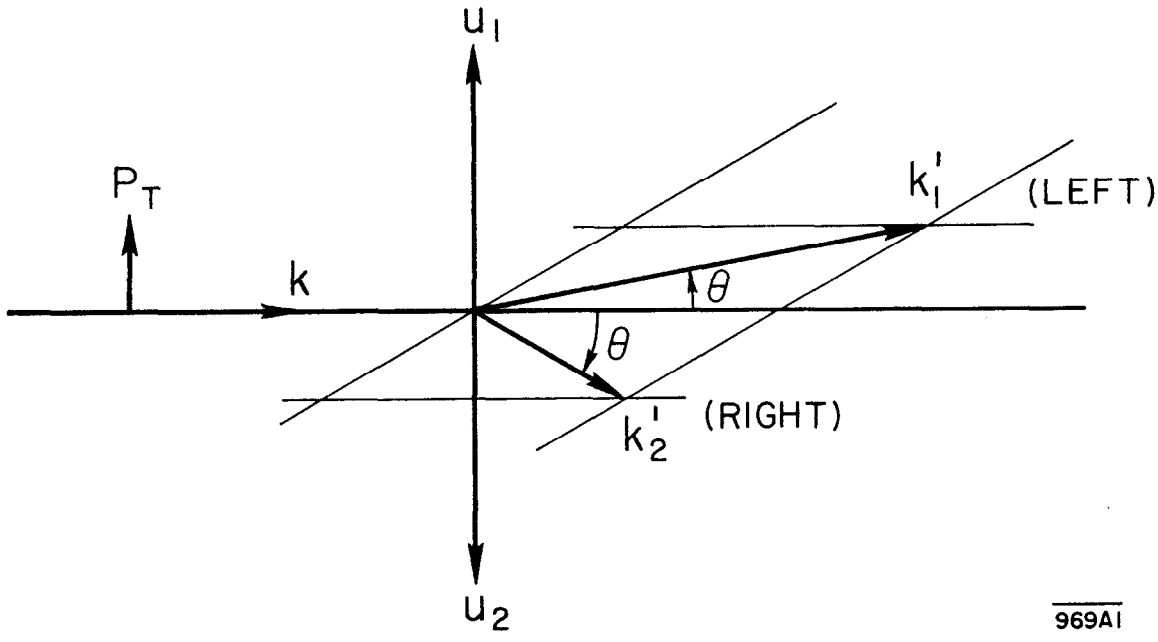

FIG. 1

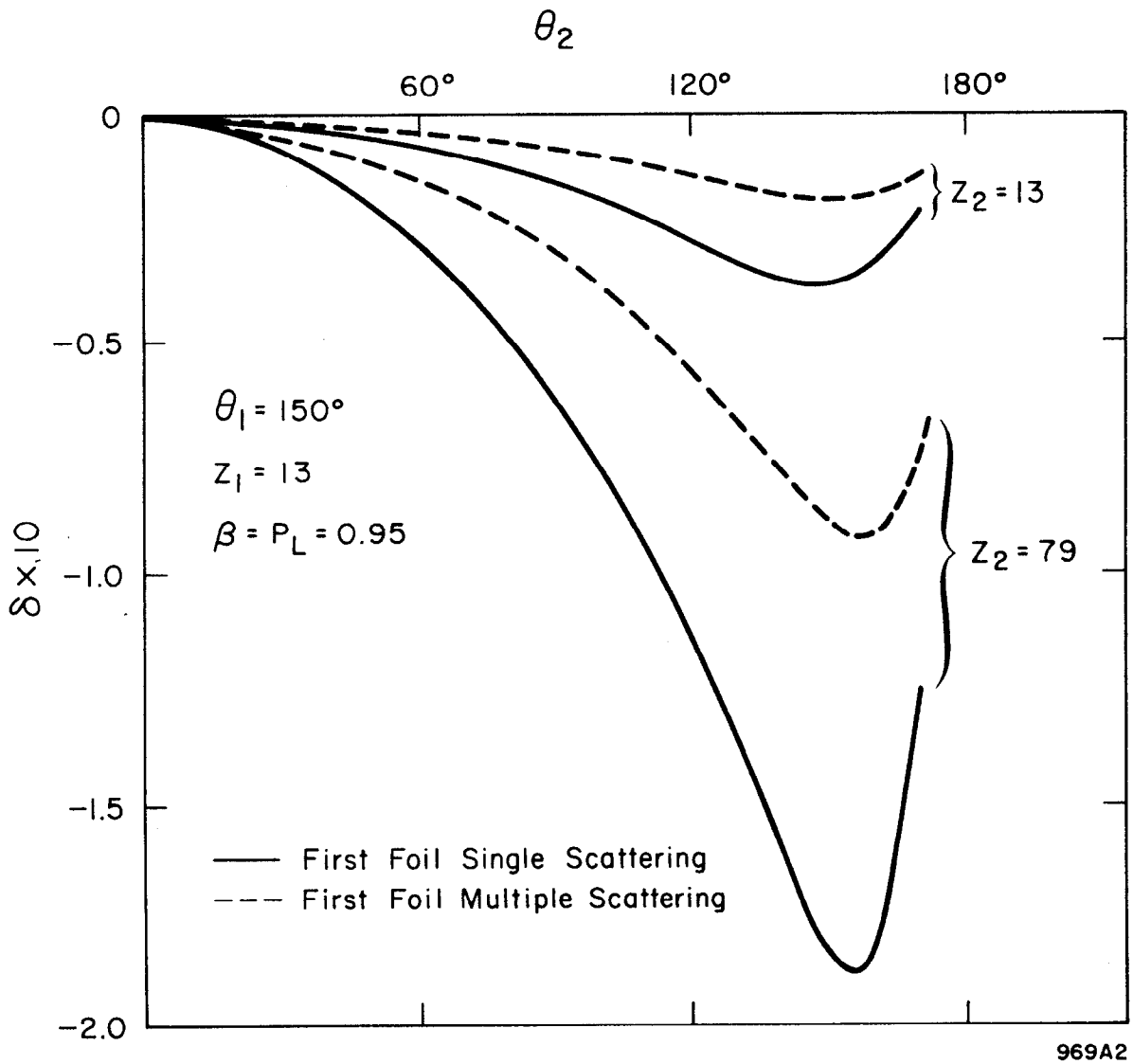
θ_2
60°
120°
180°
0
−0.5
−1.0
−1.5
−2.0
$\delta \times 10$
$\theta_1 = 150°$
$Z_1 = 13$
$\beta = P_L = 0.95$
$Z_2 = 13$
$Z_2 = 79$
First Foil Single Scattering
First Foil Multiple Scattering
969A2

FIG. 2

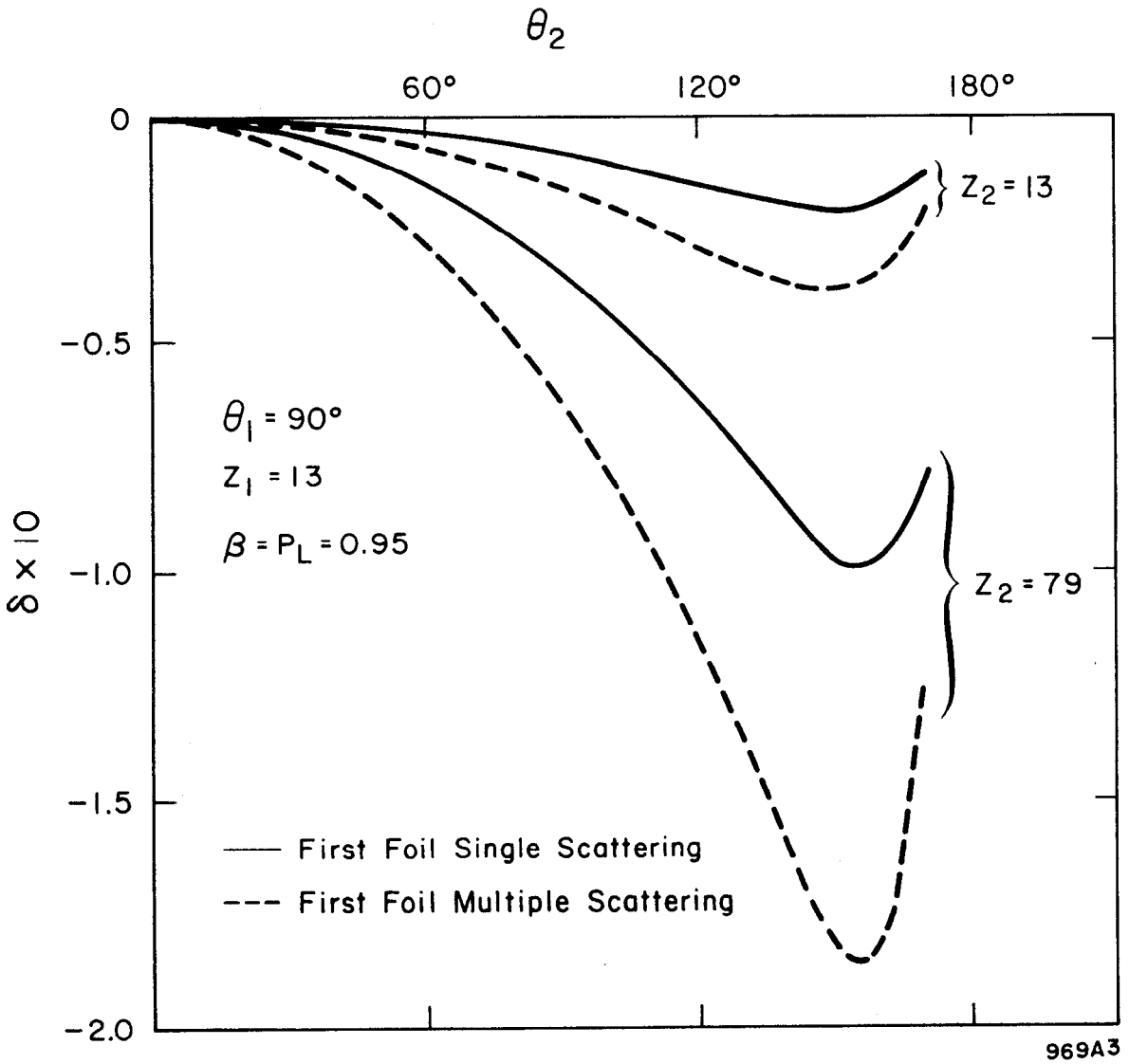
θ_2
60°
120°
180°
0
−0.5
−1.0
−1.5
−2.0
$\delta \times 10$
$Z_2 = 13$
$Z_2 = 79$
$\theta_1 = 90°$
$Z_1 = 13$
$\beta = P_L = 0.95$
First Foil Single Scattering
First Foil Multiple Scattering
969A3

FIG. 3