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Kπ SCATTERING, κ-MESON AND CURRENT ALGEBRA*

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ABSTRACT

We discuss Kπ scattering using a low-energy dominated sum rule derived from the SU(3) ⊗ SU(3) algebra of currents assuming the pionic and the kaonic pole dominance hypotheses. The exact validity of the latter assumptions implies a much larger S-wave Kπ scattering in the $I = 1/2$ channel than in the $I = 3/2$ channel. The use of the current algebra scattering lengths for threshold elastic Kπ scattering suggest that this discrimination is due to a scalar $I = 1/2$ resonance. If this is the kappa (725), a width $\geq 28 \pm 2$ MeV is predicted. If, however, the error in kaonic pole dominance is greater than 30%, the existence of the kappa would not be required.

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SECTION I - INTRODUCTION

The SU(3) $\otimes$ SU(3) algebra of charges is generally assumed to be exactly valid, and so far this assumption is consistent with all available experimental evidence. From the success of the current algebra calculation of the πN scattering lengths the pionic pole dominance hypothesis is expected to be good within 5%[1]. The question arises as to the accuracy in the corresponding hypothesis for kaons. We know from the current algebra calculation of the S-wave K^+P scattering length and effective range[2] that in the KN system it is valid within 30%. It does not necessarily follow, however, that kanoic pole dominance should have a 30% error in the $K\pi$ system. In this paper we try to examine this question by considering a sum rule for $K\pi$ scattering which is derived by using the SU(3) $\otimes$ SU(3) current algebra and the assumption that the matrix elements of the axial currents $A_\mu^{\pi,K}$ are dominated by single π,K poles via $D_5^{\pi,K} = a_{\pi,K}\varphi^{\pi,K}$ from $q^2 = m_\pi^2, m_K^2$ to $q^2 = 0$. The sum rule is dominated by low-energy $K\pi$ scattering and we shall see that the magnitude of the error in kaonic pole dominance in the $K\pi$ system depends crucially on the existence of the kappa meson.

The existence of the kappa[4] has been speculated[5] in connection with the saturation of the mesonic Adler-Weisberger sum rules[6]. These relations are not saturated very well by the vector mesons, but that lack of saturation is not by itself a compelling reason for believing in scalar mesons. The Adler-Weisberger sum rules are not particularly low energy dominated, and the discrepancy could come from the high energy part, especially from higher resonances. However, Eq. (1), derived in Section II, involves the difference of two Adler-Weisberger relations and is hence more low energy dominated. The question of the kappa is thus more important in the context of our sum rule. After evaluating its LHS numerically in Section III, we compute the vector meson contribution in Section IV and show that the contribution from higher resonances as well as any high energy elastic scattering to the integral in Eq. (1) is negligible. (One cannot show this for the ordinary Adler-Weisberger relations.) In Section V we discuss how S-wave elastic scattering may or may not contribute to our sum rule. In Section VI we derive the width of the kappa that is necessary to satisfy Eq. (1) and show that its interference with the S-wave unitarity cut is negligible. The concluding Section VII discusses the critical relation between the existence of the kappa and the theoretical uncertainty in kaonic pole dominance in the $K\pi$ system.

SECTION II - DERIVATION OF SUM RULE

The starting point of our calculation is the sum rule:

$$\frac{1}{F_\pi^2} - \frac{1}{F_K^2} = \frac{m_K^2 - m_\pi^2}{\pi} \int_{(m_K+m_\pi)^2}^{\infty} ds \, \frac{\Delta_{K\pi}(s)}{(s-m_K^2)(s-m_\pi^2)} \quad . \tag{1}$$

In Eq. (1) F_π, F_K are the usual π_{ℓ_2} and K_{ℓ_2} decay constants respectively, and $\Delta_{K\pi} = \sigma^{tot}_{K^+\pi^-} - \sigma^{tot}_{K^+\pi^+}$.

To derive the sum rule we start from the forward unsubtracted $K\pi$ dispersion relation[7]:

$$\frac{\pi}{4m_\pi m_K} \left(M^f_{K^+\pi^-} - M^f_{K^+\pi^+}\right)_{\text{thr.}} = \lim_{p^2_{\pi,K} \to m^2_{\pi,K}} \int_{(m_K+m_\pi)^2}^{\infty} ds \, \frac{\Delta_{K\pi}(s)}{[s^2-2s(p_K^2+p_\pi^2)+(p_K^2-p_\pi^2)^2]^{\frac{1}{2}}} ,$$

where $i(2\pi)^4 M_{K\pi} = \prod_i \int d^4x_i \lim_{p_i^2 \to m_i^2} (-p_i^2 + m_i^2)\, e^{\mp i p_i x_i} < 0 \,|\, T \Big\langle \varphi_4^K(x_4), \varphi_3^\pi(x_3),$ $\overset{*}{\varphi}{}_2^K(x_2), \overset{*}{\varphi}{}_1^\pi(x_1) \Big\rangle \,|\, 0 >$. The matrix element of the T-product has a pole in each p_i^2. We shall extrapolate 1) in p_π^2 from m_π^2 to zero with p_K^2 kept at m_K^2, and 2) in p_K^2 from m_K^2 to zero with p_π^2 fixed at m_π^2. For 1) using $D_5^\pi = a_\pi \varphi^\pi$, π-pole dominance and the commutation relation for $A^{\pi^\pm}$, and then smoothly going back to the mass shell, we obtain a la Weinberg (Ref. 1) that

$$\frac{\pi}{4m_\pi m_K F_\pi^2} \int d^4z\, e^{ip_\pi \cdot z} < K^+ \Big| 4p_\pi^\mu V_\mu^3(z)\delta^{(4)}(z) - i\delta(z_0)[A_0^{\pi^-}(z),\, D_5^{\pi^+}(0)] - [A_0^{\pi^+}(z),\, D_5^{\pi^-}(0)] \Big| K^+ >_{\text{thr.}} = \int ds\, \frac{\Delta_{K\pi}}{s - m_K^2}\,. \tag{1a}$$

Similarly, the use of $D_5^K = a_K \varphi^K$, K-pole dominance and the commutation relation for $A^{K^\pm}$ leads, in the second case, to

$$\frac{\pi}{4m_\pi m_K F_K^2} \int d^4z\, e^{ip_K \cdot z} < \pi^+ \Big| 4p_K^\mu V_\mu^3(z)\delta^{(4)}(z) - i\delta(z_0)[A_0^{K^-}(z),\, D_5^{K^+}(0)] - [A_0^{K^+}(z),\, D_5^{K^-}(0)] \Big| \pi^+ >_{\text{thr.}} = \int ds\, \frac{\Delta_{K\pi}}{s - m_\pi^2}\,. \tag{1b}$$

If we now use the symmetry properties of the "σ-commutators"[8], i.e. $[A_0^{\pi^-}(z), D_5^{\pi^+}(0)] = [A_0^{\pi^+}(z), D_5^{\pi^-}(0)]$, $[A_0^{K^-}(z), D_5^{K^+}(0)] = [A_0^{K^+}(z), D_5^{K^-}(0)]$, we get Eq. (1) from the difference of Eqs. (1a) and (1b).

SECTION III - F_π AND F_K EMPIRICS

We use the experimental results $F_K/m_K \sin\theta = 72.4 \times 10^{-3}$ and $F_K/F_\pi \tan\theta = .277$ obtained from $\Gamma(K \to \pi e\nu)/\Gamma(\pi \to \pi^0 e\nu)$ ignoring symmetry-breaking (which is absent to first order by the Ademollo-Gatto theorem). Alternately, we can take $\sin\theta = .21$ from hyperonic decays(9) assuming that Cabibbo theory, i.e. that there is exact SU(3) in the states, is nearly exact for baryons. Finally, there is the recent calculation of $F_K/F_\pi = 1.16$ from $SU(3)\otimes SU(3)$ spectral function sum rules and vector and scalar meson dominance(10). When substituted in $F_K/F_\pi \tan\theta$, this gives $\sin\theta = .231$. We regard the value $\sin\theta = .221 \pm .006$ as the most reliable one. From the success of the meson mass-formulas derived from current algebra considering only first order symmetry-breaking,(11) higher order SU(3)-violating effects are not expected to be very significant. The approximations made in obtaining the others are more suspect. Using $\sin\theta = .221 \pm .006$, we obtain $F_K/m_K = .327 \pm .008$, $F_\pi/m_\pi = .946 \pm .002$, and compute the LHS of Eq. (1) to be $(.371 \pm .036)/m_\pi^2$ (12).

SECTION IV - HIGH-ENERGY BEHAVIOR AND RESONANCES

To compute the RHS, we first divide up the integral as $\int_{(m_K+m_\pi)^2}^{S_0} + \int_{S_0}^{\infty}$, where S_0 is chosen to be $(2\ \mathrm{BeV})^2$, i.e. $S_0 \gg m_K^2 + m_\pi^2$, so that it is reasonable to use Regge theory to estimate the second integral. Only the ρ-trajectory contributes to $\Delta_{K\pi}$, and this contribution is twice the ρ-contribution to $\sigma^{tot}_{K^+\pi^-}$. Following Barger and Olsson(13), we write:

$$\Delta^{\rho}_{K\pi} \sim \frac{2\gamma_{\rho K}\gamma_{\rho\pi}}{q_{K\pi}s^{1/2}} \pi \frac{\Gamma(2)}{\Gamma(3/2)} \left(\frac{s-m_{\pi}^2-m_K^2}{s_1}\right)^{1/2} . \tag{2}$$

In Eq. (2) s_1 is a scaling factor taken to be 1 BeV^2, $q_{K\pi}$ is the CM momentum and $\gamma_{\rho K}$, $\gamma_{\rho\pi}$ are Regge residues. Using SU(3) and the parametrization of Ref. 13, we have $2\gamma_{\rho K} = \gamma_{\rho\pi} \simeq 2.06$. If we neglect $O(\frac{m_\pi^2 + m_K^2}{s})$ above S_0, Eq. (2) gives $\Delta_{K\pi} \sim 8.48/s$. Then the Regge contribution to the second integral is $\simeq .006/m_\pi^2$, which we can neglect for our purposes. Hence we shall ignore the integral from S_0 to ∞ in the RHS of Eq. (1).

Let us now consider the contributions from the relevant known resonances to our sum rule. The P-wave part of $K\pi$ scattering is experimentally known(14) to be completely dominated in the I = 1/2 channel by the 1^- resonance K*(890). We use the Breit-Wigner formula for the K*-contribution to $\sigma^{tot}_{K^+\pi^-}$ (15), i.e.

$$\sigma^{tot}_{K^+\pi^-}(K^*) = \frac{2}{3}\frac{4\pi}{k^2}(2J_{K^*}+1)\frac{\Gamma_{K^*}^2 m_{K^*}^2}{(s-m_{K^*}^2)^2 + m_{K^*}^2\Gamma_{K^*}^2} . \tag{3}$$

In Eq. (3) k is the CM momentum given by $4k^2m_{K^*}^2 = m_{K^*}^4 + m_K^4 + m_\pi^4 - 2m_{K^*}^2m_\pi^2 - 2m_{K^*}^2m_K^2 - 2m_\pi^2m_K^2$. The contribution of K*(890) to the RHS of Eq. (1) is then found to be $.146/m_\pi^2$. Similarly, we calculate the contribution of the 2^+ resonance K*(1420)(noting that it decays into $K^+\pi^-$ only 52% of the time) to be $.005/m_\pi^2$. This rapidly decreasing contribution with the increasing mass of the resonance justifies our neglect of contributions from any possible direct channel resonance in between $\sqrt{s}$ = 1.5 BeV and

2 BeV. For the same reason, the effect of any possible "tail" of the $K^*(890)$ on our result is not regarded as significant. Thus we still have a discrepancy of $(.220 \pm .036)/m_\pi^2$ between the LHR and RHS of Eq. (1) which is expected to come from the S-wave part.

SECTION V - S-WAVE ELASTIC SCATTERING

Since S-wave scattering dominates below the mass of $K^*(890)$, we need consider only elastic scattering for this partial wave. The forward elastic invariant amplitudes for $K^+\pi^\pm \to K^+\pi^\pm$ can be written near threshold by reducing two pions and using PCAC on $D_5^{\pi^\pm}$, after Weinberg[1], as

$$M^f_{K^+\pi^\pm} = \frac{1}{F_\pi^2}\left\{\mp\, 2p_\pi\cdot p_K - i(2\pi)^3 2p_{\pi_0}\int d^4z\ \delta(z_0)e^{ip_\pi\cdot z} < K^+\left|[A_0^{\pi^+}(z), D_5^{\pi^-}(0)]\right|K^+ >\right\}. \quad (4)$$

According to Ref. 1, the "σ-term" in Eq. (4) involving the commutator between an axial current and a divergence should not contribute to the threshold scattering lengths because of the Adler consistency condition[16]. Then $(M^f_{K^+\pi^\pm})_{thr.} = \mp\,(2m_\pi m_K/F_\pi^2)$ and $\sigma_{K^+\pi^-} = \sigma_{K^+\pi^+}$ at threshold. Since $\Delta_{K\pi}$ vanishes at threshold and the cross-sections themselves are small there, a significant contribution to our sum rule from S-wave elastic scattering can be ruled out assuming a reasonably smooth behavior in the S-wave partial amplitudes. It may be argued of course that the "σ-contribution" is not negligible since the Adler consistency condition holds only in a different region of extrapolation. In that case, one will get a positive contribution to $\Delta_{K\pi}$ and hence to the RHS of our sum rule (1).

To see that, consider the "σ-term"

$$-i \int d^4z\, \delta(z_0) e^{ip_\pi \cdot z} < K^+ \left| [A_0^{\pi^+}(z), D_5^{\pi^-}(0)] \right| K^+ >$$

in the limit $p_\pi \to 0$. This then equals

$$-i \int d^3z < K^+ \left| [A_0^{\pi^+}(z), D_5^{\pi^-}(0)] \right| K^+ > = -i \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_K - \vec{p}_n)$$

$$\left\{ < K^+ \left| A_0^{\pi^+}(0) \right| n > < n \left| D_5^{\pi^-}(0) \right| K^+ > - < K^+ \left| D_5^{\pi^-}(0) \right| n > < n \left| A_0^{\pi^+}(z) \right| K^+ > \right\}$$

$$= \sum_n (2\pi)^3 \delta^{(3)}(\vec{p}_K - \vec{p}_n)(E_n - E_K) \left\{ \left| < K^+ \left| A_0^{\pi^+}(0) \right| n > \right|^2 + \left| < n \left| A_0^{\pi^+}(0) \right| K^+ > \right|^2 \right.$$

Although the single particle contribution to this would be zero unless there is a 0^+ strange meson, the S-wave Kπ continuum could make a positive contribution. Thus it would not be totally unexpected if the conclusion $(\sigma_{K^+\pi^-})_{\text{thr.}} = (\sigma_{K^+\pi^+})_{\text{thr.}}$ were wrong. However, since the "σ-contribution" is smaller than the current algebra term by one order in the pion mass, it would be surprising if the large numerical discrepancy in our sum rule was caused by it.

SECTION VI - κ-WIDTH AND UNITARITY CUT

If we assume that the discrepancy of $(.220 \pm .036)/m_\pi^2$ between the LHS and the RHS of Eq. (1) is due to the kappa (725), by using the Breit-Wigner formula (3) we see that a width of 28 ± 2 MeV is needed. Because of our remarks on off-shell corrections in Ref. (15), we should give an inequality, i.e. $\Gamma_\kappa \geq 28 \pm 2$ MeV. We have to show, however that there is no significant interference between the S-wave resonance and the imaginary part in the elastic S-wave amplitude above threshold coming from the unitarity branch cut. In $\pi\pi$ scattering, the S-wave partial amplitude is known(17) to have a square root branch cut and the imaginary part of the scattering amplitude above threshold is proportional to the CM momentum $k = \sqrt{s-4m_\pi^2}$. In $K\pi$ scattering $k = \sqrt{\frac{(s-m_\pi^2-m_K^2)^2 - 4m_\pi^2 m_K^2}{s}}$. Assuming that the S-wave unitarity cut here is exactly the same as in $\pi\pi$ scattering, we expand the forward scattering amplitude just above threshold in s and u, consistent with crossing symmetry, keeping only linear terms as an approximation. Thus, with t=0,

$$M_f^{(\pm)} = A^{(\pm)} + B^{(\pm)}(s \pm u) + i\, C^{(+)} \left\{ \sqrt{\frac{(s-m_\pi^2-m_K^2)^2 - 4m_\pi^2 m_K^2}{s}} \pm \sqrt{\frac{(u-m_\pi^2-m_K^2)^2 - 4m_\pi^2 m_K^2}{u}} \right\}, \quad (5)$$

where $M^f_{\beta\alpha} = \delta_{\beta\alpha} M_f^{(+)} + \frac{1}{2}[\tau_\beta, \tau_\alpha] M_f^{(-)}$, $M_f^{(\pm)}(u,o,s) = M_f^{(\pm)}(s,o,u)$, and the coefficients A,B,C, are assumed to be real. Unitarity implies that

$$\lim_{k \to 0} \operatorname{Im} M^f_{K^+\pi^\pm} = \lim_{k \to 0} \frac{32\pi k}{s} \left| \frac{M_{K^+\pi^\pm}}{16\pi} \right|^2 . \quad (6)$$

Keeping only the lowest power of k in Eq. (5) and comparing with Eq. (6), we obtain:

$$\frac{1}{16\pi}\left(C^{(+)} \mp C^{(-)}\right)\left(2 + \frac{m_K}{m_\pi} + \frac{m_\pi}{m_K}\right)^{\frac{1}{2}} = \frac{1}{m_K + m_\pi}\,\frac{1}{128\pi^2}\left|M_{K^+\pi^\pm}\right|^2_{thr.} \quad (7)$$

We now use the Weinberg prediction that $M_{K^+\pi^\pm\ thr.} = \mp (2m_\pi m_K/F_\pi^2)$(18) to evaluate $C^{(\pm)}$ in Eq. (7), and then substituting in Eq. (5), we derive

$\mathrm{Im}\, M^f_{K^+\pi^\pm} = \frac{k}{2\pi}\frac{m_K^2}{F_\pi^2}\frac{1}{m_K + m_\pi} + O(k^2)$. If we assume that this is the only significant imaginary part in the S-wave partial amplitude at $s = m_\kappa^2$, the contribution of the interference term to $\sigma^{tot}_{K^+\pi^-}$ at $s = m_\kappa^2$ becomes

$$\sigma^{intf}_{K^+\pi^-}(m_\kappa^2) = \frac{1}{2\pi}\frac{1}{m_\kappa(m_K + m_\pi)}\frac{m_K^2}{F_\pi^2} .$$

Now, even if we take the contribution of the interference term to the RHS of Eq. (1) to be $\frac{m_K^2 - m_\pi^2}{\pi}\frac{2m_\kappa \Gamma_\kappa}{(m_\kappa^2 - m_K^2)(m_\kappa^2 - m_\pi^2)}\sigma^{intf}_{K^+\pi^-}(m_\kappa^2)$, this is still $< 1\%$ of the contribution from the kappa.

SECTION VII - DISCUSSION OF RESULT

We have shown that in view of Weinberg's result $(\sigma_{K^+\pi^-})_{thr.} = (\sigma_{K^+\pi^+})_{thr.}$, the assumption of the exact validity of the pole dominance hypothesis for pions and kaons suggests through the $SU(3)\otimes SU(3)$ current algebra the existence of the kappa with a width $\gtrsim 28 \pm 2$ MeV, given a mass of 725 MeV. The error quoted in this lower limit for the width comes from the experimental error in the value used for the Cabibbo angle. However, the numerical discrepancy which requires the existence of the kappa is about 60% of the LHS in Eq. (1). If the kaonic pole dominance assumption and hence Eq. (1b) is incorrect by more than 30%, it is easy to see that, since $\frac{1}{F_K^2}/(\frac{1}{F_\pi^2} - \frac{1}{F_K^2}) \simeq 2$, a 60% discrepancy could creep into Eq. (1) in such a way that no kappa might be needed. Thus the non-existence of the kappa would be consistent with a 30% or greater error in kaonic pole dominance in the $K\pi$ system, whereas the existence of the kappa would imply that the error is less. In fact, given the mass and the width of the kappa and the discrepancy of $(.220 \pm .036)/m_\pi^2$ between the LHS and the RHS of Eq. (1), this error can be computed easily. Thus an experimental resolution of the question of the existence of the kappa meson and an accurate determination of its parameters will, in the light of the present work, be of considerable theoretical interest in estimating the uncertainty in the kaonic pole dominance hypothesis in the $K\pi$ system.

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REFERENCES

1. S. Weinberg, Phys. Rev. Letters 17, 616 (1966). There is, of course, nearly a 12% error in the Goldberger-Treiman relation, but part of it is due to the extrapolation in the argument of $g_{\pi NN}$.
2. Probir Roy, Phys. Rev. 162, 1644 (1967).
3. This sum rule was first obtained, though in a somewhat different form, by S. Rai Chowdhury and G. C. Joshi (Nuovo Cimento 46A, 680 (1967) by another argument. Their conclusion regarding K*-saturation is radically different from ours because of unjustifiable approximations involving symmetry breaking in estimating $g_{N\Lambda K}$ and inaccuracies in the used numerical value for $f_A(\Lambda)$ which affect their numerical evaluation of the LHS of the sum rule through the unnecessary use of the Goldberger-Treiman relation for strangeness-changing currents.
4. The experimental situation regarding the kappa is very confused. See A. H. Rosenfeld, et al., Rev. Mod. Phys. 39, 1 (1967), p. 44-5.
5. S. Matsuda, S. Oneda and J. Sucher, Phys. Rev. 159, 1247 (1967).
6. V. S. Mathur and L. K. Pandit, Phys. Rev. 143, 1220 (1966).
7. The corresponding relation for πN scattering is given, for example, in S. Gasiorowicz, Elementary Particle Physics (John Wiley and Sons, New York, 1966, p. 379).
8. D. J. Gross and R. Jackiew, Phys. Rev. 163, 1688 (1967), prove these from the Jacobi identity between divergences and the zeroth components of axial currents and the antisymmetry of the structure constants. The proof is valid for both the $SU(2) \otimes SU(2)$ and the $SU(3) \otimes SU(3)$ algebras.

9. N. Brene, et al., Phys. Rev. 149, 1288 (1966).

10. S. L. Glashow, H. J. Schnitzer, S. Weinberg, Phys. Rev. Letters 19, 139 (1967).

11. S. Fubini, G. Furlan, C. Rossetti, Nuovo Cimento 40A, 1171 (1965).

12. It should be mentioned, in this context, that using $\sin\theta = .21$ and .231, we later get $\Gamma_\kappa \geq 33$ and 23 MeV respectively.

13. V. Barger and M. Olsson, Phys. Rev. 146B, 1080 (1966).

14. S. Wojcicki, et al., Phys. Rev. 135B, 494 (1964).

15. In evaluating the contributions of the resonances to our sum rule (1), we have not considered any rough off-shell corrections. Adler (S. L. Adler, Ref. (17), see also V. S. Mathur and L. K. Pandit, Ref. (7)) suggested a kinematical correction of $(k_0/k)^J$ to the RHS of Eq. (3) where k_0 = same as k with the mass of the "soft" final particle set equal to zero, and J = spin of the resonance. It is easy to see that any such correction will reduce the contribution of the K*(890) to our sum rule and hence will imply an even greater Γ_κ. That is consistent with our inequality.

16. S. L. Adler Phys. Rev. 137, B1022 (1965); 139, B1638 (1965).

17. W. Zimmermann, Nuovo Cimento 21, 249 (1961).

18. The inclusion of the "σ-contribution" as a correction will not affect our general conclusion here.