PHENOMENOLOGICAL ANALYSIS OF THE $\gamma \mathrm{NNN}^{*}$ FORM FACTORS ${ }^{\dagger}$<br>A. J. Dufner and Y. S. Tsai<br>Stanford Linear Accelerator Center, Stanford University, Stanford, California

ABSTRACT

The data on electroexcitation of $N^{*}$ in the momentum transfer range $-\mathrm{q}^{2}=0.1$ to $2.33 \mathrm{BeV}^{2}$ have been analysed phenomenologically using an isobar model. Assuming only the Ml transition, we obtained a phenomenological form factor for the $\gamma \mathrm{NN}{ }^{*}$ vertex. This form factor is found to decrease much faster than the elastic nucleon form factors. This implies that the $N^{*}$ has a larger spacial extension than $N$.

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[^0]
## I. INTRODUCTION

The electroexcitation of the proton into $\mathbb{N}^{*}(1236 \mathrm{MeV})$ has been investigated both experimentally ${ }^{1,2}$ and theoretically ${ }^{3}$ by many people. The purpose of this paper is to analyse some ${ }^{1,2}$ of these recent data phenomenologically using a very simple isobar model. A simple parametrization of the problem as given in this paper is desirable for many applications such as: 1) calculation of the radiative tail due to the 3-3 resonance in order to extract information from second ( 1525 MeV ) and third (1688 MeV) resonances in the inelastic electron scattering; 2) calculation of the contribution of the 3-3 resonance to various pair production experiments; 3) estimation of the contribution of the 3-3 resonance to various sum rules; and 4) estimation of the effect of the 3-3 resonance on various other processes in which the $\mathbb{N}^{*}$ appears as a propagator in Feynman diagrams (e.g. Fig. 4). The usual analysis using dispersion techniques is not suited for this purpose because the result is too complicated. The situation is very similar to the analysis of elastic electron proton scattering where phenomenological nucleon form factors are often very useful even though no one can derive them exactly from other known physical phenomena. The major difficulties in performing the phenomenological analysis are the following:

1. It is difficult to estimate the non-resonance background in a model independent way. From the prominence of the resonance peak in the data, one expects that the background should not exceed 10 to $25 \%$ of the curve at the resonance peak. At the resonance peak the 3-3 resonance amplitude is imaginary and the background is expected to be mostly real.

Hence the background simply adds to the $3-3$ cross section. The background consists of: a) the tail of the second resonance ( 1525 MeV ); b) the three Born diagrams shown in Fig. 1 a, $b, c$ with the $I=3 / 2, J=3 / 2$ amplitudes subtracted from these diagrams, and c) the small non 3-3 amplitudes generated by the imaginary part of the $3-3$ amplitude due to dispersion relations and crossing. These statements are model dependent. The items (b) and (c) have been estimated in great detail by Zagury ${ }^{3}$. As can be seen from Zagury's numerical curves, the estimates of the background terms depend greatly upon some uncertain factors such as Gen. Hence we have chosen to estimate the background directly from the data graphs themselves. This may cause a. 10 to $15 \%$ error in the $3-3$ cross section at the peak. For most of the applications we have in mind, such errors are tolerable.
3. Even though the MI amplitude is expected to dominate ${ }^{4}$ the transition $\gamma+N \rightarrow N^{*}$, one cannot tell from the data of electroproduction exactly how much the Q2 and E2 amplitudes contribute. We have written a general expression for $N^{*}$ production including $Q 2$ and $E 2$ in addition to M1 (see Eq. (2.16)), but the formula contains too many parameters, and hence is impractical for use in our analysis. We have therefore assumed that only MI contributes to the transition and obtained Eq. (2.18). All we can say is that the data available do not contradict the cross section expressed in this form.

Assuming that only M1 contributes to the transition we have obtained a phenomenological fit to the transition form factor for $\gamma+\mathbb{N} \rightarrow \mathbb{N}^{*}$ from the most recent Stanford $^{2}$ and DESY $^{2}$ data. The result can be written as

$$
\begin{equation*}
\left(c_{3}\left(q^{2}\right) M_{p}\right)^{2}=(2.05 \pm 0.04)^{2} e^{-6.3 \sqrt{-q^{2}}}\left(1+9.0 \sqrt{-q^{2}}\right) \tag{1.1}
\end{equation*}
$$

This form factor seems to decrease much faster than the nucleon form factors ${ }^{5}$

$$
\begin{equation*}
G_{E p}^{2}=\left(\left.\frac{G_{M p}}{2.79}\right|^{2}=\left(\frac{G_{M N}}{-1.91}\right)^{2}=\frac{1}{\left(1-\frac{q^{2}}{.71 \mathrm{Gev}^{2}}\right)^{4}}\right. \tag{1.2}
\end{equation*}
$$

The static theory of Fubini, Nambu and Wataghin ${ }^{6}$ predicts that the form factor associated with $\gamma+N \rightarrow N^{*}$ is proportional to $G_{M V}=G_{M p}-G_{M V}$. But this statement is very ambiguous, because in the static limit a factor such as $E_{i}^{*}+M_{p}$ in the initial state of proton is automatically replaced by $2 \mathrm{M}_{\mathrm{p}}$. The result is that a kinematical factor such as

$$
\begin{equation*}
\left(E_{i}^{*}+M_{p}\right) / 2 M_{p}=\frac{\left(M_{p}+M_{33}\right)^{2}}{4 M_{p} M_{33}}\left(1+\frac{\left|q^{2}\right|}{\left(M_{p}+M_{33}\right)^{2}}\right) \tag{1.3}
\end{equation*}
$$

would be replaced by 1. ( $E_{i}^{*}$ is the energy of initial proton in the rest system of the $N^{*}$.) This factor is not small compared with unity when $\mid q^{2}$ is $2.35 \mathrm{GeV}^{2}$, for example. Therefore, the static model does not predict the form factor for the $\gamma N N^{*}$ vertex at high momentum transfer. From Eq. (1.1) and Eq. (1.2) one is tempted to conclude that the $\mathbb{N}^{*}$ has a larger radius than $N$, in agreement with the intuitive notion than an excited state should have a looser structure than the ground state. This observation is true even if Eq. (1.1) is multiplied by the square of the factor given by Eq. (1.3) and then the product is compared with Eq. (I.2)

In the appendix we present in detail how the multipole analysis can be carried out using the formalism of Durand, DeCelles and Marr ${ }^{7}$. This method seems to be much more simple and straightforward than the usual way of reducing the matrix elements into a C.G.I.N. type ${ }^{3,6,8}$ of decomposition.

## 2. CAICUIATIONS

In the isobar model the relevant diagram is shown in Fig. 2, which defines our notation. $P_{1}, P_{2}, P_{i}$ and $P_{f}$ are the four momenta of the incident electron, final electron, initial proton and $N^{*}$, respectively. $q$ is the momentum transfer. The final pion is denoted by $P$. Since the initial proton has isospin $1 / 2$, spin $1 / 2$ and parity + , and the $N^{*}$ has isospin $3 / 2$, spin $3 / 2$, and parity + , the transition current for $\gamma+\mathbb{N} \rightarrow \mathbb{N}^{*}$ must be isovector and has multipolarities ${ }^{7}$ Q2 (scalar quadrupole), E2 (transverse electric quadrupole) and M1 (magnetic dipole). Instead of decomposing the amplitude into $M 1, E 2$, and Q2, one could also use the helicity amplitudes $f_{0}, f_{+}$and $f_{\text {_ }}$ used by Bjorken and Walecka ${ }^{9}$; however, since in the case of $N^{*}(1236)$ both theoretical and experimental analyses indicate that the MI amplitude dominates the cross section ${ }^{4}$, it is more natural to decompose the amplitude into Q2, E2 and M1. We shall choose the $\gamma \mathrm{pN}{ }^{*}$ coupling to be the sum of three gauge invariant amplitudes ${ }^{10,11}$ :

$$
\begin{align*}
& H_{3}=\text { ie } C_{3} \bar{\psi}_{\nu}(x) \gamma_{5} \gamma_{\mu} \varphi(x) F_{\mu \nu}+h \cdot c .  \tag{2.1}\\
& H_{4}=-e C_{4} \bar{\psi}_{\nu}(x) \gamma_{5}\left(\partial_{\mu} \varphi(x)\right) F_{\mu \nu}+h \cdot c . \tag{2.2}
\end{align*}
$$

$$
\begin{equation*}
H_{5}=-e C_{5} \bar{\psi}_{\nu}(x) y_{5} \varphi(x) \partial_{\mu} F_{\mu \nu}+\text { h.c. } \tag{2.3}
\end{equation*}
$$

where $\varphi(x)$ is the proton field, $F_{\mu \nu}$ is the electromagnetic field tensor $F_{\mu \nu}=\partial_{\mu} A_{\nu}-\partial_{V} A_{\mu}$ and $\psi_{\nu}$ is the spin $3 / 2$ field of Rarita and Schwinger, satisfying the subsidiary conditions ${ }^{12}$

$$
\begin{gathered}
\partial_{\nu} \psi_{v}=0,
\end{gathered} \gamma_{\nu} \psi_{v}=0 \quad \text { and } .
$$

With this choice of couplings $H_{5}$ does not contribute to the cross section when the photon is on the mass shell. $C_{3}, C_{4}$ and $C_{5}$ are for simplicity assumed to be functions of $q^{2}$ alone (but not $M_{f}^{2}$ ) in momentum space. $C_{3}(0)$ and $C_{4}(0)$ can be obtained by comparison with photoproduction experiments. The experiments of Lynch et al. ${ }^{1}$ and of the DESY ${ }^{2}$ group detect only the energy and angle of the final electrons, hence the cross section can be written in the lab system as ${ }^{13}$

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega_{2} d_{2}}=\frac{r_{0}^{2} m_{e}^{2}}{q^{4}} 4 E_{2}^{2}\left[G_{2}\left(q^{2}, M_{f}^{2}\right) \cos ^{2} \frac{\theta}{2}+2 G_{1}\left(q^{2}, M_{f}^{2}\right) \sin ^{2} \frac{\theta}{2}\right] \tag{2.4}
\end{equation*}
$$

where ${ }^{14}$
$r_{0}=2.8 \times 10^{-13} \mathrm{~cm}, m_{e}=0.51 \times 10^{-3} \mathrm{GeV}, q^{2}=-4 \mathrm{E}_{1} \mathrm{E}_{2} \sin ^{2} \frac{\theta}{2}$ $M_{f}^{2}=\left(q+p_{i}\right)^{2}=q^{2}+M_{p}^{2}+2 q_{0} M_{p}, q_{0}=E_{1}-E_{2}$ and the functions $G_{1}$ and $G_{2}$
are defined by

$$
\begin{gather*}
T_{\mu \nu} \equiv \frac{(2 \pi)^{3}}{e^{2}} \sum_{i} \sum_{f}\langle f| J_{\mu}(0)|i\rangle^{*}<f\left|J_{\nu}(0)\right| i>\delta^{4}\left(q+P_{i}-P_{f}\right) \\
\equiv G_{1}\left(q^{2}, M_{f}^{2}\right)\left(q_{\mu} q_{\nu} q^{-2}-g_{\mu \nu}\right)+G_{2}\left(q^{2}, M_{f}^{2}\right) M_{p}^{-2}\left(p_{i \mu}-q_{\mu}\left(P_{i} \cdot q\right) q^{-2}\right) \\
\left(P_{i \nu}-q_{\nu}\left(P_{i} \cdot q\right) q^{-2}\right) \tag{2.5}
\end{gather*}
$$

|i $>$ and $\mid f>$ represent the initial proton state and final pion nucleon system, respectively. Choosing the direction of the three dimensional momentum transfer $\vec{Q}^{*}$ in the rest frame of $N^{*}$ as the $z$ axis, we have from Eq. (2.5)

$$
\begin{align*}
& G_{1}\left(q^{2}, M_{f}^{2}\right)=T_{x x}=T_{y y}=T_{++}=T_{--} \equiv T_{\perp} \quad \text { and }  \tag{2.6}\\
& G_{2}\left(q^{2}, M_{f}^{2}\right)=\frac{M_{f}^{2}}{M_{f}^{2}} \frac{q^{4}}{Q^{*}}\left(T_{00}-\frac{Q^{* 2}}{q^{2}} T_{\perp}\right) . \tag{2.7}
\end{align*}
$$

Since $G_{1}$ is invariant, $T_{\perp}$ can be evaluated in any Lorentz frame ${ }^{15}$. $T_{00}$ is evaluated in the rest frame of the $\mathbb{N}^{*}$. The three dimensional momentum transfer $\vec{Q}^{*}$ in the rest system of the $N^{*}$ is related to the corresponding quantity in the lab system by

$$
\begin{equation*}
M_{f}^{2} Q^{* 2}=M_{p}^{2} Q^{2} \tag{2.8}
\end{equation*}
$$

Integrating over the phase space of the $N^{*}$, and ignoring its width, we obtain from Eq. (2.5)

$$
\begin{align*}
T_{00} & =\frac{M_{f}}{e^{2}} \delta\left(M_{f}^{2}-M_{33}^{2}\right) \sum_{\lambda_{f} \lambda_{i}}\left|<\lambda_{f}\right| J_{0}(0)\left|\lambda_{i}>\right|^{2}  \tag{2.9}\\
T_{\perp} & =\frac{M_{f}}{e^{2}} \delta\left(M_{f}^{2}-M_{33}^{2}\right) \sum_{\lambda_{f} \lambda_{i}}\left|<\lambda_{f}\right| J_{+}(0)\left|\lambda_{i}>\right|^{2} \tag{2.10}
\end{align*}
$$

where $\lambda_{f}$ and $\lambda_{i}$ are the helicity states of the $\mathbb{N}^{*}$ and $P_{i}$, respectively. In order to take care of the finite width of the $\mathbb{N}^{*}$, the $\delta$ function in Eq. (2.9) and (2.10) is replaced by the absolute square of the denominator of the propagator ${ }^{16}$ of the $N^{*}$.

$$
\begin{equation*}
\delta\left(M_{f}^{2}-M_{33}^{2}\right) \rightarrow \frac{\Gamma M_{33} \pi^{-1}}{\left(M_{f}^{2}-M_{33}^{2}\right)+\Gamma^{2} M_{33}^{2}} \tag{2.11}
\end{equation*}
$$

The width $\Gamma$ is the transition probability of $N^{*} \rightarrow \pi+N$, and since the $\pi+N$ system is in the $p$ state, we expect ${ }^{17}$

$$
\Gamma \propto P^{* 3}
$$

where $P^{*}$ is the momentum of decaying pion in the rest system of the $N^{*}$, and can be written as

$$
P^{* 2}=\left(\frac{M_{f}^{2}-M_{p}^{2}+\mu^{2}}{2 M_{f}}\right)^{2}-\mu^{2}
$$

where $\mu=0.14 \mathrm{BeV}$.
$\Gamma \propto P^{* 3}$ if we take into account only the $p$ wave phase space of the decaying pions and ignore the form factor associated with the $\mathbb{N}^{*} \rightarrow \mathbb{N}+\pi$ vertex.

We have tried two expressions for $\Gamma\left(M_{f}^{2}\right)$ :

$$
\begin{equation*}
\Gamma\left(M_{f}^{2}\right)=.12 \mathrm{GeV}\left(\frac{P^{*}}{P_{R}^{*}}\right)^{3} \tag{2.12}
\end{equation*}
$$

and ${ }^{4}$

$$
\begin{equation*}
\Gamma\left(M_{f}^{2}\right)=.1293 \mathrm{GeV} \frac{\left(0.85 \frac{\mathrm{P}^{*}}{\mu}\right)^{3}}{1+\left(0.85 \frac{\mathrm{P}^{*}}{\mu}\right)^{2}} \tag{2.13}
\end{equation*}
$$

where $P_{R}^{*}$ is the value of $P^{*}$ at the resonance, i.e. let $M_{f}=M_{33}=1.236 \mathrm{GeV}$. The matrix elements in Eq. (2.9) and (2.10) can be written in terms of Q2, M1 and E2 as ${ }^{7}$

$$
\begin{align*}
& \left.\sum_{\lambda_{f} \lambda_{i}}\left|<\lambda_{f}\right| J_{0}(0)\left|\lambda_{i}\right\rangle\right|^{2}=\frac{(Q 2)^{2}}{5}  \tag{2.14}\\
& \left.\sum_{\lambda_{f} \lambda_{i}}\left|<\lambda_{f}\right| J_{+}(0)\left|\lambda_{i}\right\rangle\right|^{2}=\frac{(M 1)^{2}}{3}+\frac{(E 2)^{2}}{5} \tag{2.15}
\end{align*}
$$

In the appendix we compute the $Q 2, M 1$ and $E 2$ amplitudes explicitly in terms of $C_{3}, C_{4}$ and $C_{5}$. (cf. Eqs. (A.5), (A.6), and (A.7)).

Substituting Eqs. (A.5), (A.6), (A.7) and (2.5 through 2.15) into Eq. (2.4) we obtain

$$
\begin{align*}
\left.\frac{d^{2} \sigma}{d \Omega d p_{2}}\right|_{\text {lab }}= & \frac{r_{0}^{2} m_{e}^{2}}{q^{4}} 4 E_{2}^{2} \frac{M_{f} M_{33} \Gamma \pi^{-1}}{\left(M_{f}^{2}-M_{33}^{2}\right)^{2}+\Gamma^{2} M_{33}^{2}} \frac{1}{2 M_{p}\left(E_{i}^{*}+M_{p}\right)} \\
& {\left[\frac{M_{p}^{2}}{M_{f}^{2}} q^{4} \frac{4}{3}\left[-C_{3}+C_{4} M_{f}+C_{5} q_{0}^{*}\right]_{Q 2}^{2} \cos ^{2} \frac{\theta}{2}\right.} \\
& +Q^{* 2}\left\{\frac{1}{3}\left[\left(2 E_{i}^{*}+2 M_{p}+q_{0}^{*}\right) C_{3}-C_{4} M_{f} q_{0}^{*}-C_{5} q^{2}\right]_{M 1}^{2}\right.  \tag{2.16}\\
& \left.\left.+\left[q_{0}^{*} C_{3}-C_{4} M_{f} q_{0}^{*}-C_{5} q^{2}\right]_{E 2}^{2}\right\}\left(2 \sin ^{2} \frac{\theta}{2}-\frac{q^{2}}{Q^{2}} \cos ^{2} \frac{\theta}{2}\right)\right]
\end{align*}
$$

The subscripts Q2, M1 and $E 2$ at the right hand side of each square bracket identify the contributions of each multipole to the cross section. The stars represent the quantities in the rest frame of the $N^{*}$; they can be written in terms of invariant quantities as follows:

$$
\begin{aligned}
& E_{i}^{*}=\left(M_{p}^{2}+M_{f}^{2}-q^{2}\right)\left(2 M_{f}\right)^{-1}, \\
& q_{0}^{*}=\left(M_{f}^{2}-M_{p}^{2}+q^{2}\right)\left(2 M_{f}\right)^{-1}, \text { and } \\
& Q^{* 2}=q_{0}^{* 2}-q^{2} .
\end{aligned}
$$

If $Q 2=0$ and $E 2=0$, we obtain from Eq. (2.16)

$$
\begin{equation*}
C_{5}=0 \text { and } C_{4}=C_{3} M_{f}^{-1} \tag{2.17}
\end{equation*}
$$

In this case Eq. (2.16) can be simplified into

$$
\begin{align*}
\left.\frac{d^{2} \sigma}{d \Omega_{2} d p_{2}}\right|_{l a b}= & \frac{r_{0}^{2} m_{e}^{2}}{-q^{2}} \frac{E_{2}}{E_{1}}\left(Q^{2}+\left(E_{1}+E_{2}\right)^{2}\right)\left(\frac{M_{p}}{M_{f}}\right)^{2} \frac{q_{0}+M_{p}+M_{f}}{3 M_{f}} c_{3}^{2}\left(q^{2}\right) \\
& \times \frac{2 \Gamma M_{f} M_{33} \pi^{-1}}{\left(M_{f}^{2}-M_{33}^{2}\right)^{2}+\Gamma^{2} M_{33}^{2}} \tag{2.18}
\end{align*}
$$

where

$$
\begin{aligned}
& E_{1}=\text { incident electron energy } \\
& E_{2}=\text { out going electron energy } \\
& q_{0}=E_{1}-E_{2} \\
& q^{2}=-4 E_{1} E_{2} \sin ^{2} \frac{\theta}{2} \\
& Q^{2}=q_{0}^{2}-q^{2}
\end{aligned}
$$

$$
\begin{aligned}
M_{p} & =.938 \mathrm{GeV} \\
M_{33} & =1.236 \mathrm{GeV} \\
M_{f} & =\left(q^{2}+M_{p}^{2}+2 M_{p} q_{0}\right)^{\frac{1}{2}} \\
m_{e} & =0.51 \times 10^{-3} \mathrm{GeV} \\
r_{0} & =2.8 \times 10^{-13} \mathrm{~cm} \\
\Gamma & =\text { See Eqs. (2.12) and (2.13) }
\end{aligned}
$$

Eq. (2.16) has too many parameters and hence it is impractical to use it for our purpose ${ }^{14}$. We shall assume that only M 1 contributes to the cross section and use Eq. (2.18). Let us write the unknown function $C_{3}\left(q^{2}\right)$ as

$$
\begin{equation*}
C_{3}\left(q^{2}\right) M_{p}=\operatorname{AF}\left(q^{2}\right) \tag{2.19}
\end{equation*}
$$

where $F(0)=1$ and

$$
\begin{equation*}
A=C_{3}(0) M_{p}=2.05 \pm 0.04 \tag{2.20}
\end{equation*}
$$

from the Dalitz and Sutherland analysis of photoproduction experiments (see next section). We then determine $F\left(q^{2}\right)$ from the experimental data ${ }^{1,2}$. The procedure used is as follows:
i) Let $F\left(q^{2}\right)=e^{-a \sqrt{-q}\left(1+b \sqrt{-q^{2}}\right)}$, and adjust $a$ and $b$ until Eq. (2.18) reproduces the experimental cross sections at the peak as closely as possible.
ii) The experimental curves will, in general, be higher than the curves obtained above on both sides of the peak. We assume that the background consists of a flat part plus the tail of the second resonance. The
flat part is estimated by the difference between the curve obtained in i) and the experimental curve midway between the threshold and the peak. The tail of the second resonance at the 3-3 peak is estimated by drawing a reasonable resonance curve. The fraction of background at the peak is estimated together with rather generous error assignments and these are given in Col. 2 of Table 1.
iii) The experimental form factor squared, $F^{2}\left(q^{2}\right) \exp$. shown in Col. 2 of Table II is then obtained by subtracting the fraction of background from $F^{2}\left(q^{2}\right)$ in step $i$ ).
iv) The error in the experimental form factor is estimated by taking the root mean square of the errors due to estimates of background, experimental cross sections and the coefficient $A$ :

$$
\begin{equation*}
\frac{\Delta F^{2}}{F^{2}} \text { exp. }=\left[\left(\frac{\Delta(\text { Background })}{\text { Peak }}\right)^{2}+\left(\frac{\Delta \sigma}{\sigma}\right)^{2}+4\left(\frac{\Delta A}{A}\right)^{2}\right]^{\frac{1}{2}} \tag{2.21}
\end{equation*}
$$

$\Delta$ (Background)/Peak is given by the $\pm$ error in Col. 2 of Table I; the errors in experimental cross sections, $\Delta \sigma / \sigma$, are given in Col. 3; and $\Delta A / A=0.02$ as given by Eq. (2.20). Finally $\left(\Delta F^{2} / F^{2}\right)$ exp. is given in Col. 4 of Table $I$.

For comparison we give numerical values of several functions of the
 is seen that $F^{2}\left(q^{2}\right)$ exp. goes down much faster than the elastic proton form factor which is given in Col. $4\left(B=.71 \mathrm{GeV}^{2}\right.$ and $\left.n=-4\right) \cdot F^{2}\left(q^{2}\right) \exp$. seems to decrease faster than -4 th power but slower than -5 th with increasing $\left|q^{2}\right|$. The exponential form seems to fit rather nicely. Fig. 3 shows the comparison of the data with Eq. (2.18) using

$$
\begin{equation*}
c_{3}^{2}\left(q^{2}\right) m_{p}^{2}=(2.05)^{2} e^{-6.3 \sqrt{-q^{2}}}\left(1+9 \sqrt{-q^{2}}\right) \tag{2.25}
\end{equation*}
$$

and $\Gamma$ given by Eqs. (2.12) and (2.13). Fig. 3a, b, c represent Lynch et al.'s data which are given in terms of

$$
\begin{equation*}
\frac{d^{2} \sigma}{d \Omega_{2} d p_{2}}\left(\frac{\alpha}{4 \pi^{2}} \frac{K}{\left|q^{2}\right|} \frac{E_{2}}{E_{1}}\left[2+\frac{\cot ^{2} \frac{\theta}{2}}{1-q_{0}^{2} / q^{2}}\right]\right)^{-1} \tag{2.26}
\end{equation*}
$$

versus $K \equiv\left(M_{f}^{2}-M_{p}^{2}\right) /\left(2 M_{p}\right)$ for fixed $q^{2}$ and $E_{2}$. Fig. 3d, e, f represent DESY data which are given in terms of $d^{2} \sigma / d \Omega_{2} d p_{2}$ versus $P_{2}$ for fixed $\theta$ and $E_{1}$. These curves are not only the overall fit of our formulae and parameters but also give some idea about the shape of the background.

## 3. DISCUSSIONS

1) The position of a resonance peak and the shape of the resonance curve are somewhat sensitive to the form of the width function $\Gamma\left(M_{f}^{2}\right)$ chosen. If a constant $\Gamma=.12 \mathrm{GeV}$ were used, the resonance peak would occur at $M_{f}=1.236 \mathrm{GeV}$ which contradicts the data; the experimental peaks occur at $\mathrm{M}_{\mathrm{f}} \approx 1.220 \mathrm{GeV}$. Of course, a constant $\Gamma$ gives a completely wrong behavior near the $\pi \mathbb{N}$ threshold. The forms of $\Gamma\left(M_{f}^{2}\right)$ given by Eqs. (2.12) and (2.13) give a theoretically correct $P$ wave threshold behavior but give somewhat lower values than the experimental curves near threshold. This simply means that near threshold other mechanisms such as $S$ wave pion electroproduction are significant. Due to large uncertainty in the background and experimental uncertainties, it is impossible to judge whether Eq. (2.12) or Eq. (2.13) is better from our curves.
2) Comparisons of fits to Lynch, et al.'s data $\left(-q^{2}=0.1\right.$ to $\left.0.5 \mathrm{GeV}^{2}\right)$ and $\operatorname{DESY}{ }^{2}$ data $\left(-q_{1}^{2} .236 \approx 0.79\right.$ to $\left.2.35 \mathrm{GeV}^{2}\right)$ show that the experimental peaks of Lynch, et al. 's data occur at slightly lower $M_{f}^{2}$ than our peaks, whereas DESY data occur at slightly higher $M_{f}^{2}$ than our peaks. The experimental peak positions are affected by the background. In particular, the tail of the second resonance tends to shift the peak to the high $M_{f}^{2}$ side and a large $S$ wave contribution near threshold tends to shift the peak to the low $M_{f}^{2}$ side. The observed difference in the positions of the peaks between the two experimental groups may be due to the difference in the importance of the background at different $q^{2}$.
3) The static theory of F.N.W. ${ }^{6}$ and the quark model ${ }^{4}$ predict the form factor for the vertex $\gamma \mathrm{NN}^{*}$ to be proportional to the isovector part of the nucleon form factor. As mentioned in the introduction, this is a very ambiguous statement because the $q^{2}$ dependence of the proportionality constant is not specified in these theories. This seems to have caused some confusion in the literature. For example, Ash, et al. 19 and Geshkenbein ${ }^{20}$ made their comparisons with experiment using the relation

$$
\begin{equation*}
\left(1+\frac{\left|q^{2}\right|}{\left(M_{f}+M_{p}\right)^{2}}\right)^{\frac{1}{2}} C_{3}\left(q^{2}\right) \propto G_{V M}\left(q^{2}\right) \tag{3.1}
\end{equation*}
$$

Bjorken and Walecka ${ }^{9}$ inferred from the $\mathrm{F} \cdot \mathrm{N} \cdot \mathrm{W} .{ }^{6}$ result

$$
\begin{equation*}
\left(1+\frac{\mid q^{2}}{\left(M_{f}+M_{p}\right)^{2}}\right) C_{3}\left(q^{2}\right) \propto G_{V M}\left(q^{2}\right) \tag{3.2}
\end{equation*}
$$

and Salam, Delbourgo, and Strathdee predicted on the basis of $U(-6,6)$ that

$$
\begin{equation*}
c_{3}\left(q^{2}\right) \propto G_{V M}\left(q^{2}\right) \tag{3.3}
\end{equation*}
$$

Our analysis of the data shows that Eq. (3.2) is closer to the truth, but $C_{3}\left(q^{2}\right)$ still goes down faster with increasing $\left|q^{2}\right|$ than Eq. (3.2). Intuitively this may be just a manifestation of the fact that an excited state such as $N^{*}$ has a looser structure than the ground state such as p. In quark language this implies that there is a spin-spin coupling between two quarks, $c \sigma_{1} \cdot \sigma_{2}$ with a positive $c$ so that when two quark spins are parallel they repel each other and when they are antiparallel they attract.
4) Various estimates have been made for the constant $A=C_{3}(0) M_{p}$ from photoproduction experiments.
a) Gourdin and Salin ${ }^{10}$, and later Mathews ${ }^{11}$ obtained

$$
\begin{array}{ll}
C_{3}(0) M_{p}=2.49 & \text { (Gourdin and Salin) } \\
C_{3}(0) M_{p}=2.0 & \text { (Mathews) }
\end{array}
$$

b) Dalitz and Sutherland ${ }^{4}$ made a detailed analysis of the Ml excitation of the $N^{*}$ from photoproduction, and they obtained for the radiative decay width $N_{+}^{*} \rightarrow p+\gamma$ :

$$
\begin{equation*}
\Gamma_{\gamma}=\alpha Q^{* 3}\left(2 M_{p} M_{33}\right)^{-1}\left[(1.28 \pm 0.02) \frac{2}{3} \sqrt{2} \mu_{p}\right] \tag{3.4}
\end{equation*}
$$

where $\mu_{p}=2.79$. For comparison, we can use our matrix element to calculate this same width as

$$
\begin{equation*}
\Gamma_{y}=\frac{Q^{*}}{4 \pi}\left[\frac{(M 1)^{2}}{3}+\frac{(E 2)^{2}}{5}\right] \tag{3.5}
\end{equation*}
$$

Setting $E 2=0$, we have from Eqs. (A.6) and (A.7)

$$
\begin{equation*}
\Gamma_{\gamma}=\alpha c_{3}^{2}(0) \frac{2}{3} Q^{* 3} \frac{E_{i}^{*}+M_{p}}{M_{33}} \tag{3.6}
\end{equation*}
$$

Equating Eq. (3.6) to Eq. (3.4) we obtain

$$
\begin{equation*}
C_{3}(0) M_{p}=2.05 \pm 0.04 \quad \text { (Dalitz and Sutherland) } \tag{3.7}
\end{equation*}
$$

c) Su6 predicts ${ }^{4}$ that the number 1.28 in Eq. (3.4) should be replaced by 1 , hence

$$
\mathrm{C}_{3}(0) \mathrm{m}_{\mathrm{p}}=1.61 \quad \text { (Su6 prediction) }
$$

d) It is also interesting to point out that we can obtain $\mathrm{C}_{3}(0) \mathrm{M}_{\mathrm{p}}$ from Chew-Low static theory ${ }^{21}$. The relevant formula from static theory is

$$
\begin{equation*}
\sigma\left(\gamma+p \rightarrow \pi^{0}+p\right)=\left(\frac{Q^{*}}{P^{*}}\right)\left(\frac{\mu^{2} \alpha}{f^{2}}\right)\left(\frac{\mu_{p}-\mu_{n}}{4 M_{p}}\right)^{2} \sigma\left(\pi^{\circ}+p \rightarrow \pi^{\circ}+p\right) \tag{3.8}
\end{equation*}
$$

where $f^{2} \approx 0.08$ and $\alpha=1 / 137$. This formula is supposed to be correct near the $\pi^{\circ}+p$ threshold. In order to obtain $C_{3}$ from this relation, we compute the cross sections of both sides using the isobar model

$$
\begin{align*}
& \sigma\left(\gamma+p \rightarrow \pi^{0}+p\right)=\frac{M_{f}}{M_{p}} 16 \pi^{2} \frac{1}{9} Q^{*} \alpha\left(E_{i}^{*}+M_{p}\right) c_{3}^{2}(0) \delta\left(M_{f}^{2}-M_{33}^{2}\right)  \tag{3.9}\\
& \sigma\left(\pi^{0}+p \rightarrow \pi^{0}+p\right)=\frac{M_{f}^{2}}{M_{p}} 32 \pi^{2} \frac{1}{9 p^{* 2}} \Gamma \delta\left(M_{f}^{2}-M_{33}^{2}\right) \tag{3.10}
\end{align*}
$$

where $\Gamma=\frac{8}{3} \frac{f^{2}}{\mu^{2}} P^{* 3}$ near threshold, according to static theory. Hence

$$
\begin{equation*}
c_{3}^{2}(0)=\frac{2 M_{f}}{E_{i}^{*}+M_{p}} \frac{8}{3}\left(\frac{\mu_{p}-\mu_{n}}{4 M_{p}}\right)^{2} \tag{3.11}
\end{equation*}
$$

Now if the $\delta$ functions in Eqs. (3.9) and (3.10) are replaced by a BreitWigner formula, then they are also usable near threshold. Since Eq. (3.8) is more correct near threshold, we let $M_{f}-\mu=E_{i}^{*}=M_{p}$ in Eq. (3.11) and finally obtain

$$
\begin{equation*}
C_{3}(0) M_{p}=2.2 \quad \text { (Chew-Iow Static Theory) } \tag{3.12}
\end{equation*}
$$

5) Dalitz and Sutherland ${ }^{4}$ obtained a formula for $d^{2} \sigma / d \Omega_{2} d P_{2}$ from the result of Dalitz and Yennie ${ }^{22}$. Our Eq. (2.18) differs somewhat from their Eq. (2.16) and (2.16'). There is an error of a factor $4 \pi$ in their Eqs. (2.16) and (2.16') ${ }^{23}$. The forms of the Breit-Wigner formula used are different, but this is just a matter of taste.

From their Eq. (2.14) and our Eq. (3.5), the expression $M$ in Dalitz and Sutherland is related to our $C_{3}\left(q^{2}\right)$ by

$$
\begin{equation*}
|m|^{2}=\frac{4 \pi}{3} \alpha \frac{\left(E_{i}^{*}+M_{p}\right)}{M_{p}} c_{3}^{2}\left(q^{2}\right) \tag{3.13}
\end{equation*}
$$

Substituting Eq. (3.13) into their Eq. (2.16) we see that our Eq. (2.18) is equal to their Eq. (2.16) at $M_{f}=M_{33}$ except for a factor of $4 \pi$. According to Dalitz and Sutherland, their Eq. (2.16') is better than their Eq. (2.16) because the former has an extra factor $E_{i}^{*} / M_{p}$ which comes from the transformation from the rest system of the $\mathbb{N}^{*}$ to the laboratory system. This factor is a mystery to us because according to the way we computed the cross section, $G_{1}=T_{\perp}$ is invariant and hence no extra factor needs to be multiplied when we go from the $N^{*}$ rest system to the laboratory system. However, this factor has a numerical value of $.972 / .938$ at
the resonance, and hence is insignificant numerically.
6) We have completely ignored the possible contributions from Q2 and E2 in our analysis. Inspection of Eq. (2.16) shows that this is an experimentally impossible task unless one has some model to tell him the $q^{2}$ dependence of $C_{3}\left(q^{2}\right), C_{4}\left(q^{2}\right)$ and $C_{5}\left(q^{2}\right)$. If the decayed pion is detected in coincidence with the electron, one may be able to untangle this, but the theoretical details have to be worked out before one can say whether this is feasible or not. If $C_{3}, C_{4}$ and $C_{5}$ have roughly the same $q^{2}$ dependence, then we can conclude from our analysis and our Eq. (2.16) the following:
A. The $Q 2$ amplitude cannot be large because it has an extra factor of $q^{4}$ in its expression. If the Q2 amplitude were significant the form factors for the C's must decrease much faster than the one discussed in this paper and this seems unlikely.
B. The value of $\mathrm{C}_{5}\left(\mathrm{q}^{2}\right)$ must be small because it is multiplied by $\mathrm{q}^{2}$ in M1 and E 2 , and our analysis shows that the cross section goes down rather rapidly with increasing $\left|q^{2}\right|$.
7) In conclusion, if $E 2=0$ and $Q 2=0$, then $G_{1}$ and $G_{2}$ of Eq. (2.4) can be written as

$$
G_{1}\left(q^{2}, M_{f}^{2}\right)=\frac{Q^{2}}{-q^{2}} G_{2}\left(q^{2}, M_{f}^{2}\right)=\frac{\Gamma M_{33} M_{f} \pi^{-1}}{\left(M_{f}^{2}-M_{33}^{2}\right)^{2}+r^{2} M_{33}^{2}} Q^{* 2} 2 C_{3}^{2}\left(q^{2}\right) \frac{\left(E_{i}^{*}+M_{p}\right)}{3 M_{p}}
$$

Our best fit for $C_{3}\left(q^{2}\right)$ is given by Eq. (1.1). This formula is sufficient for calculating most of the applications we have in mind as mentioned in the introduction. At this stage it is natural for the reader
to ask why we went through all the trouble of decomposing $H_{3}, H_{4}$ and $H_{5}$ into multipoles instead of directly obtaining some analytical expression for $G_{1}$ by fitting the data. The reason is that there are many kinds of application of the isobar model in which it is more convenient to write expressions in terms of $C_{3}, C_{4}$ and $C_{5}$ than $G_{1}$ and $G_{2}$. For example, one may wish to evaluate diagrams such as are given in Fig. 4.
8) Finally, one is tempted to ask whether form factors associated with the second resonance ( $1525 \mathrm{MeV} I, J^{\pi}=\frac{1}{2}, \frac{3^{-}}{2}$ ) and the third resonance ( $1688 \mathrm{MeV} \mathrm{I}, \mathrm{J}^{\pi}=\frac{1}{2},{\frac{5^{+}}{2}}^{+}$) can be analysed in the same fashion. Inspection of the DESY ${ }^{2}$ data shows that it is hopeless to isolatc the higher resonance contributions from the background and estimate their cross sections to within $20 \%$ accuracy. However, for many purposes a cross section known to within $20 \%$ or even $30 \%$ can be very valuable. If one wishes to do better than this, the final states of the target system must be detected in addition to the scattered electrons.

APPENDIX

## Multipole Analysis of Feynman Diagrams

In this appendix we illustrate how to extract multipole moments, as defined covariantly by Durand, et al. ${ }^{7}$, when a relativistic vertex function is given. If the target particle has $\operatorname{spin} 1 / 2$ and the final particle has spin $S$, then angular momentum conservation tells us that there are at most 6 multipole moments $Q\left(S \pm \frac{1}{2}\right), M\left(S \pm \frac{1}{2}\right)$ and $E\left(S \pm \frac{1}{2}\right)$. Parity conservation eliminates 3 of the 6 amplitudes. For $N^{*}(1236)$ the relative parity between the nucleon and $N^{*}$ is + , hence we have Q2, MI. and E2. Iet us consider helicity amplitudes given by the Hamiltonian of Eqs. (2.1), (2.2), and (2.3):

$$
\begin{align*}
\Gamma_{\lambda_{f}, \lambda_{i}}^{(\mu)}= & \left\langle p_{f} \lambda_{f}\right| j_{\mu}\left|p_{i} \lambda_{i}\right\rangle=e \bar{\psi}_{\nu}\left(p_{f} \lambda_{f}\right) \gamma_{5}\left[c_{3}\left(\phi g_{\nu \mu}-q_{\nu} \gamma_{\mu}\right)\right. \\
& \left.+C_{L_{4}}\left(q \cdot p_{f} g_{v_{\mu}}-q_{\nu} p_{f \mu}\right)+C_{5}\left(q \cdot p_{i} g_{\nu \mu}-q_{\nu} p_{i \mu}\right)\right] \varphi\left(p_{i} \lambda_{i}\right) \tag{A.1}
\end{align*}
$$

Since the spin $3 / 2$ particle is more complicated than the spin $1 / 2$ particle, we shall evaluate everything in the rest frame of the $N^{*}$. Let us use the explicit representation

$$
\gamma_{5}=\left[\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right], \quad \gamma_{0}=\left[\begin{array}{cc}
1 & 0 \\
0 & -1
\end{array}\right], \quad \vec{\gamma}=\left[\begin{array}{cc}
0 & \vec{\sigma} \\
-\vec{\sigma} & 0
\end{array}\right]
$$

Then

$$
\varphi\left(p_{i} \lambda_{i}\right)=\frac{1}{\sqrt{N}}\left[\begin{array}{c}
1 \\
\vec{\sigma} \cdot \vec{p}_{i} \\
E_{i}+M_{p}
\end{array}\right] \chi_{\lambda_{i}} \quad \text { where } \quad \chi_{\frac{1}{2}}=\alpha \equiv\left[\begin{array}{l}
1 \\
0
\end{array}\right]
$$

$$
\chi_{-\frac{1}{2}}=\beta \equiv\left[\begin{array}{l}
0 \\
1
\end{array}\right] \quad \text { and } \quad N=\frac{2 M_{p}}{E_{i}^{*}+M_{p}}
$$

In the rest frame of the $N^{*}, \psi_{V}\left(p_{f} \lambda_{f}\right)$ can be written (because of the subsidiary conditions) as

$$
\Psi_{V}\left(0, \lambda_{f}\right)=\left|s,-s_{z}\right\rangle
$$

Hence:

$$
\begin{aligned}
& \Psi_{v}\left(0, \frac{3}{2}\right)=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \hat{e}_{-} \beta \\
& \Psi_{v}\left(0, \frac{1}{2}\right)=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \quad\left(\sqrt{\frac{2}{3}} \hat{e}_{z} \beta+\sqrt{\frac{1}{3}} \hat{e}_{-} \alpha\right) \\
& \Psi_{v}\left(0,-\frac{1}{2}\right)=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \quad\left(\sqrt{\frac{1}{3}} \hat{e}_{+} \beta+\sqrt{\frac{2}{3}} \hat{e}_{z} \alpha\right) \\
& \Psi_{v}\left(0,-\frac{3}{2}\right)=\left[\begin{array}{l}
1 \\
0
\end{array}\right] \hat{e}_{+} \alpha
\end{aligned}
$$

In the rest frame of the $N^{*}$ we have $\vec{Q}^{*}=-\vec{P}_{i}^{*}=\hat{e}_{z} Q^{*}$ and we may evaluate the helicity amplitudes $\Gamma_{\lambda_{f} \lambda_{i}}^{(\mu)}$ for any combination of $\mu, \lambda_{f}, \lambda_{i}$ immediately from these formulae. Now, using Eq. (109), (119) and (120) of D.D.M. ${ }^{7}$, we have

$$
\Gamma_{\lambda_{f} \lambda_{i}}^{(0)}=\left(\begin{array}{lll}
\frac{3}{2} & 2 & \frac{1}{2}  \tag{A.2}\\
\lambda_{f} & 0 & \lambda_{i}
\end{array}\right)(Q 2)
$$

$$
\begin{align*}
& \Gamma_{\lambda_{f}}^{(+1)}=\left(\begin{array}{lll}
\frac{3}{2} & 1 & \frac{1}{2} \\
\lambda_{f} & 1 & \lambda_{i}
\end{array}\right)(M 1)-\left(\begin{array}{ccc}
\frac{3}{2} & 2 & \frac{1}{2} \\
\lambda_{f} & 1 & \lambda_{i}
\end{array}\right)(E 2)  \tag{AB}\\
& \Gamma_{\lambda_{f} \lambda_{i}}^{(-1)}=\left(\begin{array}{ccc}
\frac{3}{2} & 1 & \frac{1}{2} \\
\lambda_{f} & -1 & \lambda_{i}
\end{array}\right)(M 1)-\left(\begin{array}{ccc}
\frac{3}{2} & 2 & \frac{1}{2} \\
\lambda_{f} & -1 & \lambda_{i}
\end{array}\right) \tag{A.4}
\end{align*}
$$

where $\left(\begin{array}{lll}S_{f} & J & S_{i} \\ \lambda_{f} & \mu & \lambda_{i}\end{array}\right)$ is Wigner's 3 J symbol. We may arbitrarily let $\lambda_{i}=\frac{1}{2}$, remembering $\lambda_{f}=-\left(\mu+\lambda_{i}\right)$, and solve for $Q 2$, MI and $E 2$. The results are:

$$
\begin{align*}
& Q 2=-\sqrt{10} \Gamma^{(0)}-\frac{1}{2}, \frac{1}{2}=\sqrt{\frac{20}{3}} \frac{e Q^{* 2}}{N^{\frac{1}{2}}\left(E_{i}^{*}+M_{p}\right)}\left[-C_{3}+C_{4} M_{f}+C_{5} q_{0}^{*}\right]  \tag{A.5}\\
& \left.\begin{array}{rl}
M 1=\frac{1}{2}\left(\sqrt{3} \Gamma_{\frac{1}{2}, \frac{1}{2}}^{(-)}-3 \Gamma(+)\right. \\
-\frac{3}{2}, \frac{1}{2}
\end{array}\right)=  \tag{A.6}\\
& \frac{e Q^{*}}{\sqrt{N}\left(E_{i}^{*}+M_{p}\right)}\left[\left(2\left(E_{i}^{*}+M_{p}\right)+q_{0}^{*}\right) C_{3}\right. \\
& \\
& \\
& \left.-C_{4} M_{f} q_{0}^{*}-C_{5} q^{2}\right]
\end{align*}
$$

From these expressions we observe the following:

1. Our expressions for the multipoles have the correct threshold behavior; namely, $Q 2 \propto Q^{* 2}, M 1 \propto Q^{*}$, and $E 2 \propto Q^{*}$.
2. When the photon is real, $\mathrm{C}_{5}$ does not contribute to the cross section.
3. Since for real photons $\left(q_{0}^{* 2}-Q^{2}=q^{2}=0\right)$, $E 2$ is known to be at most a few percent of Ml. Setting $E 2=0$ we have

$$
\begin{equation*}
C_{4}(0)=\frac{C_{3}(0)}{M_{f}} \tag{A.8}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\operatorname{MI}\left(q^{2}=0\right)=\frac{2 Q^{*} e}{\sqrt{N}} c_{3}\left(q^{2}=0\right) \tag{A.9}
\end{equation*}
$$

4. The procedure of multipole decomposition described above can be applied to higher resonances. As long as the target particle has spin $1 / 2$, we have at most 3 multipoles no matter what the spin of the excited state may be. When the target particle has spin greater than $1 / 2, s_{i}>1 / 2$, we have more multipoles than 3 ; but we will have more $\lambda_{i}$ to choose from and hence will always have enough equations like (A.2), (A.3) and (A.4) to determine all the multipoles.

The matrix element squared summed over $\lambda_{i}$ and $\lambda_{f}$ can then we written down using Eq. (135.1) and Eq. (135.2) of D.D.M. In our example we have

$$
\begin{equation*}
\sum_{i} \lambda_{f}\left|\Gamma_{\lambda_{f} \lambda_{i}}^{(0)}\right|^{2}=\frac{1}{5}(Q 2)^{2} \tag{A.10}
\end{equation*}
$$

$$
\begin{align*}
\sum_{i} \lambda_{f}\left|\Gamma_{\lambda_{f} \lambda_{i}}\right|^{2} & =\sum_{\lambda_{f} \lambda_{i}}\left|\Gamma_{\lambda_{f}}(-)\right|_{i}^{2}=\sum_{f_{i} \lambda_{i}}\left|\Gamma_{\lambda_{f} \lambda_{i}}(x)\right|^{2}=\left.\sum_{\lambda_{f} \lambda_{i}}\left|\Gamma_{\lambda_{f} \lambda_{i}}\right|^{(y)}\right|^{2} \\
& =\frac{(M I)^{2}}{3}+\frac{(E 2)^{2}}{5} \tag{A.11}
\end{align*}
$$

These expressions can always be checked against the similar expressions obtained by using traces and projection operators. For example, in our case:

$$
\begin{align*}
\sum_{\lambda_{f} \lambda_{i}}\left|\Gamma_{\lambda_{f} \lambda_{i}}(x)\right|^{2}= & \left.-\operatorname{Tr} \frac{\not p_{i}+M_{p}}{2 M_{p}} \pi_{v}(x) \gamma_{5} \frac{\not p_{f}+M_{f}}{2 M_{f}}\right\}_{\nu_{\mu}}-\frac{2}{3} \frac{P_{f \mu} P_{f \nu}}{M_{f}^{2}} \\
& \left.+\frac{1}{3 M_{f}}\left(P_{f \nu} \gamma_{\mu}-P_{f \mu} \gamma_{\nu}\right)-\frac{\gamma_{\nu} \gamma_{\mu}}{3}\right\} \gamma_{5} \pi_{\mu}^{(x)} \tag{A.12}
\end{align*}
$$

where

$$
\begin{equation*}
\pi_{v}^{(x)}=C_{3}\left(\not \subset g_{v_{x}}-q_{v} \gamma_{x}\right)+c_{4}\left(q \cdot P_{f} g_{v x}-q_{v} P_{f x}\right)+C_{5}\left(q \cdot P_{i} g_{v_{x}}-q_{v} P_{i x}\right) \tag{A.13}
\end{equation*}
$$

It is probably worth mentioning that the method using Eqs. (A.10) and (A.11) takes much less effort than the one using Eq. (A.12) unless the trace in the latter is taken by computer. We have used all methods checking both by hand calculations and by the computer program of A. C. Hearn ${ }^{24}$.

## FOOTNOTES AND REFERENCES

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5. See, for example, R. Taylor's report in the Proceedings of the International Conference on Electrons and Photons at High Energies at SLAC (1967).
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7. L. Durand, P. DeCelles and R. Marr, Phys. Rev. 126, 1882 (1962). In this reference, the multipole analysis is done in the brick wall system. Their results can easily be transformed into the rest system of the $N^{*}$.
8. G. F. Chew, M. L. Goldberger, F. E. Low, Y. Nambu, Phys. Rev. 106, 1345 (1957).
9. J. D. Bjorken and J. D. Walecka, Ann. Phys. (N.Y.) 38, 35 (1966).
10. M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963); Nuovo Cimento 27, 309 (1963); Ph. Salin, Nuovo Cimento 32, 521 (1964).
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12. These subsidiary conditions merely tell us that the explicit representation of $\Psi_{v}$ in the rest frame of the $N_{*}$ can be given in the form
shown in the appendix (see equations following (A.1)).
13. We write the cross section in terms of $G_{1}$ and $G_{2}$ anticipating that our formula can be used for calculating many other things such as the radiative tail due to the $3-3$ resonance, see Y. S. Tsai, Nuclear Structure, ed. by R. Hofstadter and L. Schiff (Stanford University Press, Stanford, California, 1964) p. 221; or the contribution of the 3-3 resonance to pair production, see S. D. Drell and J. D. Walecka, Ann. Phys. (N.Y.) 28, 18 (1964). Notice that the normalization used for $G_{1}$ and $G_{2}$ in this paper is slightly different from that of the above two references.
14. The metric used in this paper is such that $P_{1} \cdot P_{2}=E_{1} E_{2}-P_{1} \cdot P_{2}$. The units used are $\hbar=c=1$ and $e^{2} / 4 \pi=\alpha$. Bosons are normalized such that there are 2 E particles per $\mathrm{cm}^{3}$ and the fermions are normalized such that there are $E / M$ particles per $\mathrm{cm}^{3}$, where $E$ is the energy of the particle and $M$ is its mass.
15. $G_{1}=T_{\perp}$ as long as $P_{i}$ does not have any transverse components. This is true in the laboratory system, in the rest system of $N^{*}$, or in the brick wall system.
16. The numerator on the right hand side of Eq. (2.11) is determined by requiring that the integration with respect to $M_{f}^{2}$ gives unity.
17. If we assume the matrix element for $\mathbb{N}^{++} \rightarrow p+\pi^{+}$to be $g M_{p}^{-1} \varphi\left(p_{f}^{-} p\right)$ $p_{\mu} \psi_{\mu}\left(p_{f}\right)$, we obtain $\Gamma=\frac{l}{12 \pi} \frac{E^{*}+M_{p}}{M_{f}} \frac{g^{2}}{M_{p}^{2}} P^{* 3}$ where $E^{*}$ is the energy of the decayed proton. If $\Gamma=.12 \mathrm{GeV}$ at resonance, we obtain $\frac{\mathrm{g}^{2}}{4 \pi}=16.4$ which is very close to the pion nucleon coupling constant $\frac{g^{2}}{4 \pi}=14.7$.
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22. R. H. Dalitz and D. R. Yennie, Phys. Rev. 105, 1598 (1957).
23. From their Eqs. (2.13) and (2.14) their unit of $e^{2}$ must be $e^{2} / 4 \pi=\alpha$. If this is so, their Eq. (2.16) and Eq. (2.16') are both wrong by a factor of $4 \pi$.
24. A. C. Hearn, Comm. of the A.C.M. 2, 573 (1966). See also "REDUCE User's Manual", Stanford Institute of Theoretical Physics Report No. ITP-247 (unpublished).

## TABLE I

## Estimates of Background and Errors

| $-q^{2}$ | $\frac{\text { Background }}{\text { Peak }}$ | Experimental <br> Error <br> $\%$ | $\left(\Delta F^{2} / F^{2}\right)$ |
| :--- | :---: | :---: | :---: |

$$
\begin{aligned}
& \begin{array}{l}
e^{-6.3 \sqrt{-q^{2}}\left(1+9 \sqrt{-q^{2}}\right)} \\
0.524 \\
0.188 \\
0.0856 \\
0.0332 \\
0.00454 \\
0.00095
\end{array} \\
& \begin{array}{l}
\left|1-\frac{q^{2}}{.75}\right|^{-5} \\
0.535 \\
0.186 \\
0.0778 \\
0.02 .1 \\
0.00350 \\
0.000837
\end{array} \\
& \begin{array}{l}
1-\left.\frac{q^{2}}{.77}\right|^{-5} \\
0.543 \\
0.193 \\
0.0819 \\
0.029 \\
0.00383 \\
0.000924
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& \begin{array}{ll}
\left.1-\frac{q^{2}}{.59}\right)^{-4}\left(1-\frac{q^{2}}{.54}\right)^{-4} 1 \\
0.535 & 0.507 \\
0.193 & 0.171 \\
0.0858 & 0.0727 \\
0.0331 & 0.0269 \\
0.00553 & 0.00426
\end{array}
\end{aligned}
$$

## FIGURE CAPIIONS

Fig. 1. The Born diagrams which contribute to the background.

Fig. 2. Electroexcitation of the $N^{*}$.

Fig. 3. Comparison of the fits using Eqs. (2.18), (2.25) and the experimental data: (a), (b) and (c) are Lynch, et al.'s data which are given in terms of Eq. (2.26) versus $K \equiv\left(M_{f}^{2}-M_{p}^{2}\right) /\left(2 M_{p}\right)$ for fixed $q^{2}$ and $E_{2} ;(d)$, (e) and (f) represent Brasse, et al.'s data which are given in terms of $d^{2} \sigma / d \Omega_{2} d p_{2}$ versus $p_{2}$ for fixed $\theta$ and $E_{1}$. Two forms of the width function $I$ are illustrated. The dotted lines are obtained by using Eq. (2.12) in the cross section (2.18), and the solid lines are from Eq. (2.13). The arrow on each abscissa indicates the position of the $N^{*}(1.236)$ resonance. In (e) an example is given of the radiative corrections which have been applied by Brasse, et al. to the data of (e), (f) and (g).

Fig. 4. Examples of Feynman diagrams which can be calculated easier in terms of $C_{3}, C_{4}$ and $C_{5}$ than in terms of $G_{1}$ and $G_{2}$.


FIG. I


FIG. 2




[^0]:    ${ }^{\dagger}$ supported by the U. S. Atomic Energy Commission.

