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KINEMATICAL RESOLUTION OF A STREAMER CHAMBER FOR EVENTS WITH UNOBSERVED INTERACTION VERTEX*

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I. INTRODUCTION

The completion and first successful test of the SLAC large streamer chamber (STC) magnet system[1] in a high energy photon beam led us to examine systematically related problems connected with the analysis of multiprong events; we focused our interest especially on 5-prong 3-constraint events and 3-prong 0-constraint events. Employing a chain of analysis procedures SYBIL-TEUTA which have been newly developed (and which will be briefly described later), we investigated the question of the momentum resolution and the angular resolution in a 3-camera system. In particular we studied the spectrum of possible kinematical decisions, when the film data were taken from a system where:

(a) The geometrical analysis has to deal with a non-uniform magnetic field (~ 30%),

(b) The interaction vertex is not observed, (since the interaction takes place in a target (H_2) where no streamers are being formed),

(c) The error correlation between tracks of the same event are large,

(d) The incident photon energy is not measured.

II. DATA INPUT

The study has been done using simulated data input. The analysis of simulated events permits--since the correct result is already known--a general study of the

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kinematical resolution of the STC magnet system with respect to random input errors. Thus the results obtained from this investigation are valid under the assumption that for the case of real events possible systematic errors are small compared with the random errors.

In Section V, the result of the full analysis of a small sample of measured real events obtained with the SLAC STC is given, concerning the error of the fitted parameters and the absolute calibration of the STC, which confirm the assumptions made for the fake events. With the help of a fake program[3] three views of film coordinates with Gaussian errors have been generated, which correspond to streamer chamber tracks in a non-uniform magnetic field; the optical parameters for the data simulation have been chosen closely to the real parameters of the SLAC large streamer chamber system.[2] Figure 1 shows one of the first pictures taken with the SLAC STC. The fit of the optical parameters, including large optical distortions and the first preliminary analysis of a few measured multiprongs, indicate (with limited statistics) that the average measurement error in the film coordinates (u,v) is consistent with (35-mm film)

$$\Delta u = \Delta v = \pm 7-9 \text{ microns} .$$

The demagnification between real space and filmplane is about 75. For the simulated film data and the geometry program, a setting error of ± 8 microns in filmplane has been assumed. The STC has dimensions of $60 \times 150 \times 200$ cm. The assumed magnetic field (15 kG, 30% non-uniformity and up to 40% radial component) corresponds to the actual field in the large STC magnet.

Streamer Chamber Analysis

The following two event types have been simulated and analyzed:

(1) $\gamma + p \longrightarrow p + \rho^{\circ} + \pi^{+} + \pi^{-} \longrightarrow p\pi^{+}\pi^{-}\pi^{+}\pi^{-}$

(2) $\gamma + p \longrightarrow p + \omega \longrightarrow p\pi^{+}\pi^{-}\pi^{\circ}$

The production amplitude was assumed to be constant. The t distribution of the vector mesons was sampled from an exponential distribution, the γ energy from a bremsstrahlung spectrum. S wave and phase-space distributions have been assumed for ρ° and ω decays, respectively. The invariant masses of the vector mesons were sampled from Breit-Wigner distributions.

The interest in this event type is related to the copious production of vector mesons in 3 and 5 prongs, which can be expected in high energy photoproduction on hydrogen.

III. ANALYSIS PROCEDURES, SYBIL AND TEUTA

A method of reconstruction of high energy trajectories and of the interaction vertex in a non-uniform magnetic field has been reported in a previous paper.[4] A newly developed computer procedure, SYBIL, performs the least square fit to the film planes of a 3-camera system.

By fitting the projections of the special trajectories directly to the measurements on the three filmstrips (rather than fitting a special trajectory to spatially reconstructed points (x, y, z)), one approximates better the usual assumption of having Gaussian distributed and uncorrelated data input. In the following, we list the essential features of the geometry program SYBIL.

(a) One minimizes $X^2 = d^T G d$. d is the perpendicular distance of a measured point in the film plane to the projection of the corresponding track. T stands for transposed. G is the weight matrix.

(b) All charged tracks and the unmeasured vertex are optimized in 3 (or 2) film planes simultaneously.

(c) The hypothetical tracks and their projections into the film plane are represented by the numerical solution of the equation of motion of a charged particle in a magnetic field.

(d) By solving numerically the differential equation of motion, the procedure SYBIL can be used in principle for any arbitrary magnetic field.

(e) Computation of corresponding points in different views or rotation into appropriate stereo systems are not required. The procedure uses the measurements in all film views independently.

(f) The optimized parameters are

$K_i = 1/p_i$

p_i = momentum of the i^{th} particle

λ_i = dip angle of the i^{th} particle

φi = azimuth of the i^{th} particle

$i = 1, 2, 3, \ldots N$

N = number of tracks

Xo, Yo, Zo = vertex, common to all N tracks

The number of degrees of freedom is equal to the total number of all measured points minus the number of fitted parameters, which is $N_p - (3N + 3)$. The weight factor in G corresponds to the measurement error in the film plane. N_p is the number of all points on N tracks.

The error distribution in spatial coordinates of the fitted vertices of event type (1) is shown in Fig. 2. The spatial reconstruction of the vertex itself reproduced the random vertex distribution which was generated for the simulated events, indicating the degree of precision in reconstructing the vertex and the absence of obvious bias.

The kinematical constraint will be imposed by using the new least square procedure TEUTA, which has been developed to determine the mass assignment which is consistent with the geometrically reconstructed momenta, angles and their errors. TEUTA minimizes the expression:

$$(X^m - X^f)_i \, H_{ij}^{-1} (X^m - X^f)_j$$

under the constraints of conservation of momentum and energy at the reconstructed vertex. X^m are the input quantities, $1/p$, λ, f, obtained from the geometrical optimization; H_{ij} is the error matrix including all correlations; X^f are fit-parameters; i,j, are indices going from 1 to 3N, where N is the number of measured tracks. Since TEUTA is treating incoming and outgoing particles, as well as their correlations, on the same footing, it is straightforwardly applicable for inelastic colliding beam interactions. For details about the program and the applied mathematical tools, see Ref. 4.

IV. RESULT OF THE ANALYSIS OF SIMULATED EVENTS

We analyzed 100 simulated events of type (1) and 50 events of type (2) through analysis system SYBIL-TEUTA. Assuming that the mass of a final state particle could be proton, kaon or pion mass, 14 different hypotheses consistent with baryon number, charge, and strangeness conservation, had to be tested. We noticed that computing the mass square of the incident particle (photon) before starting the actual fitting of a given hypothesis gave a very useful criterion. Setting even broad intervals for the invariant mass square of the incident photon ($-2.5 < M_\gamma^2 < 2.0$ BeV^2), 85% of the wrong hypotheses could be eliminated. (The distribution of fitted real events (3-prong, 3 constraints) measured from STC pictures showed that even a limit such as $-.05 \leq M_\gamma \leq .05$ (BeV)^2 contains 90% of the 3 C fits with probabilities greater than 1%.) It is assumed that the angles of the incident photon beam are known within ± 1 mradian, which constitutes a powerful kinematical constraint.

A summary of the results of the analysis of event type (1) can be represented as follows: (We denote by "right" hypothesis the known mass assignment of the fake event and by "wrong" hypothesis any other mass assignment.)

93 events had either only one good fit ("right one"), or besides the right fit only fits with a χ^2 probability at least 100 times smaller than for the right hypothesis.

2 events had a probability for the right hypothesis which was four times bigger than for one of wrong hypothesis.

2 events had a χ^2-probability for the right hypothesis which was comparable to the wrong ones.

2 events had only wrong solutions but with a negligible probability.

1 event did not give any solution.

For all ambiguous events, the incident photon had an energy of about 15 BeV. The χ^2-probability of the geometric-reconstruction for the three cases where only a wrong hypothesis did fit, was already small.

The events of the type (2) have been fitted as 3-prong 3-constraint events in order to test how often one could mistake zero constraint events with a real 3-prong 3-constraint event. From 50 events, 48 were rejected or had probability less than 10^{-4}. The remaining had two solutions with probability of 0.09.

Within the limits of our statistics and assumptions, we could identify 95% of the events unambiguously purely by kinematical tests. This has bearing on the fact that it is not clear yet to what extent, in an STC, ionization of the tracks can be used for particles identification.

For all accepted fits of type (1), we analyzed the relevant errors of the fitted quantities, which of course depend on the assumption that the measurements on film have a standard error of about 8 microns. The distribution of the percentage error of the unknown momentum of the incident photon is plotted in Fig. 3. It appears that the unmeasured energy of the incident photon E_γ can be determined to 1%. The error distribution of the fitted momenta of all tracks is plotted in Fig. 4. The average error of the momenta of the kinematically fitted particles appears to be about 1%. The corresponding average error of the fitted angles is about 1 - 2 mradians. In order to test the sensitivity for missing particles, we plotted in Fig. 5 the error on the total CM energy. The error is less than 1%, indicating that up to a CM energy of about 5 GeV, one should get only a small contamination of missing mass events. Figure 6 shows the mass resolution for the dipion effective mass in percent, indicating a halfwidth error of about 0.6%. Considering the fact that the incident momentum of the simulated event was sampled from a bremsstrahlung spectrum (3 BeV $\leq E_{\gamma} \leq$ 15 BeV), the errors on all the quantities listed above constitute a "weighted" average for γ-p interactions between 3 and 15 BeV.

As discussed before, the trajectories are not fitted individually, which is usually done in a bubble chamber reconstruction procedure, but fitted simultaneously, imposing the constraint that all tracks originate from the same vertex. This constraint is necessary for STC events, since the interaction vertex is usually not visible. Consequently strong correlations between the fitted geometrical quantities of different tracks are to be expected. We studied this effect by comparing the result of the kinematic fit (A) including correlations between the tracks and (B) neglecting these correlations.

For about 15% of the fitted tracks, we observe differences in the fitted momenta from (A) and (B), which are greater than 4%, whereas the distribution of the differences (A) - (B) is centered around zero and has a halfwidth of .1% (Fig. 7). The dip angle and the azimuthal angle in more than 12% of the tracks showed differences greater than two degrees, which is also not consistent with the error distribution of the fitted quantities. This makes the use of one of the existing bubble chamber procedures, such as SQAW or GRIND[6] for the analysis of events without an observed vertex difficult, since they are not prepared for the input of a complete covariance matrix.

V. SOME RESULTS FROM REAL MEASURED STC EVENTS

We analyzed a small sample of V°'s which were observed in a test run of the STC letting a pencil photon beam traverse a gaseous H_2 target. The hydrogen is contained in a plastic tube suspended along the center line of the STC. The vertices, in some of these special cases visible, have not been measured. The magnetic field in this test run was only about 5 kG. The V°'s were analyzed, assuming the K° or Λ hypothesis. The distribution of the calculated K° mass (0 constraint) is plotted in Fig. 8. Two of the measured V°'s fit only the Λ hypothesis. One notices that the distribution is centered on the tabulated 498 MeV within about .5% and 62% of the events are within a bin of about 12 MeV. The average measuring error in the film plane was estimated to be 6 microns.

Assuming the observed V° was a K°, we obtained from a kinematical 1C fit a χ^2 distribution which is plotted in Fig. 9b. For the geometrical fit, we plotted the quantity

$$\sqrt{\chi^2/\text{Degrees of freedom}}$$

in Fig. 9a. This distribution reflects the fact that the assumption of a measuring error of ± 6 microns was appropriate.

In summing up the results of the analysis of simulated high energy events, we gained confidence that the SLAC streamer chamber magnet system offers a high kinematical resolution, when one accounts for the non-uniformity of the magnetic field by numerical track integration and when one uses the full error correlation from the geometrical fit for the kinematical optimization. The described analysis procedures are now used in a standard way to analyze STC pictures from a multiprong photoproduction experiment!

This study is a part of the effort of Group D (SLAC) (M. Davier, I. Derado, D. Drickey, D. Fries, R. Mozley, A. Odian, F. Villa, D. Yount) to prepare and carry out a photoproduction experiment using a large streamer chamber magnet system.

We gratefully acknowledge helpful discussions with our colleagues.

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   F. Beck, R. K. Böck, GRIND, T.C. Program Library, CERN, Geneva, Switzerland.

Figure Captions

1. Strange particle observed in the SLAC streamer chamber.

2. Distribution of errors from geometrical fit (SYBIL) of the reconstructed interaction vertex.

3. Distribution of the error of the fitted and unmeasured incident photon energy.

4. Distribution of the errors of the momenta of the fitted fake events.

5. Errors on the total CM energy of fitted fake events.

6. Mass resolution for dipion (invariant) masses, including all possible combinations (fake events).

7. Distribution of the difference A - B where

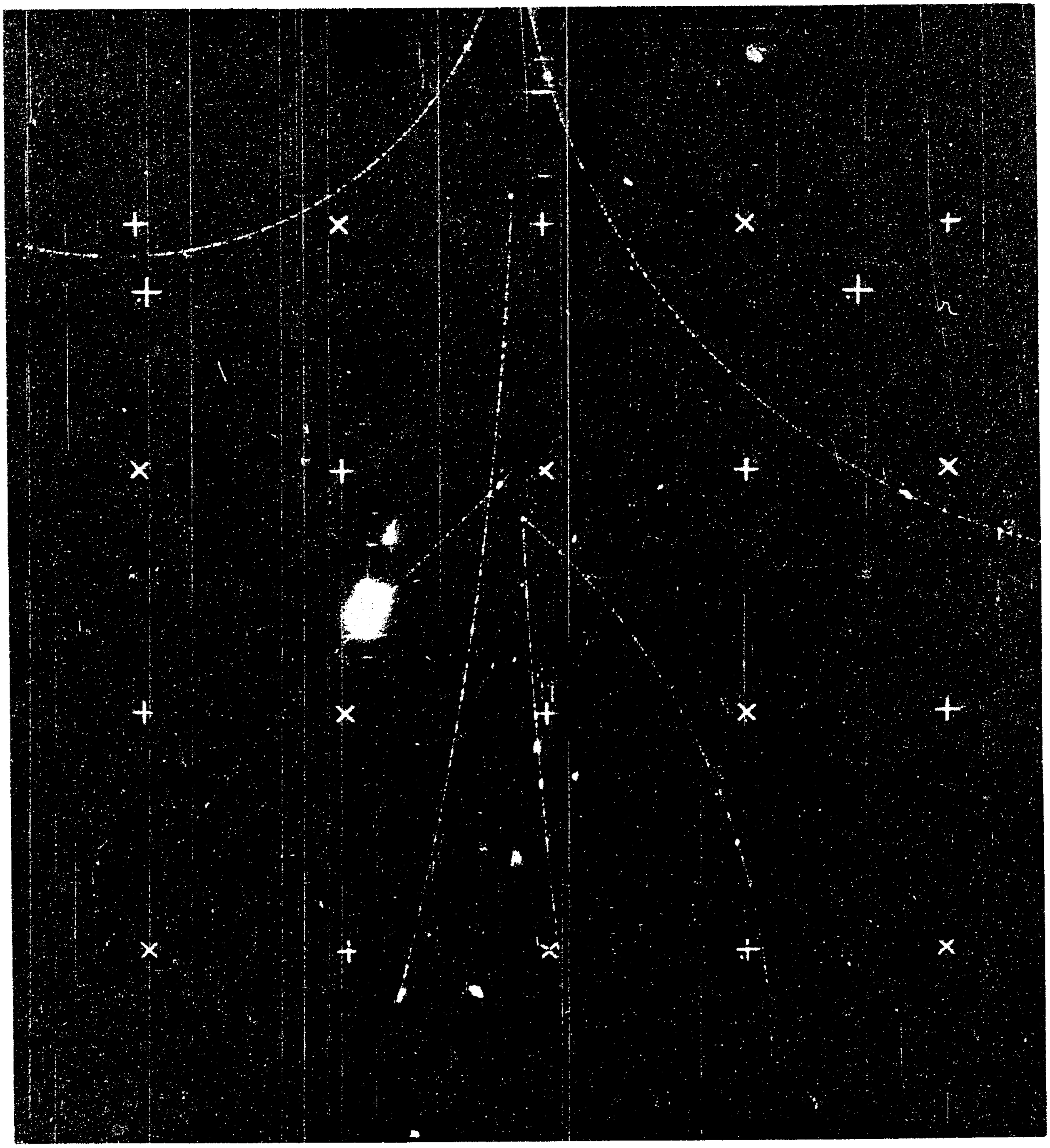
   A = momenta with full error correlations (between all tracks of the same event)

   B = momenta with partial error correlations (only due to parameters of the same track)

8. Distribution of the invariant mass of a small sample of V°'s observed in the streamer chamber (5 kG), which fit the K° hypothesis.

9a. Distribution of $\sqrt{\chi^2/N}$ from the geometrical reconstruction of the measured V°'s, where N is the number of degrees of freedom.

9b. χ^2 distribution of the one-constraint fit of the measured K°'s.



945A1

FIG. 1

DISTRIBUTION OF ERRORS OF FITTED VERTICES OF FAKE 5 PRONGS

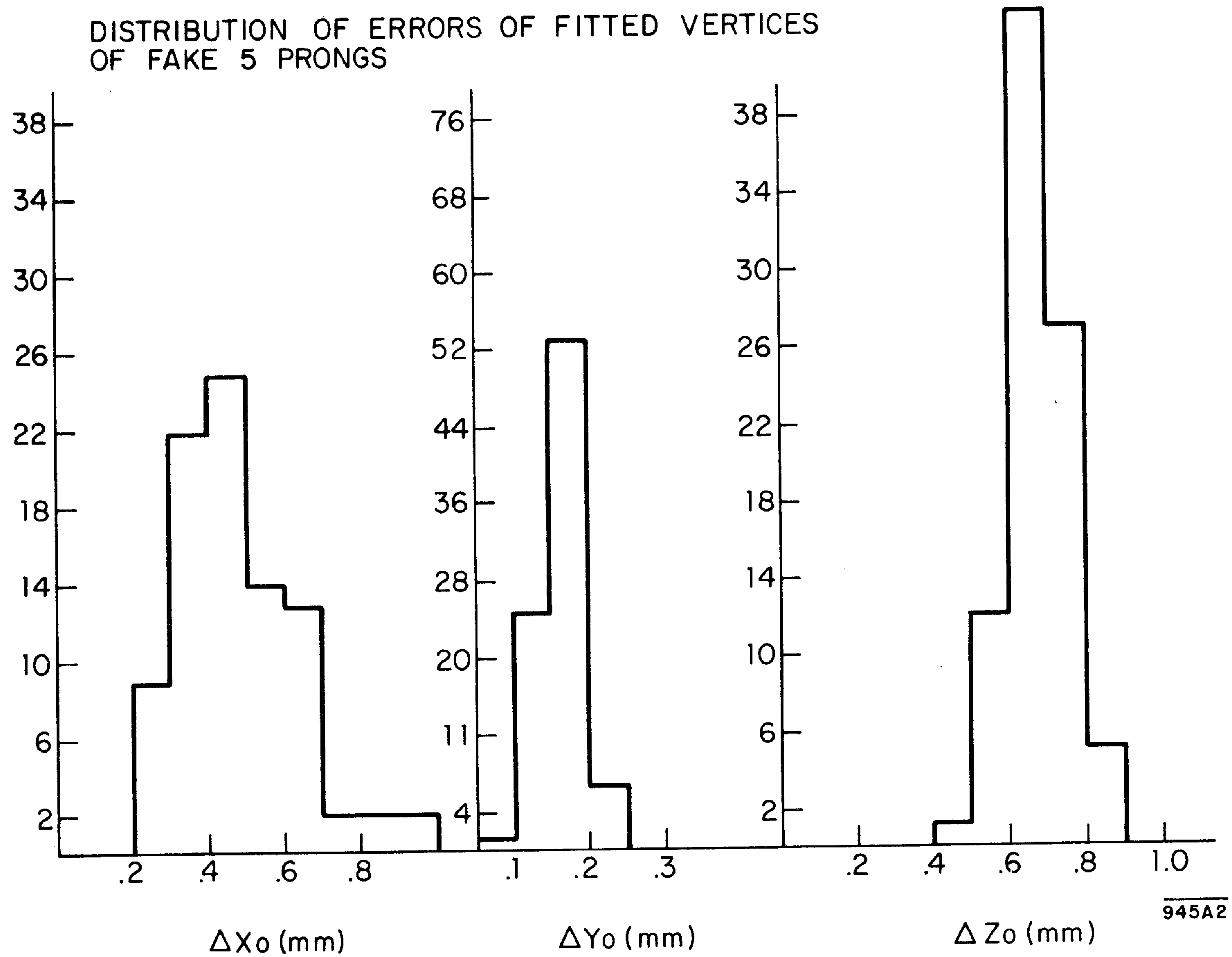
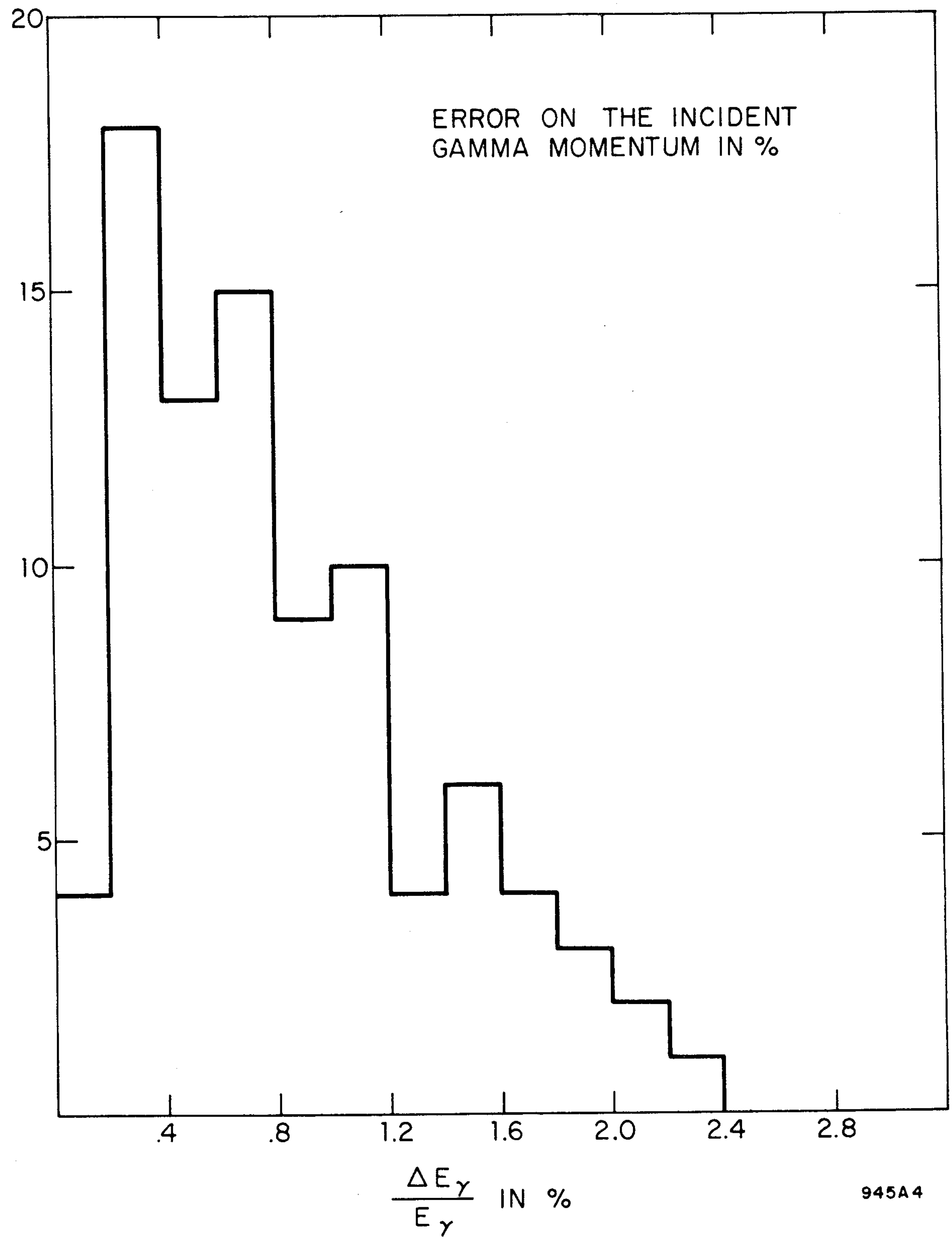
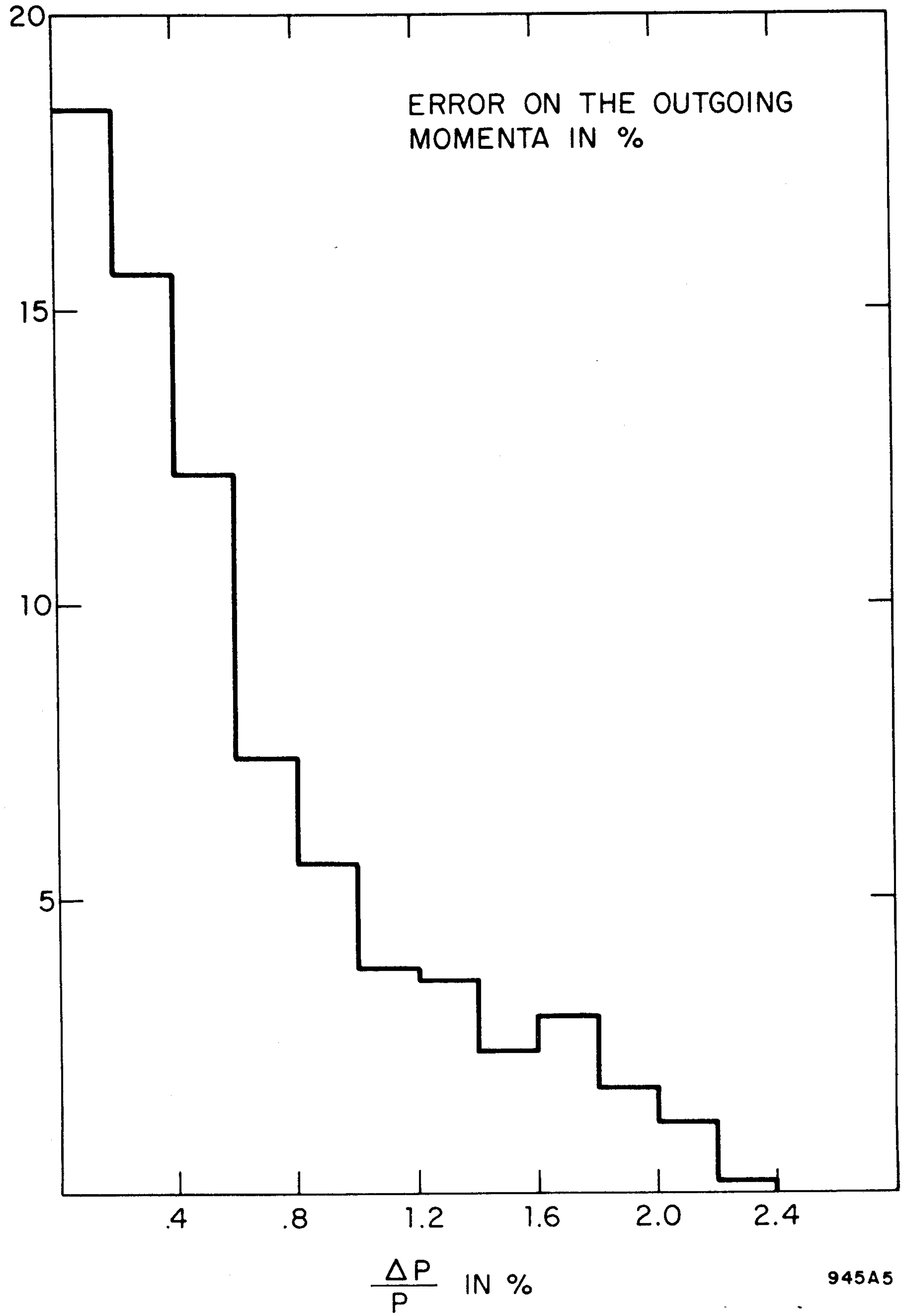


FIG. 2



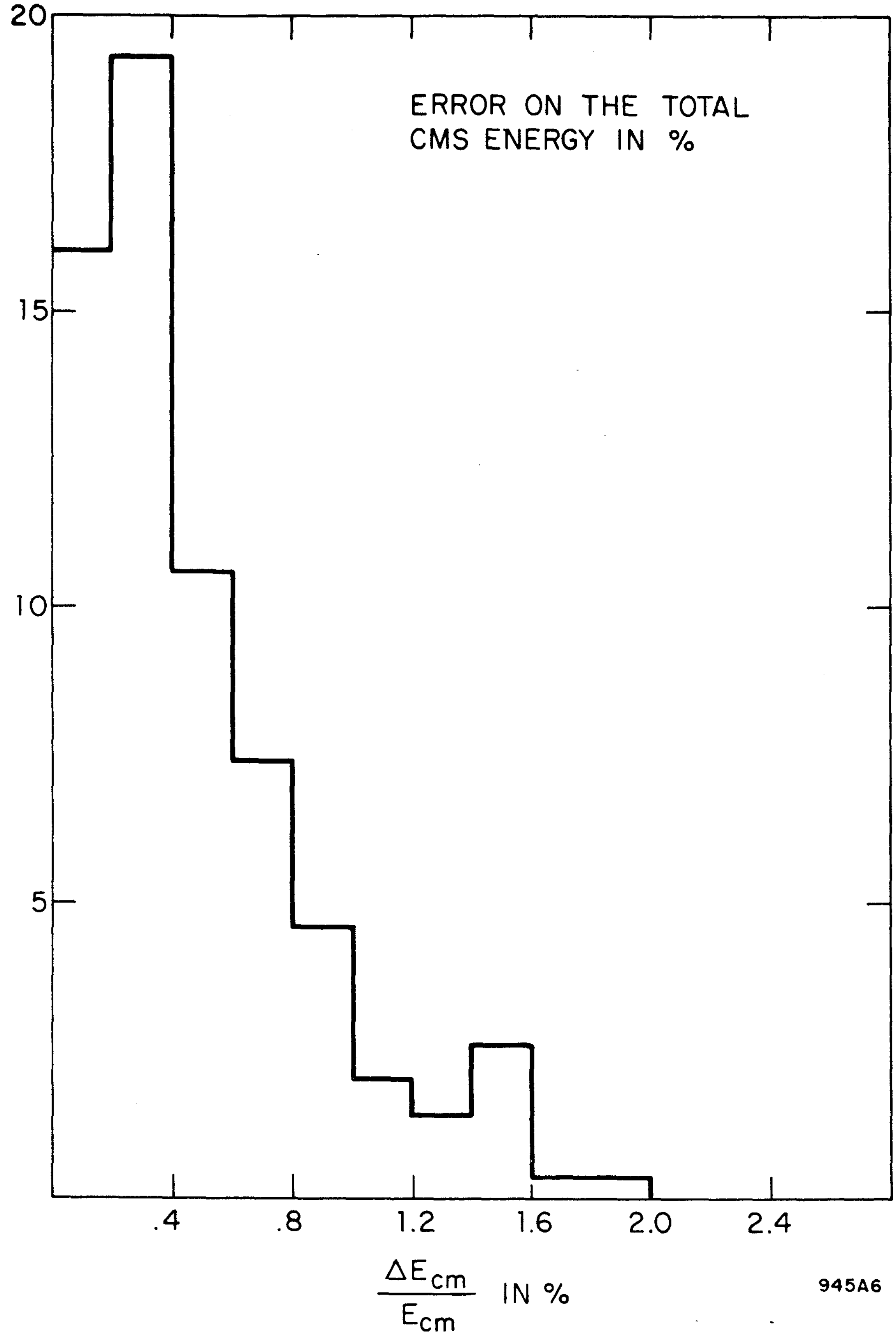
ERROR ON THE INCIDENT GAMMA MOMENTUM IN %
20
15
10
5
.4
.8
1.2
1.6
2.0
2.4
2.8
$\frac{\Delta E_\gamma}{E_\gamma}$ IN %
945A4

FIG. 3



20
15
10
5
ERROR ON THE OUTGOING MOMENTA IN %
.4
.8
1.2
1.6
2.0
2.4
$\frac{\Delta P}{P}$ IN %
945A5

FIG. 4



ERROR ON THE TOTAL
CMS ENERGY IN %
20
15
10
5
.4
.8
1.2
1.6
2.0
2.4
$\frac{\Delta E_{cm}}{E_{cm}}$ IN %
945A6

FIG. 5

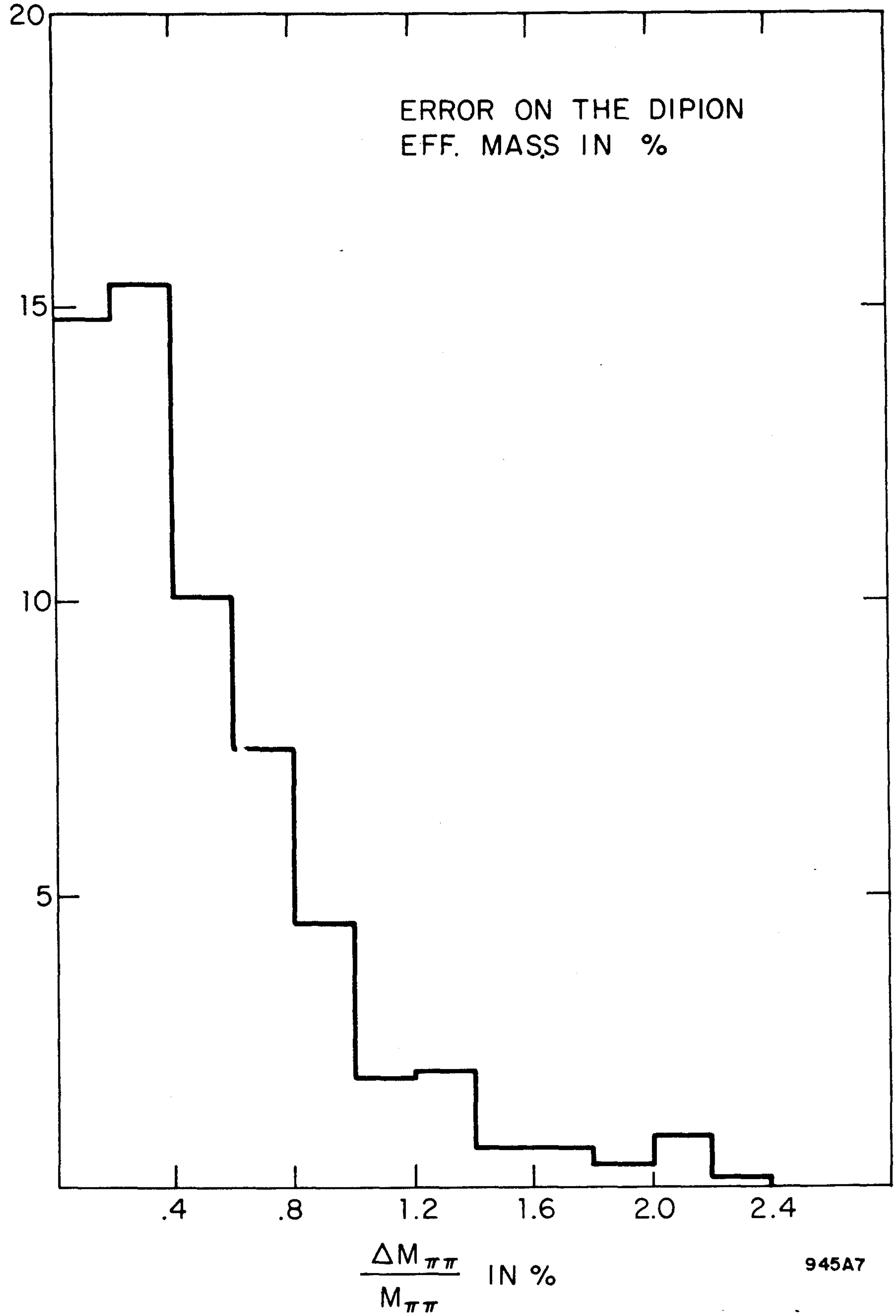
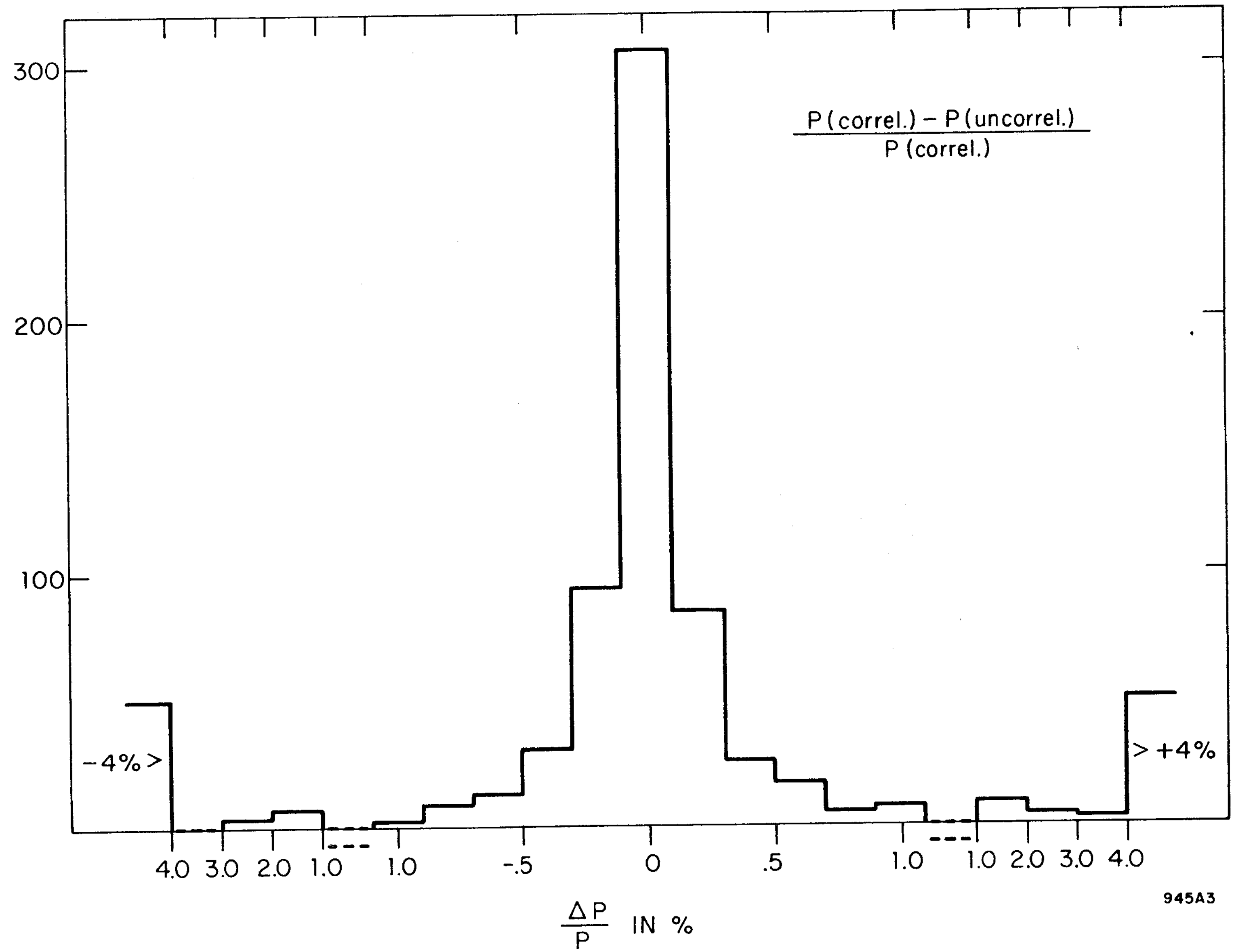

20
15
10
5
ERROR ON THE DIPION
EFF. MASS IN %
.4
.8
1.2
1.6
2.0
2.4
$\frac{\Delta M_{\pi\pi}}{M_{\pi\pi}}$ IN %
945A7

FIG. 6



300
200
100
P (correl.) – P (uncorrel.)
P (correl.)
–4% >
> +4%
4.0 3.0 2.0 1.0
1.0
-.5
0
.5
1.0
1.0 2.0 3.0 4.0
$\frac{\Delta P}{P}$ IN %
945A3

FIG. 7

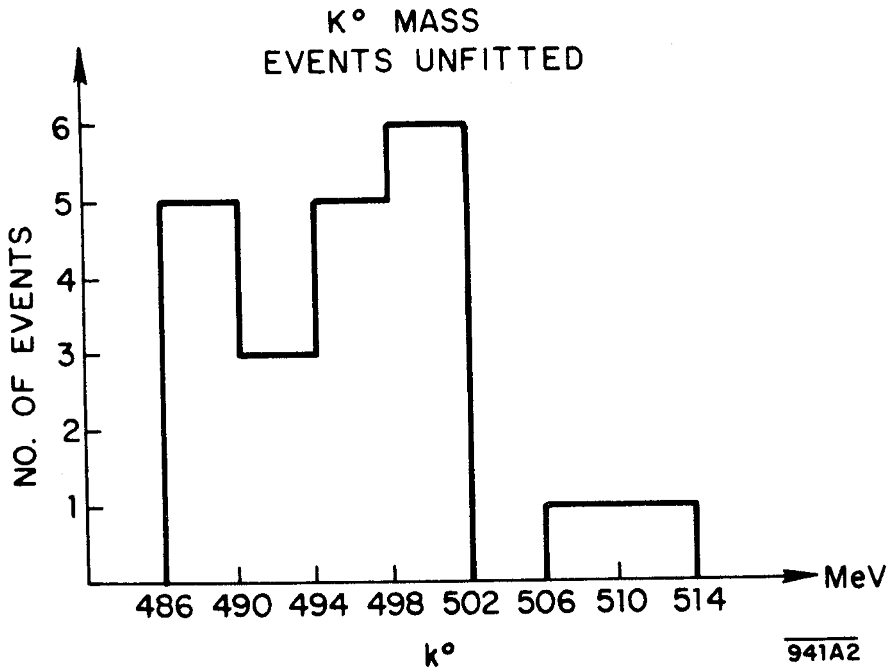

K° MASS
EVENTS UNFITTED
NO. OF EVENTS
6
5
4
3
2
1
486 490 494 498 502 506 510 514
MeV
k°
941A2

FIG. 8

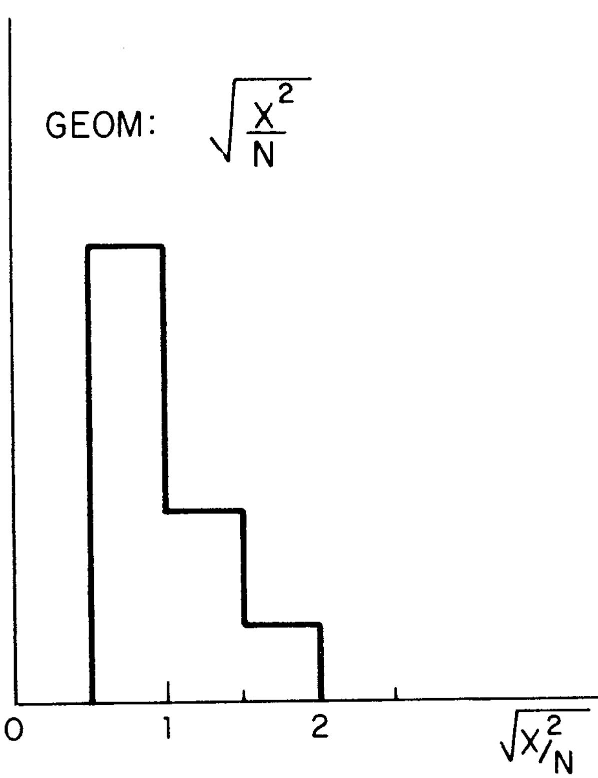


FIG. 9a

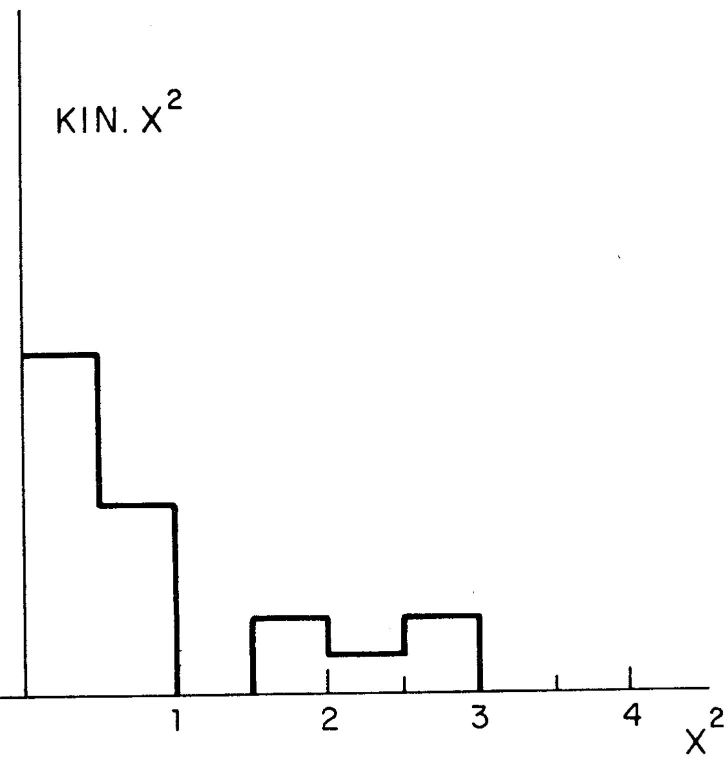


FIG. 9b

945A9