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SHRINKAGE EFFECTS AND ONE PION EXCHANGE *

G. Wolf
Stanford Linear Accelerator Center, Stanford, California †

The experimental data on quasi two-body final states in π^+p interactions have recently been studied over a fairly large π^+ momentum range.[1] In particular the energy behaviour of the cross sections and of the four momentum transfer distributions have been investigated.

For the process

$$\pi^+p \rightarrow N^{*++}\rho^o \quad (1)$$

it was found that (a) the width of the forward peak in the differential cross section $d\sigma/dt$ (where t is the square of the four momentum transfer between the incoming π^+ and the outgoing ρ^o) shrinks with increasing momentum p of the incident π^+; (b) the cross section for the process (1), $\sigma_{N^{*++}\rho^o}$, seems to behave like $\sim p^{-0.5}$. According to the authors their results disagree with the assumption that the process (1) proceeds via a one meson exchange.

It is the purpose of this note to show that the experimental data on $d\sigma_{N^{*++}\rho^o}/dt$ and on the energy behaviour of $\sigma_{N^{*++}\rho^o}$ are well described by a one-pion-exchange (OPE) model if one takes the finite widths of N^{*++} and ρ^o into account. The study is part of a rather general investigation of one-pion-exchange contributions to three and four body final states.[2]

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The OPE Model

Figure 1 shows the OPE diagram which is supposed to dominate the process (1). The contribution of this diagram will be calculated in a form factor approach, which is equivalent to a calculation based on the absorption model as long as one integrates over the N^{*++} and ρ^o decay angles. The differential cross section for the diagram in Fig. 1 is given by[3]

$$(1) \quad \frac{d\sigma}{d|t|\, d\,m dM} = \frac{1}{4\pi^3 p^{*2} s} \; \frac{G^2(t)}{(\mu^2 - t)^2} \; m^2 q_t \, \sigma_{\pi^+\pi^-}(m,t) \; M^2 Q_t \, \sigma_{\pi^+ p}(M,t)$$

where

s	square of the cms rest energy.
p*	cms momentum in the initial state.
t	square of the momentum transfer between incoming π^+ and outgoing $\pi^+\pi^-$ system. (See Figure 1)
m	$\pi_a^+\pi^-$ effective mass.
M	$p\pi_b^+$ effective mass.
q_t, Q_t	momentum of the exchanged pion in the $\pi_a^+\pi^-$ and $p\pi_b^+$ rest frame respectively.
$\sigma_{\pi^+\pi^-}(m,t)$, $\sigma_{\pi^+p}(M,t)$	cross sections for the reactions $\pi^+\pi^- \to \pi^+\pi^-$ and $\pi^+p \to \pi^+p$ respectively, where one of the pions has a mass squared of t.
G(t)	is a so-called form factor.

Dürr and Pilkuhn[4] have suggested to relate the off-shell cross sections, $\sigma(m,t)$, to the corresponding on-shell cross sections, $\sigma(m)$, in the following way: Consider the case where the $\pi_a^+\pi^-$ and $p\pi_b^+$ systems come from pure ρ^o and

N^{*++}_{33} decay respectively, then

$$(2) \qquad q_t\, \sigma_{\pi^+\pi^-}(m,t) = \left(\frac{q_t}{q}\right)^2 \frac{1+R_\rho^2 q^2}{1+R_\rho^2 q_t^2}\; q\, \sigma_{\pi^+\pi^-}(m)$$

and

$$(3) \qquad Q_t\, \sigma_{\pi^+\rho}(M,t) = \frac{(M+m_p)^2 - t}{(M+m_p)^2 - \mu^2} \left(\frac{Q_t}{Q}\right)^2 \frac{1+R_{N^*}^2 Q^2}{1+R_{N^*}^2 Q_t^2}\; Q\, \sigma_{\pi^+p}(M)$$

with q and Q being the momentum of π^+_a and the proton in the $\pi^+_a\pi^-$ and $p\pi^+_b$ rest frame respectively. m_p is the proton mass.

The values of the elastic scattering cross sections $\sigma_{\pi^+\pi^-}(m)$ and $\sigma_{\pi^+p}(M)$ are taken from experiment.[5,6] Figure 2 shows $\sigma_{\pi^+\pi^-}(m)$ and $\sigma_{\pi^+p}(M)$ for the ρ^o and N^* region respectively, as used in the calculation. R_ρ and R_{N^*} are parameters which in nonrelativistic scattering are defined as the radii of interaction.

It is obvious from the formulae (1) to (3) that the off-shell corrections for given values of m, M depend only on t, and <u>not</u> on the incoming momentum.

From a fit of the OPE model to experimental data, on the processes $\bar{p}p \to N^{*++}_{33}\overline{N^{*++}_{33}}$, $pp \to N^{*++}_{33}\, n$, $\pi^- p \to n\rho^o$ and $\pi^+ p \to N^{*++}\rho^o$ the following results on G(t), R_ρ and R_{N^*} have been obtained:[2]

With the parametrization $G(t) = \dfrac{c - \mu^2}{c + t}$

$$c = (2.29 \pm 0.27)\ \mathrm{GeV}^2$$

$$R_\rho = (8.28 \pm 2.0)\ \mathrm{GeV}^{-1} = (1.64 \pm 0.40)\ f^{-1}$$

$$R_{N^*} = (3.97 \pm 0.11)\ \mathrm{GeV}^{-1} = (0.79 \pm 0.02)\ f$$

A detailed description of the procedure adopted will follow.[2]

Comparison with Experiment

The comparison with experiment was done for the differential cross section for all events in the experimentally chosen N^*, ρ^o mass region.[7] The OPE cross section has been integrated over exactly the same N^*, ρ^o mass region. In Fig. 3a-d, the experimental data for the process (1) are given for incoming π^+ momenta of 2.35 GeV/c, 3.5 GeV/c, 4 GeV/c and 6.95 GeV/c.[8] The result of the OPE calculation is shown by the curves in Fig. 3. The curves are in close agreement with the experimental points. Both the t and the energy dependence of the experimental data are reproduced by the OPE model. No shrinkage effects other than what are predicted by the OPE model are observed. Hence, we conclude that the experimental data on the process (1) are in agreement with the assumption that one-pion exchange is the dominant production mechanism.

The apparent shrinkage in the OPE model comes about by:

1. The large N^* and ρ^o mass intervals over which the data have been taken: The minimum value of $|t|$ changes considerably over the N^*, mass region and with incoming energy (see Table I).
2. The differential cross section $d\sigma/d|t|$ does not follow an exponential law $\sim e^{Bt}$; the slope is steeper at small values of $|t|$ and less steep at larger $|t|$ values. With increasing energy the maximum of $d\sigma/d|t|$ moves towards smaller values of $|t|$, hence the fall of $d\sigma/d|t|$ becomes steeper.

We have compared the OPE model also to the process

$$\bar{p}p \longrightarrow \overline{N^{*++}} \quad N^{*++} \qquad (2)$$

which is similar to $N^{*++}\rho^o$ production in that it is likely to be dominated by OPE and it involves the production of broad resonances. The differential cross sections $d\sigma/d|t|$ (where t is now the square of the four momentum transfer between incoming proton and outgoing isobar) measured at 3.6 GeV/c and 5.7 GeV/c are shown in Fig. 3e, f.[9] The width of the forward peak shrinks by going from 3.6 GeV/c to 5.7 GeV/c. The OPE calculation as shown by the curves in Fig. 3e, f are in remarkable agreement with the data. Therefore, the observed shrinkage for $\overline{N^{*++}}\ N^{*++}$ production can be understood in the same way as outlined above for the process (1).

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REFERENCES AND FOOTNOTES

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3. See for example: E. Ferrari and F. Selleri, Nuovo Cimento 27, 1450 (1963).
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6. A compilation of π^+p cross sections was made by M.N. Focacci and G. Giacomelli, CERN Report 66-18 (1966).
7. By integrating the theoretical cross section over the N^*, ρ^0 mass region, partial waves other than the p waves from N^* and ρ^0 respectively are contributing too. In the case of the N^* region their effect on the off-shell corrections is supposed to be negligible. In the case of the ρ^0 meson, the $T = 0, 2$ S-wave contributions to $\sigma_{\pi^+\pi^-}(m)$ have been estimated from a $\pi\pi$ phase shift analysis.[5] Instead of (3), the following correction formula was used:

$$\text{(3a)} \quad q_t\, \sigma_{\pi^+\pi^-}(m,t) = \left[c_o + c_1 \left(\frac{q_t}{q}\right)^2 \frac{1 + q^2 R_\rho^{\;2}}{1 + q_t^2 R_\rho^{\;2}} \right] q\, \sigma_{\pi^+\pi^-}(m)$$

with $c_1 = 0.8 + (m - 0.6)/2$ m in units of GeV

$c_o = 1 - C_1$

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TABLE I

Minimum Four-Momentum Transfer Squared, $|t|_{min}$, for $\pi^+ p \longrightarrow N^{*++} \rho^o$ for different ρ and N^* mass values.

Lab. momentum of incoming π^+ (GeV/c)	m_{ρ^o} (GeV)	$M_{N^{*++}}$ (GeV)	$\lvert t \rvert_{min}$ (GeV^2)
2.35	0.675	1.185	0.079
	0.825	1.285	0.225
4.0	0.66	1.12	0.027
	0.86	1.32	0.122
6.95	0.64	1.12	0.013
	0.88	1.42	0.081

FIGURE CAPTIONS

1. OPE diagram for reaction $\pi^+ p \rightarrow p \pi^+ \pi^+ \pi^-$.

2. (a) Elastic $\pi^+ \pi^-$ scattering cross section in the ρ region;

   (b) Elastic $\pi^+ p$ scattering cross section in the N^*_{33} region.

3. Differential cross sections $d\sigma/d|t|$ for events in the N^{*++}_{33} , ρ^o mass region:

   (a) at 2.35 GeV/c (0.675 GeV < m < 0.825 GeV, 1.185 GeV < M < 1.285 GeV);

   (b) at 3 – 4 GeV/c (0.68 GeV < m < 0.86 GeV, 1.12 GeV < M < 1.32 GeV);

   (c) at 4 GeV/c (0.66 GeV < m < 0.86 GeV, 1.12 GeV < M < 1.32 GeV);

   (d) at 6.95 GeV/c (0.64 GeV < m < 0.88 GeV, 1.12 GeV < M < 1.42 GeV)

   Differential cross section $d\sigma/d|t|$ for events in the $\overline{N^{*++}}$, N^{*++} mass region:

   (e) at 3.6 GeV/c (1.13 GeV < $M_{\overline{N^*}, N^*}$ < 1.33 GeV);

   (f) at 5.7 GeV/c (1.15 GeV < $M_{\overline{N^*}, N^*}$ < 1.35 GeV).

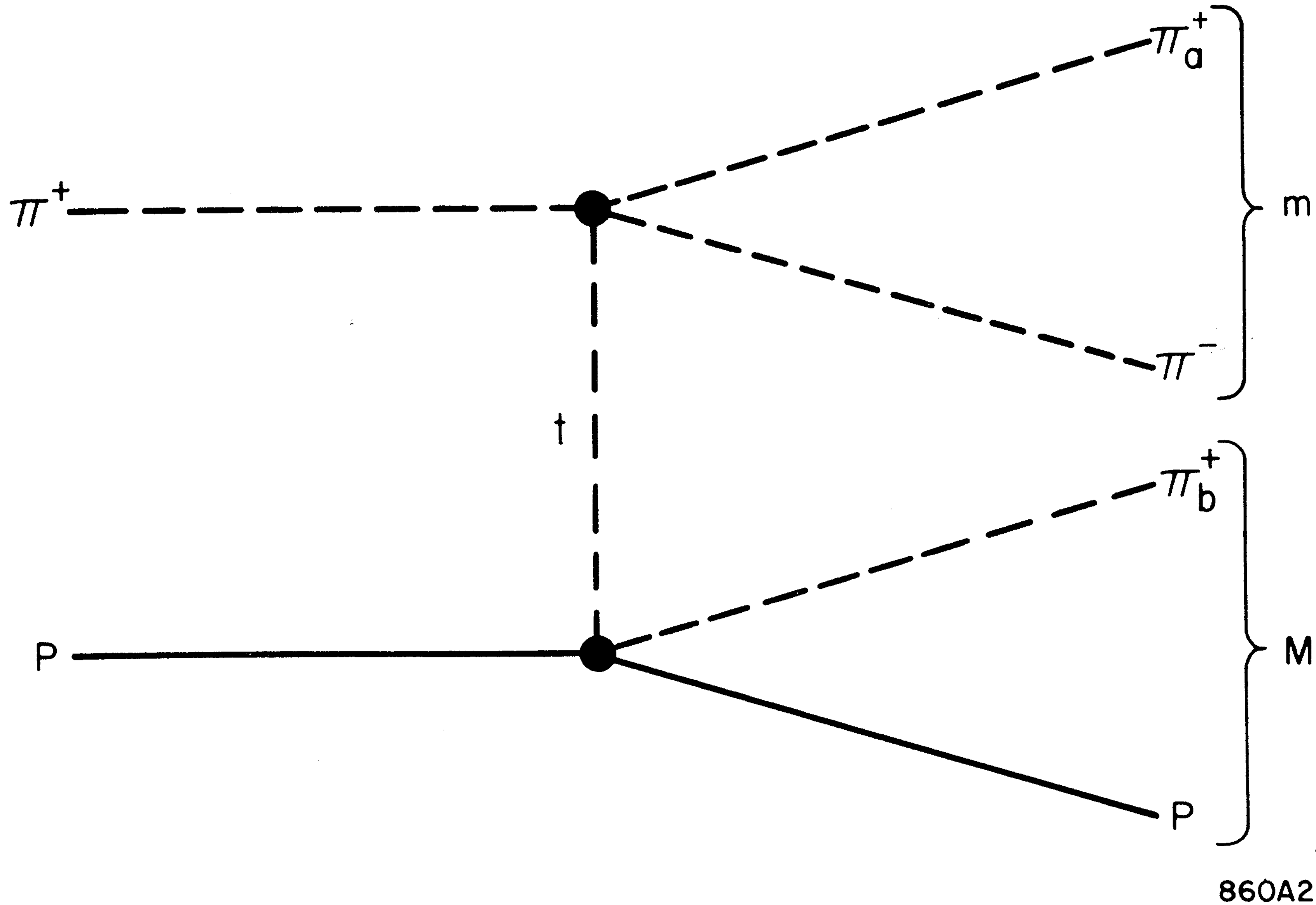
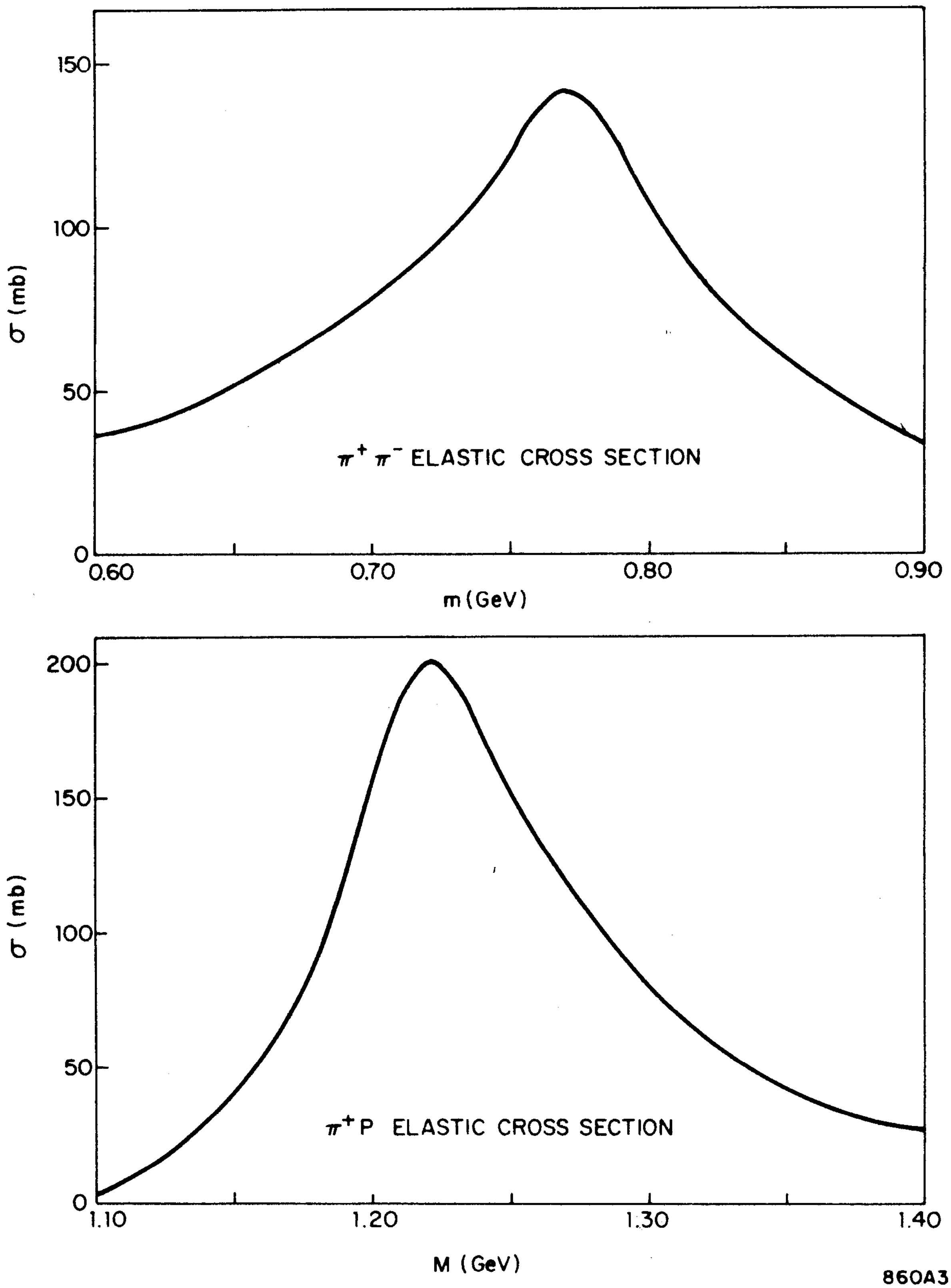
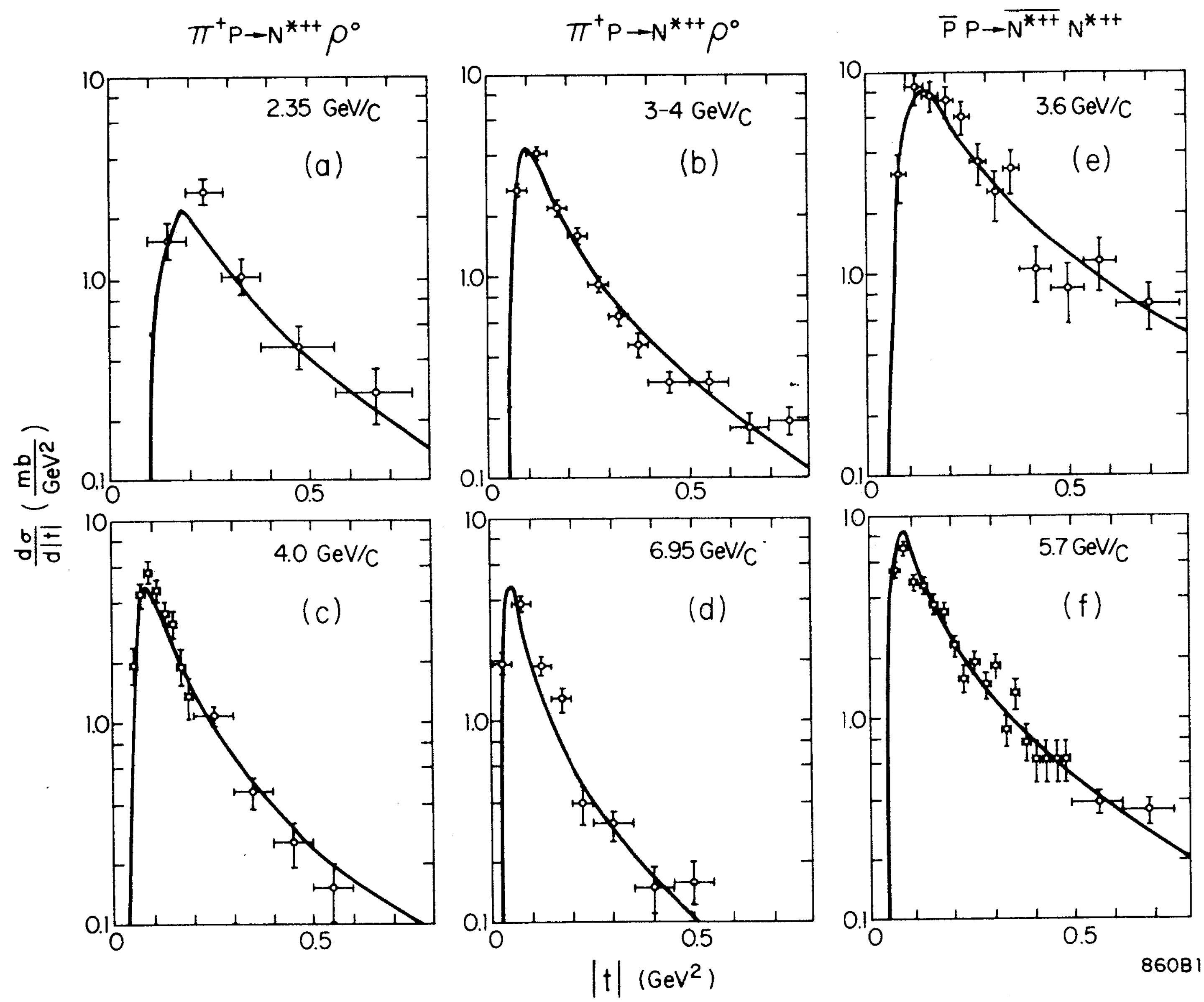


Fig. 1



150
100
50
0
σ (mb)
π⁺π⁻ ELASTIC CROSS SECTION
0.60
0.70
0.80
0.90
m (GeV)
200
150
100
50
0
σ (mb)
π⁺P ELASTIC CROSS SECTION
1.10
1.20
1.30
1.40
M (GeV)
860A3

Fig. 2



π⁺P→N*⁺⁺ρ°
π⁺P→N*⁺⁺ρ°
P̄ P→N̄*⁺⁺ N*⁺⁺
2.35 GeV/C
(a)
3-4 GeV/C
(b)
3.6 GeV/C
(e)
4.0 GeV/C
(c)
6.95 GeV/C
(d)
5.7 GeV/C
(f)
dσ/d|t| (mb/GeV2)
|t| (GeV²)
10
1.0
0.1
0
0.5
860B1

Fig. 3