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MAGNETIC CONTRIBUTION TO THE PROTON-NEUTRON MASS DIFFERENCE AND THE ROLE OF THE NUCLEON PSEUDORESONANCE *

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ABSTRACT

It is found that the case for an appreciable "magnetic" n-p mass difference as advanced recently by Cornwall and Patil is seriously weakened when the dependence of such a calculation on strong interaction dynamics is properly taken into account. The unobservable nucleon "pseudoresonance" in the renormalization function ($M_p \simeq 1230$ MeV, $\Gamma_p \simeq 160$ MeV) is shown to play a dominant role in this connection; in the absence of the Roper pole-mechanism of Cornwall and Patil it still produces a "magnetic" mass difference, $M_n - M_p \simeq 0.9$ MeV.

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I. INTRODUCTION

Recently, Cornwall and Patil[1] asserted that there could be a quite appreciable "magnetic" n-p mass difference were proper account taken of the Roper (P_{11}) resonance at 1400 MeV. Their calculation of $M_n - M_p \simeq 2.4$ MeV without the corrections for Coulomb self-energy is based on three assumptions, (a) that unsubtracted dispersion relations hold for the nucleon proper self-energy part,[2] (b) the intermediate states are dominated by the Nγ state, (c) the "magnetic" NNγ proper vertex functions are dominated by the Roper resonance. Although Cornwall and Patil[1] note that assumption (a) is closely related to the hypothesis of vanishing nucleon wave-function renormalization, nowhere in their actual calculation do they make use of this connection. (Indeed, their expression for the imaginary part of the "magnetic" proper self-energy [Eq. (8) of Ref. 1] is written in terms of the improper magnetic vertex function [the form factors $F_M^{n,p}(S)$] instead of the proper vertex.) In this paper, we find, after making use of this connection and refining the considerations of Ref. 1 somewhat, that the resultant sensitivity of this type of calculation to strong-interaction dynamics (by way of the necessary introduction of the nucleon "pseudoresonance"[3]) must seriously weaken the case for so naive a model of the n-p mass difference.

II. THE CALCULATION

Inspection of, say, the ppγ proper vertex in Bincer's[4] form,

$$e\bar{u}_p(p)\Gamma_\mu(p,p+\ell) \equiv e\bar{u}_p(p)\Big\{\Big[\gamma_\mu - i\sigma_{\mu\nu}\frac{\ell^\nu}{2M}F_2^p(W) + \ell_\mu F_3^p(W)\Big]Z(W)\Lambda_+(p\!\!\!/ + \ell\!\!\!/)$$

$$+\Big[\gamma_\mu - i\sigma_{\mu\nu}\frac{\ell^\nu}{2M}F_2^p(-W) + \ell_\mu F_3^p(-W)\Big]Z(-W)\Lambda_-(p\!\!\!/ + \ell\!\!\!/)\Big\}, \qquad (1)$$

where

$$\left[S'_F(\pm W)\right]^{-1} = Z(\pm W)(\pm W - M) \tag{2}$$

indicates that an initial refinement of the calculation of Ref. 1, would consist in the simple replacement

$$F_M^{n,p}(\pm W) \longrightarrow \Gamma_M^{n,p}(\pm W) = F_M^{n,p}(\pm W)\, Z(\pm W) \tag{3}$$

in the dispersion integral for $(M_n - M_p)$ magnetic in the W-representation. Thus,

$$(M_n - M_p)_{\text{magnetic}} = -\frac{\alpha}{16\pi M^2}\int_M^\infty \frac{dW}{W-M}\,\frac{(W^2-M^2)^3}{W^3}\left\{\left|F_M^n(W)\right|^2 - \left|F_M^p(W)\right|^2\right\}\left|Z(W)\right|^2$$

$$+\frac{\alpha}{16\pi M^2}\int_M^\infty \frac{dW}{W+M}\,\frac{(W^2-M^2)^3}{W^3}\left\{\left|F_M^n(-W)\right|^2 - \left|F_M^p(-W)\right|^2\right\}\left|Z(-W)\right|^2. \tag{4}$$

Now, from Ida's[3] analysis of the nucleon renormalization function $Z(W)$, we know that $|Z(-W)|^2 \ll 1$ for $W \geq M$ except in the neighborhood of the weak reflection of the physically unobservable "pseudoresonance" in $Z(W)$ at $W = M_P = 1230$ MeV. The resultant strong damping of the contribution to $(M_n - M_p)_{\text{magnetic}}$ from the "negative branch" leads us to regard it as a perturbation on our calculation and it is neglected henceforth. A calculation of the contribution to $(M_n - M_p)_{\text{magnetic}}$ from the "positive branch" in the "narrow-resonance" limit[5] now shows the effect of the real Roper pole to be enhanced by a factor $|Z(M_R)|^2 \approx 2$; the inclusion of the contribution from the narrow $(\Gamma_p(M_p) = 160$ MeV) nucleon pseudoresonance will only serve to further amplify this result. This suggests that we ought to examine more closely the assumption of Cornwall and Patil[1] that

$$F_M^{n,p}(s) = \mu_{n,p}\left(\frac{M^2 - M_R^2 + i\Gamma_R M_R}{s - M_R^2 + i\Gamma_R M_R}\right). \tag{5}$$

If we put Bincer's representation for the improper (proton) vertex into "Gordon" form, so that,

$$e\bar{u}_p(p)F_\mu(p,p+\ell) \equiv e\bar{u}_p(p)\left\{\left[\frac{1}{M+W}(2p_\mu+\ell_\mu) - \frac{1}{M+W}\, i\sigma_{\mu\nu}\ell^\nu\left(1+F_2(W)\right)\right.\right.$$

$$\left. + \ell_\mu F_3(W)\right]\Lambda_+(\underline{p}+\underline{\ell}) + \left[\frac{1}{M-W}(2p_\mu+\ell_\mu) - \frac{1}{M-W}\, i\sigma_{\mu\nu}\ell^\nu\left(1+F_2(-W)\right)\right.$$

$$\left.\left. + \ell_\mu F_3(-W)\right]\Lambda_-(\underline{p}+\underline{\ell})\right\}, \tag{6}$$

then it seem to us equally reasonable to take, for example,

$$F_M^p(W) = \frac{2M}{W+M}\left\{1 + \kappa^p\left(\frac{M-M_R+i\frac{\Gamma_R}{2}}{W-M_R+i\frac{\Gamma_R}{2}}\right)\right\}, \tag{7}$$

that is we embed the Roper resonance in $F_2(W)$ only. Substituting our expression (7) for $F_M^p(W)$ and an analogous one for $F_M^n(W)$ along with a one-resonance-pole fit to $Z(W)$,[6]

$$Z(W) \simeq \frac{M-M_P + i\frac{\Gamma_P}{2}}{W-M_P + i\frac{\Gamma_P}{2}}, \tag{8}$$

into the one-branch-approximation to $(M_n-M_p)_{\text{magnetic}}$, we find[7]

$$(M_n-M_p)_{\text{magnetic}} \simeq \alpha\frac{(M_P^2-M^2)^2}{16M^2M_P^3}(M_P+M)\left[\frac{(M_P-M)^2+\frac{\Gamma_P^2}{4}}{\left(\frac{\Gamma_P}{2}\right)}\right]$$

$$\times\left[\left|F_M^p(M_P)\right|^2 - \left|F_M^n(M_P)\right|^2\right]$$

$$+\frac{\alpha(M_R^2-M^2)^2}{4M_R^3}\left|Z(M_R)\right|^2\frac{(\kappa^p-\kappa^n)\Gamma_R}{(M_R+M)} \tag{9}$$

in the narrow-resonance limit. Numerically,

$$(M_n - M_p)_{magnetic} \simeq (1.9 + 0.6) = 2.5 \text{ MeV}, \tag{10}$$

with the enhanced Roper term accounting for less than 25 % of the result. Thus there is the likelihood that the correct value of $M_n - M_p$ is to emerge in this model as the difference between two dynamically sensitive numbers. On the other hand it is amusing to see that if we neglect the effect of a Roper resonance entirely, then

$$(M_n - M_p)_{magnetic} \simeq 0.9 \text{ MeV}, \tag{11}$$

with this contribution wholly a consequence of the pseudoresonance.

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REFERENCES AND FOOTNOTES

1. J. M. Cornwall and S. H. Patil, Phys. Rev. Letters 18, 757 (1967).

2. K. Nishijima, Phys. Rev. Letters 12, 39 (1964).

3. M. Ida, Phys. Rev. 136, B1767 (1964).

4. A. Bincer, Phys. Rev. 118, 855 (1960).

5. The "narrow-resonance" limit is characterized by the approximation,

$$\left| s - M_R^2 - iM_R\Gamma_R \right|^{-2} \longrightarrow (M_R\Gamma_R)^{-1}\,\pi\delta(s - M_R^2).$$

In our reconstruction of the calculation of Ref. 1 we find $M_n - M_p \simeq 2.7$ MeV in this limit; since this result differs from that quoted in Ref. 1 by only 13%, the introduction of a width-function $\Gamma_R(s)$ reflecting the p-wave character of the Roper resonance does not seem waranted.

6. This fit automatically satisfies the additional requirements, $Z(M) = 1$, $Z(\infty) = Z = 0$.

7. We neglect the terms proportional to $(\kappa^p)^2 - (\kappa^n)^2$.