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SOME DIFFICULTIES IN THE FIELD-THEORETIC APPROACH TO THE PROTON-NEUTRON MASS DIFFERENCE

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ABSTRACT

Some consequences of a manifest lack of gauge-invariance in the usual field-theoretic approach to the calculation of the proton-neutron mass difference are pointed out. In particular, it is shown that the assumption $Z = 0$ (where Z is the strong wavefunction renormalization constant) is inessential to this problem of gauge-dependence. A practical calculation of $\delta M = M_p - M_n$, assuming $Z = 0$ and relying on the method of low-mass dominance of Drell and Pagels, is carried out. The cutoff for this calculation, $\lambda^2 = 3.6$ for $(g^2/4\pi) = 15$, is determined by requiring that $Z = 0$ be satisfied $O(g^2)$. If we accept the possibility that the prescription adopted by Bassetto and Paccanoni (BP) for extracting a gauge-independent result has some basis, then we find, on including the effects of the anomalous nucleon moments, that $\delta M = -0.4$ MeV, in rough agreement with their earlier result. However, inclusion of the vertex corrections $O(\alpha)$, which are neglected by BP, leads to the unsatisfactory result $\delta M = +0.8$ MeV.

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1. INTRODUCTION

Recently, it was argued by Bassetto and Paccanoni[1] that the conceptually simple approach to the calculation of the proton-neutron mass difference afforded by field-theoretic sum rules[2] in the limit of vanishing nucleon wave function renormalization[3,4] could yield a mass difference with the right sign. The smallness of the result obtained[1,5] was attributed in part to an abbreviated treatment there of corrections $O(\alpha)$, which neglected the electromagnetic corrections to the strong vertices; indeed, it was conjectured in Ref. 1 that were computational difficulties overcome, this "method would be reliable also on quantitative grounds".

Although the first purpose of this paper is to provide a more quantitative test of this conjecture which does not at the same time require "drastic" off-shell approximations[1], we will in the course of our discussion be obliged to comment on the persistent infra-red problem which appears to beset the field-theoretic approach[1,6] to the proton-neutron mass difference, and which is connected to the ordinarily gauge-dependent results derived from such an approach. On the other hand, if we accept the possibility that the prescription adopted by Bassetto and Paccanoni[1] for extracting a gauge-independent result for δM may have some basis, the method of low-mass dominance developed by Drell and Pagels[7] allows us, for reasons indicated in the next section, to go ahead and calculate the vertex corrections $O(\alpha)$ as well. [We remark that the more difficult calculation of the contribution from photon exchange, which is a problem in "Jost enhancement"[8], is specifically omitted here (as it was, for that matter, in Pagel's recent work[9] on the proton-neutron mass difference).]

2. $Z = 0$[10] CALCULATION WITHOUT VERTEX CORRECTIONS

To make clear that the assumption $Z = 0$ is inessential to the problem of gauge dependence in the field-theoretic approach, we observe that the $Z = 0$ calculation of the proton-neutron mass difference, $\delta M \equiv M_p - M_n$, may be carried out either by taking the limit $Z \to 0$ in the expression derived by Fried and Truong[6] for the case $Z \neq 0$ [either Eq. (15) in the first or Eq. (11) in the last of Ref. 6, after including the essential e^2-contribution from $Z_{p,n}$.[11]], so that

$$\delta M = \lim_{Z \to 0} \frac{Z^{-1}\delta(e^2) - Z^{-2}S\Delta(e^2)}{1 - Z^{-1}G + Z^{-2}S(\xi_p - \xi_n)} = - \frac{\Delta\ (e^2)}{\xi_p - \xi_n}, \tag{2.1}$$

where[2,12]

$$Z = 1 - \frac{1}{\pi}\int_R dW[\tau_+(W) + \tau_-(W)], \tag{2.2}$$

$$S = - \frac{1}{\pi}\int_R dW[(W-M)\tau_+(W) - (W+M)\tau_-(W)], \tag{2.3}$$

$$G\delta M = - \frac{1}{\pi}\int_{R_p} dW\left\{[(W-M_p)\tau_+^p(W)-(W+M_p)\tau_-^p(W)]-[(W-M_n)\tau_+^n(W)-(W+M_n)\tau_-^n(W)]\right\}, \tag{2.4}$$

$$\delta(e^2) = - \frac{1}{\pi}\int_{M+\lambda_{min}}^{\infty} dW[(W-M)\tau_+^e(W) - (W+M)\tau_-^e(W)], \tag{2.5}$$

or, alternatively, by requiring, as in Ref. 1, that

$$Z_p - Z_n = 0, \tag{2.6}$$

with

$$\begin{aligned} Z_p &= Z + \xi_p\delta M + \Delta(e^2), \\ Z_n &= Z + \xi_n\delta M. \end{aligned} \tag{2.7}$$

Note that the ordinary (Lorentz gauge) mode of calculation (see below), $\Delta(e^2)$ is infra-red divergent.

By relying on the method of low-mass dominance[7], we are able to consider a "practical" calculation of δM which avoids the off-shell approximations of Ref. 1. The rationale for this sort of approach is furnished by the "zeroth approximation"[13],

$$Z = 1 - \frac{3g^2}{8\pi^2} \int_M^{\lambda M} dW \, \frac{q}{W} \left\{ \frac{(E-M)}{(W-M)^2} + \frac{(E+M)}{(W+M)^2} \right\}$$
$$= 1 - \frac{3g^2}{32\pi^2} (\ln \lambda^2 + \frac{1}{\lambda^2} - 1) = 0, \tag{2.8}$$

which determines a cutoff[14] $\lambda^2 = 3.6$ for $(g^2/4\pi) = 15$; this result seems not too unreasonable when compared with Pagel's[9] value of $\lambda^2 \sim 3$.

Were we now, as in Ref. 1, to neglect the electromagnetic splittings of the pion-nucleon couplings $g_{N\pi}$, together with the contributions from the nucleon anomalous moments, keeping only the driving term corresponding to the intermediate state (p,γ), we would find[15]

$$0 = Z_p - Z_n = \frac{\delta M}{M} \left(\frac{g^2}{4\pi^2}\right)\left(\frac{1}{\lambda^2} - 1\right) - \frac{\alpha}{4\pi} \left[\left(\ln \lambda^2 + \frac{1}{\lambda^2} - 1\right) \right.$$
$$\left. + 4 \ln \left(\frac{2\lambda^2}{\lambda^2 - 1}\right) \right] - \frac{\alpha}{\pi} \ln \frac{\lambda_{min}}{M} , \tag{2.9}$$

which expression exhibits a characteristic infra-red divergence. This divergence is not encountered in Ref. 1 because current conservation (and hence, gauge invariance[16]) has been put into the approximation to the off-shell electromagnetic proper vertex,

$$\Gamma^{\nu}_{pp\gamma}(p,p_1,q) = \frac{M}{W}\,(\gamma^{\nu} - p_1^{\nu}\,\frac{\gamma^{\mu}q_{\mu}}{p_1\cdot q}) \;, \qquad (2.10)$$

"by hand". If this prescription for a meaningful result has some basis, then its application to the calculation leading to (2.9) yields the result (now including the effect of the anomalous moments[17] as well),

$$\frac{\delta M}{M} = [(\frac{g^2}{4\pi^2})(\frac{1}{\lambda^2} - 1)]^{-1}\,\frac{\alpha}{4\pi}\left\{(\ell n\,\lambda^2 + \frac{1}{\lambda^2} - 1) + \frac{k}{2}\,[(\lambda^2 - 1) + (\frac{1}{\lambda^2} - 1)]\right\} \simeq -\,0.4\ \mathrm{MeV}, \qquad (2.11)$$

which is in rough agreement with the result of Ref. 1.[18]

3. Z = 0 CALCULATION WITH VERTEX CORRECTIONS $O(\alpha)$

In Ref. 1 it is suggested that the electromagnetic splitting of the pion-nucleon coupling constants may be calculated from the vertex sum rule which derives from the high-energy behavior of the proper vertex function,[2,19]

$$\begin{aligned} 0 &= \lim_{W\to\infty} \Gamma_{(j)}(W) \\ &= \lim_{W\to\infty}\left\{(g + \delta g_{(j)}) + \frac{(W-M_j)}{\pi}\int_R \frac{dW'\,\mathrm{Im}\Gamma_{(j)}(W')}{(W'-M_j)(W'-W)} - \frac{(W-M_j)}{\pi}\int_R \frac{dW'\,\mathrm{Im}\Gamma_{(j)}(-W')}{(W'+M_j)(W'+W)}\right\} \\ &= (g + \delta g_{(j)}) - \frac{1}{\pi}\int_R \frac{dW\,\mathrm{Im}\Gamma_{(j)}(W)}{W-M_j} - \frac{1}{\pi}\int_R \frac{dW\,\mathrm{Im}\Gamma_{(j)}(-W)}{W+M_j}\;. \qquad (3.1) \end{aligned}$$

Because of our requirement that the condition Z = 0 be satisfied only to $\Theta(g^2)$, we are able to neglect the effects on the $\delta g_{(j)}$ from coupling constant and mass-shift feedback[20], and thus write the unsubtracted dispersion relation,

$$\delta g_{(j)} = \frac{1}{\pi}\int_{M+\lambda_{\min}}^{\lambda M} dW \frac{[\mathrm{Im}\Gamma_{(j)}(W)]_{(N\gamma)}}{W-M} + \frac{1}{\pi}\int_{M}^{\lambda M} \frac{dW[\mathrm{Im}\Gamma_{(j)}(-W)]_{(N\gamma)}}{W+M} \tag{3.2}$$

$$\equiv \delta\Gamma_{(j)}(M);$$

moreover, as explicitly indicated in (3.2), we limit consideration to the contribution to the absorptive part of Γ coming from baryon-one-photon intermediate states only. Since the proper vertex couples, through its absorptive part, only to the crossed photoproduction amplitude for which current conservation fails [That is, $\ell^{\mu}T_{\mu}$ ($N\gamma$; crossed) $\neq 0$ where ℓ^{μ} is the four-momentum of the real photon.], this calculation is also not gauge-invariant in the confines of two-particle unitarity. [For example, one finds in the Lorentz gauge, neglecting the contribution from the anomalous nucleon moments,

$$\left(\frac{g_{p\pi^o} - g_{n\pi^o}}{g}\right) = \frac{\alpha}{\pi}\left\{\frac{1}{2}\ln\lambda^2 + \frac{1}{\lambda^2-1}\ln\lambda^2 + \ln\left(\frac{2\lambda^2}{\lambda^2-1}\right) + \ln\frac{\lambda_{\min}}{M}\right\}, \tag{3.3}$$

which is, not surprisingly, infra-red divergent.][21] On the other hand, since the improper vertex (or form factor) couples through its absorptive part to the complete photoproduction amplitude, for which $\ell^{\mu}T_{\mu}(N\gamma) = 0$, no such problem in encountered[9]. If, as in Ref. 9, we choose to derive the electromagnetic splitting of pion-nucleon couplings from an appropriate assumption about the high-energy bevarior of the form factors, $K_{(j)}(W)$, so that,[9]

$$\delta g_{(p,j)} - \delta g_{(n,k)} = \frac{1}{\pi}\int_{M}^{\lambda M} dW \left\{\frac{\mathrm{Im}K(W)}{W-M} + \frac{\mathrm{Im}K(-W)}{W+M}\right\}_{[(p,j)-(n,k)]}, \tag{3.4}$$

with[22]

$$\frac{\delta g^{o}}{g} \equiv \left(\frac{g_{p\pi^{o}} - g_{n\pi^{o}}}{g}\right) = \frac{\alpha}{\pi}\left\{\frac{1}{2}\left[\left(\frac{\lambda^{2}+1}{\lambda^{2}-1}\right) \ln \lambda^{2} - 1 - \frac{1}{4}\left(\ln \lambda^{2} + \frac{1}{\lambda^{2}} - 1\right)\right]\right.$$

$$\left. - \frac{K}{8}\left[3\left(\frac{1}{\lambda^{2}} - \ln \lambda^{2}\right) + 5 - 2\Phi(1-\lambda^{2}) - \frac{8}{\lambda^{2}-1} \ln \lambda^{2}\right]\right\}, \qquad (3.5)$$

$$\frac{\delta g^{\pm}}{g} \equiv \left(\frac{g_{p\pi^{+}} - g_{n\pi^{-}}}{g}\right) = \frac{\alpha}{2\pi}\left\{\left[\ln\left(\frac{\lambda^{2}-1}{\lambda^{2}}\right) - \frac{1}{2}\left(\frac{1}{\lambda^{2}} - 1\right)\right]\right.$$

$$\left. - \frac{K}{4}\left[-3 \ln \lambda^{2} + \left(\frac{1}{\lambda^{2}} - 1\right) - 4\Phi(1-\lambda^{2})\right]\right\}, \qquad (3.6)$$

then, from

$$\delta(Z_{p} - Z_{n})_{\pi N \text{ coupling}} = - \frac{2g}{32\pi^{2}}\left(2\delta g^{\pm} + \delta g^{o}\right)\left(\ln \lambda^{2} + \frac{1}{\lambda^{2}} - 1\right)$$

$$= -2\left[\frac{2}{3}\left(\frac{\delta g^{\pm}}{g}\right) + \frac{1}{3}\left(\frac{\delta g^{o}}{g}\right)\right], \qquad (3.7)$$

we find

$$\delta M = +0.8 \text{ MeV}, \qquad (3.8)$$

which seems hardly promising.

REFERENCES

1. A. Bassetto and F. Paccanoni, Nuovo Cimento 44A, 839 (1966).
2. M. Ida, Phys. Rev. 136, B1767 (1964).
3. For an extensive review and bibliography on this subject we refer the interested reader to K. Hayashi, M. Hirayama, T. Muta, N. Seto, and T. Shirafugi, "Compositeness Criteria of Particles in Quantum Field Theory and S- Matrix Theory," (to appear in Fortschritte der Physik).
4. We remind the reader that the quantitative study of the LSZ inequality made in Ref. 2 found a numerical result not inconsistent with the assumption, $Z_N = 0$.
5. $\delta M \equiv M_p - M_n = -0.2$ MeV.
6. H. M. Fried and T. N. Truong, Phys. Rev. Letters 16, 559 (1966); 16, 884(E) (1966); Phys. Rev. 152, 1467 (1966).
7. S. D. Drell and H. R. Pagels, Rhys. Rev. 140, B397 (1965).
8. See for example, J. Gillespie, Final State Interactions, (Holden-Day, Inc., San Francisco, 1964); J. D. Jackson, Nuovo Cimento 25, 1038 (1962).
9. H. R. Pagels, Rhys. Rev. 144, 1261 (1966).
10. Z denotes the value of $Z_{p,n}\big|_{(e^2 = 0)}$ in the isotopic limit; Z is thus the strong wavefunction renormalization.
11. Bassetto and Paccanoni (Ref. 1) rightly observe that the omission of this contribution led to the erroneous result reported in Ref. 6, namely, that δM vanishes when $Z = 0$. However, it is still necessary to show that the two methods (of Ref. 1 and 6) lead to the identical expression [Eq. (2.1)] in the limit $Z = 0$. The term $\Delta(e^2)$ is gauge-dependent (see below).

12. For simplicity, the contributions from the anomalous nucleon moments and from the e^2 corrections to $\tau^p_\pm(W)$ have been omitted. The quantities $\xi_{p,n}$ and $\Delta(e^2)$ derive from $Z_{p,n}$ and are defined in Eq. (2.7).

13. The "zeroth approximation" refers to the limit of isotopic symmetry with $(\mu/M) \to 0$ and to lowest order in g^2.

14. This is the cutoff in units of M^2, where M is a mean nucleonic mass.

15. We calculate according to the well known Feynman-Cutkosky rules (Ref. 7) in the Lorentz gauge $[\sum_\lambda \epsilon_\mu(\lambda)\epsilon_\nu(\lambda) = - g_{\mu\nu}]$, making use of the formulae, valid $O(g^2)$,

$$Z_{p,n} = 1 + \frac{1}{\pi}\int_R dW \; \frac{\mathrm{Im}\Sigma_{p,n}(W + i\epsilon)}{(W-M_{p,n})^2} + \frac{1}{\pi}\int_R dW \; \frac{\mathrm{Im}\Sigma^*_{p,n}(-W-i\epsilon)}{(W+M_{p,n})^2} ,$$

$$\Sigma(\underline{p}) = \Sigma(W)\left(\frac{W + \underline{p}}{2W}\right) + \quad (-W)\left(\frac{W - \underline{p}}{2W}\right),$$

with

$$\underline{p} = p_\mu \gamma^\mu = W\gamma^0 .$$

16. Of course, this insures that their result for δM is gauge-invariant.

17. $K = 1.85$ is an "average" (absolute) anomalous moment.

18. Also, for $K = 0$, we find $\delta M = - 0.1$ MeV.

19. The subscript j arises from isotopic dissymmetry. It is interesting to point out here that for $e^2 = 0$, the unsubtracted relation

$$0 = \lim_{W \to \infty} \Gamma(W) \equiv Z_1 g = g - \frac{1}{\pi}\int_{R\ \mathrm{strong}} dW \left\{\frac{\mathrm{Im}\Gamma(W)}{W-M} + \frac{\mathrm{Im}\Gamma(-W)}{W+M}\right\} ,$$

is not at all satisfied $O(g^2)$; one finds

$$Z_1 = 1 - \frac{1}{4\pi}\left(\frac{g^2}{4\pi}\right)\left\{\frac{\lambda^2 + 1}{\lambda^2 - 1}\ \ln \lambda^2 - 2\right\}.$$

20. See Appendix III of Ref. 9. These effects are $O(g^4)$ in this calculation; their inclusion would necessitate adjustment of the cutoff λ^2 for consistency.

21. But note that, the sum of all Feynman graphs $O(\alpha g^2)$ would, by extension of Thirring's theorem (W. Thirring, Phil. Mag. 41, 1193 (1956)), not appear to suffer from this disease.

22. We undertook to recalculate these perturbations after finding some errors in Appendix III of Ref. 9. We wish to thank Mr. R. Longley for checking some of the preliminary stages in this effort.