# SUM RULES FROM THE ALGEBRA OF FIELDS* 

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Recently, an algebra of gauge vector fields was proposed, ${ }^{1}$ and it was shown that this algebra may be used to rederive the results of current algebra, ${ }^{2}$ while also making definite statements about Schwinger terms. ${ }^{3}$ Especially, these statements were used to complete the proof of Weinberg's spectral function sum rules. ${ }^{4}$ It was also argued in Ref. 1 that the recent calculation of the $\pi^{+}-\pi^{0}$ mass difference ${ }^{5}$ supports the idea that the sources coupled to the photon are not given by the hadronic currents $J_{a}^{\nu}$ but by the hädronic gauge fields $\phi_{a}^{\nu}{ }^{6}$

In this note we derive an additional sum rule for the spectral functions of the vector and axial-vector $\mathrm{SU}_{2} \times \mathrm{SU}_{2}$ gauge fields, namely

$$
\begin{equation*}
\int_{0}^{\infty} \mathrm{m}^{2}\left[\rho_{\mathrm{V}}\left(\mathrm{~m}^{2}\right)-\rho_{\mathrm{A}}\left(\mathrm{~m}^{2}\right)\right] \mathrm{dm}^{2}=0 \tag{1}
\end{equation*}
$$

and we use this sum rule to bring one more argument in support of the idea ${ }^{1,6}$ that the photon is coupled to the hadronic gauge fields, and not to the hadronic currents.

The sum rule Eq. (1) is derived within the gauge fields theory of Ref. 1, assuming that:
(a) The vacuum expectation values of the Schwinger terms of the vector and the axial-vector $\mathrm{SU}_{2} \times \mathrm{SU}_{2}$ hadronic currents are equal.
(b) Both the vector and axial-vector $\mathrm{SU}_{2} \times \mathrm{SU}_{2}$ currents are conserved. ${ }^{7}$ The sum rule Eq. (1), when combined with the other two sum rules for the spectral functions of the vector and axial-vector gauge fields, ${ }^{1}$

$$
\begin{gather*}
\int_{0}^{\infty} \frac{\rho_{\mathrm{V}}\left(\mathrm{~m}^{2}\right)-\rho_{\mathrm{A}}\left(\mathrm{~m}^{2}\right)}{\mathrm{m}^{2}} \mathrm{dm}^{2}=\mathrm{F}_{\pi}^{2}  \tag{2}\\
\int_{0}^{\infty}\left[\rho_{\mathrm{V}}\left(\mathrm{~m}^{2}\right)-\rho_{\mathrm{A}}\left(\mathrm{~m}^{2}\right)\right] \mathrm{dm}^{2}=0  \tag{3}\\
-1-
\end{gather*}
$$

and the same saturation assumptions as in Ref. 4, namely saturating the vector spectral function $\rho_{\mathrm{V}}\left(\mathrm{m}^{2}\right)$ by the $\rho$ meson and the axial-vector spectral function $\rho_{\mathrm{A}}\left(\mathrm{m}^{2}\right)$ by the $\pi$ and $\mathrm{A}_{1}$ mesons, lead to a contradiction, since we then obtain $\mathrm{F}_{\pi}=0$ and $\mathrm{m}_{\rho}=\mathrm{m}_{\mathrm{A}_{1}}$.

One possibility to resolve this contradiction is to blame the assumption of saturation by the low lying states, especially for the sum rule Eq. (1), which has an extra $\mathrm{m}^{2}$ factor in the integrand. Another possibility, which seems to us more attractive, is to abandon assumption (a) (assumption (b) seems to be inessential, ${ }^{7}$ and is supposed to change results only by contributions of order $\left.\left(\mathrm{m}_{\pi}^{2} / \mathrm{m}_{\rho}^{2}\right)\right)^{4}$ which then invalidates the sum rule Eq. (1). However, if the photon is coupled to the hadronic currents $J_{a}^{\nu}$, assumption (a) is needed in order to eliminate the quadratic divergence in the $\pi^{+}-\pi^{o}$ mass difference calculation of Ref. 5. Thus a possible way to resolve our contradiction is to postulate that the photon couples to the hadronic gauge fields, ${ }^{1,6}$ and not to the hadronic currents.

We now outline the proof of Eq. (1). Our assumption about the Schwinger terms of the hadronic currents $\mathrm{J}_{\mathrm{a}}^{\nu}$ is ${ }^{8}$

$$
\begin{equation*}
\langle 0|\left[J_{\mathrm{a}}^{\mathrm{o}}(\overrightarrow{\mathrm{x}} \mathrm{t}) \mathrm{J}_{\mathrm{b}}^{\mathrm{k}}(\overrightarrow{\mathrm{y}} \mathrm{t})\right]|0\rangle \propto \delta_{\mathrm{ab}} \tag{4}
\end{equation*}
$$

where $\mathrm{a}, \mathrm{b}$ are $\mathrm{SU}_{2}$ indices, including the vector and axial-vector labeling. This, together with the gauge fields theory of Ref. 1, implies

$$
\begin{equation*}
\langle 0|\left[\partial^{r}\left(\partial_{r} \phi_{a o}(\vec{x} t)-\partial_{o} \phi_{a r}(\vec{x} t)\right), \partial^{\mu}\left(\partial_{\mu} \phi_{b k}(\vec{y} t)-\partial_{k} \phi_{b \mu}(\vec{y} t)\right)\right]|0\rangle \propto \delta_{a b} \tag{5}
\end{equation*}
$$

which implies the sum rule Eq. (1). In passing from Eq. (4) to Eq. (5), we need the equal-time commutators of Ref. 1, and also several additional ones, which
also follow from the canonical quantization. These are

$$
\begin{align*}
& {\left[\phi_{\mathrm{ao}}(\overrightarrow{\mathrm{x}} \mathrm{t}), \partial_{\mathrm{o}} \phi_{\mathrm{bk}}(\overrightarrow{\mathrm{y}} \mathrm{t})\right]=\mathrm{i} \frac{\mathrm{~g}_{\mathrm{o}}}{\mathrm{~m}_{\mathrm{o}}^{2}} \mathrm{C}_{\mathrm{abc}}\left(\partial_{\mathrm{o}} \phi_{\mathrm{ck}}(\overrightarrow{\mathrm{x}} \mathrm{t})\right) \delta^{(3)}(\overrightarrow{\mathrm{x}}-\mathrm{y})}  \tag{6}\\
& {\left[\partial^{\mathrm{o}} \phi_{\mathrm{ao}}(\overrightarrow{\mathrm{x}} \mathrm{t}), \phi_{\mathrm{bk}}(\overrightarrow{\mathrm{y}} \mathrm{t})\right]=0} \tag{7}
\end{align*}
$$

and the commutator $\left[\partial_{0} \phi_{a k}(\overrightarrow{\mathrm{x}} \mathrm{t}), \partial_{\mathrm{o}} \phi_{\mathrm{br}}(\overrightarrow{\mathrm{y}} \mathrm{t})\right]$, which follows from

$$
\begin{equation*}
\left[\mathrm{F}_{\mathrm{aok}}(\overrightarrow{\mathrm{x}} \mathrm{t}), \mathrm{F}_{\mathrm{bor}}(\overrightarrow{\mathrm{y}} \mathrm{t})\right]=0 \tag{8}
\end{equation*}
$$

The latter is another commutator following from canonical theory.
In passing from Eq. (4) to Eq. (5), we encounter expressions of the form

$$
\begin{equation*}
\left.\mathrm{C}_{\mathrm{acd}} \mathrm{C}_{\mathrm{bgh}} \mathrm{C}_{\mathrm{meh}} \mathrm{C}_{\mathrm{cef}}\langle 0| \phi_{\mathrm{g} \alpha} \phi_{\mathrm{m} \beta} \phi_{\mathrm{f} \gamma} \phi_{\mathrm{d} \delta}\right|^{0\rangle} \tag{9}
\end{equation*}
$$

and similar expressions with derivatives of the fields. These may be shown to give the same contribution for the indices $a, b$ being both vector or both axialvector, which is what we need. Similar considerations apply to other expressions that appear during the calculation passing from Eq. (4) to Eq. (5), e.g., to expressions of the form

$$
\begin{equation*}
\mathrm{C}_{\mathrm{acd}} \mathrm{C}_{\mathrm{bgh}} \mathrm{C}_{\mathrm{dgm}}\langle 0| \phi_{\mathrm{c} \alpha} \phi_{\mathrm{m} \beta} \phi_{\mathrm{h} \gamma}|0\rangle \tag{10}
\end{equation*}
$$

or of the form

$$
\begin{equation*}
\mathrm{C}_{\mathrm{acg}} \mathrm{C}_{\mathrm{bgh}}\langle 0| \phi_{\mathrm{c}} \phi_{\mathrm{h}}|0\rangle_{\mathrm{o}} \tag{1i}
\end{equation*}
$$

The actual calculation is somewhat lengthy, and will not be presented here.
Similar considerations may be applied to the case of groups higher than $\mathrm{SU}_{2} \times \mathrm{SU}_{2} \cdot$

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## REFERENCES

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7. We feel that this assumption is not essential, and that a proof may be found for the same result without it. However, within this assumption, the pion is generated by a spontaneous symmetry breakdown mechanism, as in Ref. 4.
8. Our notation follows that of Ref. 1. Our Lorentz metric is (1, $-1,-1,-1$ ). Roman Lorentz indices refer to spacial components.
