CONTRIBUTION OF SMALL ANGLE NUCLEAR SCATTERING TO ERROR FORMULAE IN BUBBLE CHAMBER ANALYSIS*

William B. Johnson

Stanford Linear Accelerator Center, Stanford University, Stanford, California

ABSTRACT

The contribution of small angle nuclear scattering to uncertainties in bubble chamber measurements is re-examined to include experiments at high momenta (above 4 GeV/c). Use is made of recent high energy elastic scattering data to estimate these uncertainties for $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$. Finally the contribution from interference between coulomb and nuclear scattering is evaluated for $\pi^{\pm}$ and p.

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Wm. B. Johnson

Stanford Linear Accelerator Center
Stanford University, Stanford, California

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1. Introduction:

The science of reconstructing the direction and momenta of elementary particles from bubble chamber photographs has developed rapidly in the past ten years. A particular part of that development is the estimation of uncertainties in direction and momenta from measurement errors and multiple scattering in the bubble chamber liquid. A very complete summary of this portion of the analysis was presented by Gluckstern[1] in 1963. The applicability of these formulae to the problem has been amply demonstrated in bubble chamber experiments up to 3 or 4 GeV/c.

Lately, however, very high energy exposures have been made in the Brookhaven 80-inch hydrogen bubble chamber. In such exposures several groups have noted the breakdown of the term due to multiple nuclear scattering.[2] The purpose of this note is to correct for this and extend the range of validity of the expression for multiple nuclear scattering to include the high energies presently available at accelerators.

2. The General Problem at High Energies:

Gluckstern parameterizes multiple scattering errors by a quantity K which represents the r.m.s. projected scattering angle per unit thickness

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of material traversed and which is defined by

$$K = N \int (1 - \cos\theta) \left(\frac{d\sigma}{d\Omega}\right)_{LAB} d\Omega_{LAB}. \qquad \ldots\ldots (1)$$

For nuclear scattering he integrates this expression from $\theta = 0$ to $\theta = \theta_K = 0.02$ radians (the angle at which a kink would become apparent in the track). The scattering cross section $\left(\frac{d\sigma}{d\Omega}\right)_{LAB}$ is assumed constant over the region of integration and is evaluated in terms of the total cross section from the optical theorem. This procedure yields the formula

$$K = \frac{N\pi}{4} \left(\frac{p\,\sigma_T}{4\pi\hbar}\right)^2 \theta_K^4 \qquad \ldots\ldots (2)$$

where p is the momentum of the particle, σ_T is the total scattering cross section, and N is the number of scattering centers per unit volume. It is clear from this formula that K increases without bound at high energies.

The source of the breakdown lies in the limits of integration. At high energies the upper limit of integration is not set by the presence of a kink in the track but rather by the appearance of a proton recoil in the bubble chamber. At 10 GeV/c, for example, scattering through 0.02 radians would be accompanied by a proton with approximately 200 MeV/c and range of 3.5 cm in liquid hydrogen.

Since the upper limit is now set by the momentum of the recoiling proton, it is useful to rewrite expression (1) in terms of momentum-transfer-squared:

$$t = (E - E')^2 - (\vec{p} - \vec{p}\,')^2 \qquad \ldots\ldots (3)$$

where E, E' are the total laboratory energies of the incident particle before and after scattering respectively and $\vec{p}$, $\vec{p}'$ are the momenta before and after scattering; for very forward scattering angles θ, $E \approx E'$, $|\vec{p}| \approx |\vec{p}'|$ to a very good approximation, and

$$- t \equiv \tau = 2p^2 (1 - \cos \theta). \qquad \ldots\ldots (4)$$

Rewriting (1),

$$K = \left(\frac{N}{2p^2} \right) \int_0^{\tau_0} \tau \frac{d\sigma}{d\tau} d\tau \qquad \ldots\ldots (5)$$

where τ_0 is that value of τ for which the recoiling proton is detectable by a scanner.

Many experiments on diffraction scattering at high energies have been performed in the past three years.[3] From these data is is apparent that at high energies and small τ, $\frac{d\sigma}{d\tau}$ may be written to a very good approximation as

$$\frac{d\sigma}{d\tau} = \sigma_0 \exp(-b\tau) \qquad \ldots\ldots (6)$$

Inserting this expression in (5) we obtain

$$K = (N\sigma_0/2p^2b^2) \; [1 - \exp(-b\tau_0) - b\tau_0 \exp(-b\tau_0)]. \qquad \ldots\ldots (7)$$

The constant b is on the order of 10^{-5} $(\mathrm{MeV}/c)^{-2}$. For $\tau_o < 10^4$ $(\mathrm{MeV}/c)^2$ the exponential can be expanded to first order as a fair approximation to give

$$K \cong (N\sigma_o / 4p^2)\, \tau_o^2 \,. \qquad \ldots\ldots (8)$$

This expression is appealing since it is independent of b. Evidently this is a more satisfactory expression for K since it goes to zero as p^{-2} for large p.

Finally, expression (8) should be joined continuously to expression (2) by noting that below some cut-off value p_c no recoil proton will be observed for a scattering angle θ_K, and we must integrate (5) from $\tau = 0$ to $\tau = (p\theta_K)^2$:

$$K \cong (N\, \sigma_o\, \theta_K^4)\, p^2/4, \; p < p_c. \qquad \ldots\ldots (9)$$

where $\quad p_c = \dfrac{\tau_o}{\theta_K}$.

3. <u>Interference Effects</u>:

In Gluckstern's paper [1] interference effects between nuclear scattering and coulomb scattering are not treated. At the present time there is some data concerning the interference term [4] for pions and protons scattering in hydrogen from which one may estimate its importance and include to a fair approximation this contribution at high energies.

If one assumes that the real part of the nuclear scattering amplitude has the same t dependence as the imaginary part, one may write

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{Interference}} = \pm \frac{4Ze^2}{\beta} \alpha \sqrt{\frac{\sigma_o \pi}{1 + \alpha^2}} \frac{e^{-\frac{b}{2}\tau}}{\tau} \quad \ldots (10)$$

where α is the ratio of the real to the imaginary amplitude for nuclear scattering, Z is the charge on the nuclei in the chamber liquid (for hydrogen Z = 1), $\beta = \frac{P}{E}$ for the incident particle, and σ_o, b and τ are defined as in the previous section. The choice of sign depends upon constructive or destructive interference between the coulomb and nuclear amplitudes.

Inserting this expression into equation (5) and integrating from $\tau = 0$ to $\tau = \tau'$, we obtain

$$K_I = \pm 2NZe^2 \alpha \sqrt{\frac{\pi \sigma_o}{1 + \alpha^2}} \frac{1}{p^2\beta} \left(\frac{2}{b}\right) \left[1 - e^{-\frac{b}{2}\tau'} \right]$$

$$\cong \pm 2NZe^2 \alpha \sqrt{\frac{\pi \sigma_o}{1 + \alpha^2}} \frac{\tau'}{p^2\beta} \quad \ldots\ldots (11)$$

As in section 2., we choose

$$\tau' = \begin{cases} p^2\theta_K^2 & p < p_c \\ \tau_o & p > p_c \end{cases} \quad \ldots\ldots (12)$$

and

$$K_I = \begin{cases} \pm 2NZe^2 \alpha \sqrt{\frac{\pi \sigma_o}{1 + \alpha^2}} \frac{\theta_K^2}{\beta} & p < p_c \\ \pm 2NZe^2 \alpha \sqrt{\frac{\pi \sigma_o}{1 + \alpha^2}} \frac{\tau_o}{p^2\beta} & p > p_c. \end{cases} \quad \ldots\ldots (13)$$

It is useful to define a parameter r which is the ratio of K_I to K

$$r = \left|\frac{K_I}{K}\right| = \frac{8\sqrt{\pi}\ Ze^2\alpha}{\sqrt{\sigma_o(1+\alpha^2)}}\ \frac{1}{\beta\tau'} \qquad \ldots\ldots (14)$$

For $p > p_c$, $\beta \approx 1$, $\sigma_o \sim 70$ mb/(BeV/c)^2, and $\alpha \sim 0.2$

$$r \sim 0.3 \qquad \ldots\ldots (15)$$

It is clear from this ratio that inclusion of the interference term is moderately important at high energies.

4. Numerical Results:

The results of section 2. may be summarized as follows:

$$K = \begin{cases} A/p^2\ , & p > p_c \\ Bp^2\ , & p < p_c \end{cases} \qquad \ldots\ldots (16)$$

If we assume σ_o is roughly constant as a function of p, the coefficients A and B depend only upon the kind of particle undergoing scattering. Table I contains a summary of these coefficients obtained by taking rough averages of the cross sections from reference 3 and using the following values for the other parameters:

$$N = 3.4 \times 10^{22} \text{ atoms/cm}^3$$

$$\tau_o = 6.4 \times 10^3 \text{ (MeV/c)}^2$$

$$\theta_K = 0.02 \text{ radians}$$

Momenta are assumed to be MeV/c. From these values we obtain

$$p_c = 4000 \text{ MeV/c}.$$

The above value of τ_o corresponds to a recoil proton of about 1.5 mm.

For the work of section 3. one may prepare a similar set of formulae:

$$K_I = \begin{cases} C/(p^2\beta) & p > p_c \\ D/\beta & p < p_c \end{cases} \qquad \ldots\ldots \quad (17)$$

From the data of reference (4) we can evaluate the coefficients C and D for protons and pions in hydrogen. This information is shown in Table II. Values for N, τ_o, θ_K and p_c are those cited above.

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TABLE I

Coefficients A and B parameterizing the projected scattering angle from nuclear scattering per cm in liquid hydrogen for $\pi^{\pm}$, $K^{\pm}$, p, and $\bar{p}$. (See Eq. 16)

Particle	A $(MeV/c)^2(cm)^{-1}$	B $(MeV/c)^{-2}(cm)^{-1}$
π^-	1.2×10^{-2}	0.45×10^{-16}
K^-	0.9×10^{-2}	0.35×10^{-16}
$\bar{p}$	6.3×10^{-2}	2.45×10^{-16}
π^+	1.0×10^{-2}	0.41×10^{-16}
K^+	0.7×10^{-2}	0.27×10^{-16}
p	2.8×10^{-2}	1.09×10^{-16}

TABLE II

Coefficients C and D parameterizing the projected scattering angle from nuclear-coulomb interference per cm in liquid hydrogen for $\pi^{\pm}$ and p. (See Eq. 17)

Particle	C $(MeV/c)^2(cm)^{-1}$	D $(MeV/c)^{-2}(cm)^{-1}$
π^-	-0.45×10^{-2}	-0.28×10^{-9}
π^+	$+0.19 \times 10^{-2}$	$+0.12 \times 10^{-9}$
p	$+0.93 \times 10^{-2}$	$+0.58 \times 10^{-9}$