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GENERALIZED PDDAC, CURRENT ALGEBRA AND S-WAVE K^+P SCATTERING*

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A test of generalized PDDAC (pole-dominance-of-the-divergence-of-the-axial-current) for $A_\mu^{K^\pm}$ is made by calculating the S-wave scattering length and effective range for K^+P scattering using current-algebra techniques. First-order corrections in the kaon four-momentum are shown to be zero for the scattering length and small for the effective range. From a comparison with experiment, PDDAC for the K-meson seems to be good within 30%.

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INTRODUCTION

The traditional way to test the pole-dominance hypothesis for the divergence of the axial current $A_\mu^{K^\pm} = A_\mu^4 \mp i A_\mu^5$ would be to examine directly the equivalent of the Goldberger-Treiman relation linking $f_A(\Lambda)$ to $g_{N\Lambda K}$. Both of these quantities are, however, poorly known, at present, from direct experiment. The value of $g_{N\Lambda K}$ from K-nucleon forward scattering dispersion calculations depends very sensitively on the correct parametrization of the experimental data on low-energy $\bar{K}N$ scattering and different authors[1] have proposed different values of $g^2_{N\Lambda K}/4\pi$ ranging from 4.8 ± 1.0 to 15.3 ± 1.5. On the other hand, the rate for $\Lambda \rightarrow Pe\nu$ hyperonic decay[2] is $(2.96 \pm .55) \times 10^{-7}$ sec., whereas from an analysis of the angular distribution it has been found[3] that $(G_A/G_V)_\Lambda = 1.14 {+0.23 \atop -0.33}$, hence $f_A(\Lambda) = 1.21 \pm .35$. The Goldberger-Treiman relation then leads to $g^2_{N\Lambda K}/4\pi = 19.9 \pm 10.3$, as compared with the above values. In view of the uncertainties and the large errors involved, an independent test of generalized PDDAC is necessary in order to determine the reliability of the hypothesis. The data on low-energy K^+P scattering, which is resonance-free, are quite accurate and a comparison with experiment of the values of the scattering length and the effective range at threshold, calculated from current-algebra, would test PDDAC for $A_\mu^{K^\pm}$. Balachandran[4] et al. have calculated the scattering length, but they ignored symmetry-breaking in their numerical computation, so that their value cannot be used for this test. In this paper, we calculate the S-wave K^+P scattering length and effective range taking the violation of SU(3) into account, except in some correction-terms where symmetry-breaking effects will not be regarded as significant. Apart from extrapolations in the kaon four-momentum connected with PDDAC, our only assumption is that $\theta_V = \theta_A$[5,13]

for the bare parameters in the Cabibbo Hamiltonian. Our results agree with experiment within about 30%, and we argue that theoretical uncertainties, introduced in our values by approximations involved in the extrapolation, are of similar order.

In Section I we separate out the current-algebra term from the invariant amplitude and call the remainder the "weak amplitude". We relate the S-matrix to the $\ell = 0$ partial wave amplitude $\bar{f}_0$ and express the scattering length and the effective range in terms of f_0. In Section II we discuss the details of the extrapolation in the kaon four-momentum necessary in the use of PDDAC, and include a first order correction to the soft K ($k \to 0$) limit that involves evaluating the "weak amplitude" by taking the single-particle contributions. In section III we show that the scattering length is given solely by the current-algebra term to first order in k and estimate the order of magnitude of the quadratic correction. In Section IV we calculate the current-algebra contribution to the effective range and also the corrections due to the terms that are first order in k in the "weak amplitude". In Section V the different parameters that appear in our formulas are evaluated from various data and our results are compared with experiment. The final Section VI summarizes the conclusions.

I - S-MATRIX AND KINEMATICS

The non-trivial part of the scattering amplitude for $K^+P \to K^{+'}P'$ can be written, following Weinberg[6], as

$$S_{fi} - \delta_{fi} = - \frac{(k^2 - m_K^2)(k'^2 - m_K^2)}{|a_K|^2} \left(\frac{1}{2\pi}\right)^3 \frac{1}{\sqrt{4k_0 k_0'}} (2\pi)^4 \delta^{(4)}(p' + k' - p - k)$$

$$\int d^4z\, e^{+ik'\cdot z} < p' | k'^{\mu} k^{\nu} T\left\{A_\mu^{K^+}(z), A_\nu^{K^-}(0)\right\} - ik'^{\mu}\delta(z_0)[A_\mu^{K^+}(z), A_0^{K^-}(0)] | p > ,$$

where we have ignored the commutator between a current-density and a divergence (σ-term) invoking the Adler consistency condition.[7] In the above expression $a_K = f_K m_K^3$, where

$$\partial^\mu A_\mu^{K^+} = a_K \varphi^{K^+} \text{ and } \langle 0 | A_\mu^{K^+}(0) | K^+ \rangle = \left(\frac{1}{2\pi}\right)^{3/2} \frac{i}{\sqrt{2k_0}} \; m_K f_K k_\mu .$$

From the SU(3) $\times$ SU(3) algebra of current densities, as proposed by Gell-Mann[8], we have

$$[A_\mu^{K^+}(z), A_0^{K^-}(0)] = - V_\mu^3(z)\delta^{(3)}(\vec{z}) - \sqrt{3}\, V_\mu^8(z)\delta^{(3)}(\vec{z}). \qquad (Z_0 = 0)$$

Thus

$$S_{fi} - \delta_{fi} = - \left(\frac{1}{2\pi}\right)^3 \frac{1}{\sqrt{4k_0 k_0'}} (2\pi)^4 i\delta^{(4)}(p' + k' - p - k) \frac{(k^2 - m_k^2)(k'^2 - m_k^2)}{|a_K|^2} \tag{1}$$

$$\int d^4z \, e^{ik'\cdot z} \langle p' | \frac{1}{i} k'^\mu k^\nu T\left\{A_\mu^{K^+}(z), A_\nu^{K^-}(0)\right\} + k_\mu'\left[V^{\mu^3}(z) + \sqrt{3}\, V^{\mu^8}(z)\right]\delta^{(3)}(\vec{z}) | p \rangle,$$

where $k_\mu'\left[V^{\mu^3}(z) + \sqrt{3}\, V^{\mu^8}(z)\right]\delta^{(3)}(\vec{z})$ is the current-algebra term and $\frac{1}{i} k'^\mu k^\nu T\{A_\mu^{K^+}(z), A_\nu^{K^-}(0)\}$ is the "weak amplitude" contribution. Eq. (1) can be written as

$$S_{fi} - \delta_{fi} = -\left(\frac{1}{2\pi}\right)^6 \frac{M_N}{\sqrt{4k_0 k_0' p_0 p_0'}} (2\pi)^4 i\delta^{(4)}(k' + p' - k - p)(T^C + T^R)_{fi}$$

where T^C is the current-algebra contribution and T^R is the remainder. If $T = T^C + T^R$ has the form

$$\bar{u}(p')\left(- A + B \frac{\not{k} + \not{k}'}{2}\right) u(p) , \tag{2}$$

then the S-wave contribution to the scattering amplitude is given by[9]

$$f_0 \equiv \frac{e^{i\delta_0}\sin\delta_0}{q} = \frac{1}{8\pi}\int_{-1}^{1} d(\cos\theta)\left[\frac{E+M_N}{2W}\left\{A-(W-M_N)B\right\} - \frac{E-M_N}{2W}\cos\theta\left\{A+(W+M_N)B\right\}\right], \qquad (3)$$

where δ_0= S-wave phase shift, q = relative momentum in the CM frame, E = nucleon energy in the same, W = invariant mass of the system. If a and r are the S-wave scattering length and the effective range respectively, we have the well-known relation

$$q \cot\delta_0 = \frac{1}{a} + \frac{1}{2}q^2 r,$$

hence

$$a = \left(f_0\right)_{q=0} \text{ and } r = \frac{2}{f_0^{\,3}}\left(\frac{df_0}{dq}\right)^2 - \frac{1}{f_0^{\,2}}\frac{d^2 f_0}{dq^2}$$

which we shall evaluate at $q = 0$.

II. EXTRAPOLATIONS AND LOWEST ORDER CORRECTIONS

The principle of PDDAC for $A_\mu^{K^+}$ includes the relation $\partial^\mu A_\mu^{K^+} = a_K \varphi^{K^+}$ plus suitable smooth extrapolations of the matrix element concerned in the kaon four-momentum. First, let us consider the limit of T_{fi}, under K-pole dominance, as $k \to 0$. In that limit $k' \to p - p'$, a space-like four-vector. The "weak amplitude" term vanishes only under this strict limit. We now extrapolate the matrix element from $k'^2 = (p - p')^2$ to $k'^2 = 0$, and then assume a smooth continuation in the first power k_0' to its time-like value on the mass-shell, while k'^2 is kept at zero because of K'-pole dominance. Thus only the current algebra term contributes in this approximation, and we write

$$T^C = (2\pi)^3 \frac{\sqrt{p_0 p_0'}}{M_N}\ \frac{m_K^4}{|a_K|^2}\int d^4z\ e^{ik'\cdot z}\ k_\mu'\delta^{(3)}(\vec{z}) < p'\left| V^\mu_3(z) + \sqrt{3}\,V^\mu_8(z)\right| p > ,$$

where everything is now on the mass-shell.

If we want to estimate the first order correction to the soft $K(k \rightarrow 0)$ limit, we should keep quantities linear in k in the "weak amplitude" and then extrapolate these back to the mass-shell. To first order in k, we can evaluate the "weak amplitude" term by taking only the single-particle contributions. Schnitzer[10] has shown that the intermediate single particles that can contribute must have spin-parity $1/2^{\pm}, 3/2^{+}$. Thus the known nearby resonances, whose contributions we have to consider, are: Λ, Σ^o, $Y_0^*(1405)$ and $Y_1^*(1385)$. Now, neglecting the widths of the resonances, we can write:

$$T^R = (2\pi)^3 \frac{\sqrt{p_0 p_0'}}{M_N} \frac{m_K^4}{|a_K|^2} \sum_{\substack{n=\Lambda,\Sigma^o,\\ Y_0^*, Y_1^*}} \frac{1}{i} < p' \left| k'^{\mu} k^{\nu} T \left\{ A_{\nu}^{K^-}(0) \right| n > < n \left| A_{\mu}^{K^+}(z) \right\} \right| p >$$

with the understanding that we keep terms linear in k only. Since Y_0^* is $1/2^-$ and Y_1^* is $3/2^+$, the single-particle contributions to T^R equal

$$\bar{u}(p')\Bigg[\sum_{j=\Lambda,\Sigma^o} f_A(j)^2 \not{k}\, \gamma_5 \frac{\not{p}' - \not{k} + M_j}{(p'-k)^2 - M_j^2} \not{k}' \gamma_5 + f_A(Y_0^*)^2 \not{k} \frac{\not{p}' - \not{k} + M_{Y_0^*}}{(p'-k)^2 - M_{Y_0^*}^2} \not{k}'$$

$$+ g_A(Y_1^*)^2 \frac{\not{p}' - \not{k} + M_{Y_1^*}}{(p'-k)^2 - M_{Y_1^*}^2} \Bigg\{ k\cdot k' - \frac{1}{3} \not{k} \not{k}' - \frac{2}{3M_{Y_1^*}^2} (p'-k)\cdot k (p-k')\cdot k' \tag{4}$$

$$+ \frac{k\cdot(p'-k)\not{k}' - k'\cdot(p-k')\not{k}}{3M_{Y_1^*}} \Bigg\}\Bigg] u(p),$$

(See Fig. 1)

Here $k'^{\mu} A_{\mu}^{K^+}(j) = f_A(j) \bar{\Psi}_j \not{k}' \gamma_5 \Psi_p$ for $j = \Lambda, \Sigma^o$, $k'^{\mu} A_{\mu}^{K^+}(Y_0^*) = f_A(Y^*{}_0) \bar{\Psi}_{Y_0^*} \not{k}' \Psi_p$

and $k'^{\mu} A_{\mu}^{K^+}(Y^*_1) = i\, g_A(Y_1^*)\overline{\psi}^{\sigma}_{Y_1^*} k'_{\sigma}\psi_p$ (11), with all the form-factors evaluated at zero momentum-transfer. The expression (4) can be written as

$$\bar{u}(p')\left[-\sum_{j=\Lambda,\Sigma^o} 2\, f_A(j)^2 \frac{(k\cdot p' - M_N \not{k})}{M_j - M_N} + 2\, f_A(Y_0^*)^2 \frac{(k\cdot p' - M_N\not{k})}{M_{Y_0^*} + M_N} + \frac{g_A(Y_1^*)^2}{M_{Y_1^*} - M_N}\right.$$

$$\left.\left\{-k\cdot(p-p') + \frac{1}{3}\not{k}(\not{p}-\not{p}') + \frac{2}{3M_{Y_1^*}^2} k\cdot p' p\cdot(p-p') + \frac{k\cdot p'(\not{p}-\not{p}')-(p-p')\cdot p\not{k}}{3M_{Y_1^*}}\right\}\right]u(p)+O(k^2).$$

We write now:

$$T^C = \frac{m_K^4}{|a_K|^2} k'_{\mu}\, \bar{u}(p')\left[2F_1(0)\gamma^{\mu} + \frac{iF_2(0)\mu_p}{M_N}\sigma^{\mu\nu}(p' - p)_{\nu}\right]u(p) ,$$

where $F_1(0) = 1$, $F_2(0) = 1$, $\mu_p = 1.79$, and the arguments of F_1, F_2 have gone to zero in the extrapolation connected with PDDAC. Moreover,

$$T^R = \frac{m_K^4}{|a_K|^2} \bar{u}(p')\left[-2\sum_{j=\Lambda,\Sigma^o} f_A(j)^2 \frac{k\cdot p' - M_N\not{k}}{M_j - M_N} + 2\, f_A(Y_0^*)^2 \frac{k\cdot p' - M_N\not{k}}{M_{Y_0^*} + M_N}\right.$$

$$+ \frac{g_A(Y_1^*)^2}{M_{Y_1^*} - M_N}\left\{-k\cdot(p-p') + \frac{1}{3}\not{k}(\not{p}-\not{p}') + \frac{1}{3M_{Y_1^*}^2} k\cdot p'p\cdot(p-p')\right.$$

$$\left.\left. + \frac{k\cdot p'(p-p')-(p-p')\cdot p\not{k}}{3M_{Y_1^*}}\right\}\right]u(p) + O(k^2).$$

Thus, comparing with Eq. (2), we have

$$A^C = \left(\frac{m_K^2}{|a_K|}\right)^2 \mu_p \frac{k'\cdot(p + p')}{2M_N} ,$$

$$B^C = \left(\frac{m_K^2}{|a_K|}\right)^2 (2 + \mu_p) ,$$

$$A^R = \left(\frac{m_K^2}{|a_K|}\right)^2 \left[2\sum_{j=\Lambda,\Sigma^o} f_A(j)^2 \frac{k\cdot p'}{M_j - M_N} - 2\, f_A(Y_0^*)^2 \frac{k\cdot p'}{M_{Y_0^*} + M_N} + \frac{g_A(Y_1^*)^2}{M_{Y_1^*} - M_N} \right.$$

$$\left. \left\{ k\cdot(p-p') + \frac{2}{3}\, k\cdot p' - \frac{2}{3M_{Y_1^*}^2}\, k\cdot p' p\cdot(p-p') \right\} \right] + O(k^2) \ ,$$

$$B^R = \left(\frac{m_K^2}{a_K}\right)^2 \left[2M_N \sum_{j=\Lambda,\Sigma^o} \frac{f_A(j)^2}{M_j - M_N} - 2M_N \frac{f_A(Y_0^*)^2}{M_{Y_0^*} + M_N} + \frac{g_A(Y_1^*)^2}{3(M_{Y_1^*} - M_N)} \right.$$

$$\left. \left\{ 2M_N + \frac{p\cdot(p-p')}{M_{Y_1^*}} \right\} \right] + O(k^2) \ .$$

III. SCATTERING LENGTH

At threshold,

$$A^C = \left(\frac{m_K^2}{|a_K|}\right)^2 \mu_p\, m_K = \frac{\mu_p}{f_K^2 m_K}$$

$$B^C = (2 + \mu_p)\left(\frac{m_K^2}{|a_K|}\right)^2 = \frac{2+\mu_p}{f_K^2 m_K^2} \ .$$

Thus from Eq. (3),

$$a^C_{I=1} = a^C_{K^+P} = -\frac{1}{2\pi}\, \frac{1}{1 + \frac{m_K}{M_N}}\, \frac{1}{f_K^2 m_K} \ .$$

Similarly, $a^C_{I=0} = 0$ and $a^C_{K^oP\to K^+N} = \frac{1}{2}\, a^C_1$, so that $\sigma(K^oP \to K^+N)/\sigma(K^+P \to K^+P)$ should be 1/4 at threshold.

We also have, at threshold,

$$A^R - (W-M_N)B^R = 0 + O(k^2).$$

Thus the current-algebra calculation of the scattering length is correct to first order in k and the lowest order correction comes from the k^2 term in the "weak amplitude". There is no justification for evaluating the quadratic correction by taking only the single-particle states in the "weak amplitude", but we assume that we can estimate the error from the quadratic contribution to the Born term. Since

$$\bar{u}(p')\not{k}\,\gamma_5\,\frac{\not{p}'-\not{k}+M_j}{(p'-k)^2-M_j^2}\,\not{k}'\,\gamma_5\,u(p)$$

$$=\bar{u}(p')\,\frac{2(M_N+M_j)(k\cdot p'-M_N\not{k})+(2p'\cdot k-k^2)\not{k}-(M_N+M_j)k^2}{M_N^2-M_j^2-2p'k+k^2}\,u(p)$$

at threshold, the quadratic term is $\sim k^2/(M_N+M_j)$. Thus the Born part of the quadratic correction is $\sim m_K/(M_N+M_j)$, i.e. $\sim 25\%$, relative to the current algebra term, and this is the order of magnitude of the theoretical uncertainty in our result.

IV. EFFECTIVE RANGE

Off threshold, we write $k = (\omega,\vec{q})$, $p = (E,-\vec{q})$, $k' = (\omega,\vec{q}\,')$, $p' = (E,-\vec{q})$ in the CM frame where $\hat{q}\cdot\hat{q}' = \cos\theta = x$. Thus

$$A^C = \left(\frac{m_K^2}{|a_K|}\right)^2 \frac{\mu_p}{2M_N}\;2E\omega + q^2(1+x)\quad,$$

$$B^C = \left(\frac{m_K^2}{|a_K|}\right)^2 (2+\mu_p)\;,$$

$$A^R = \left(\frac{m_K^2}{|a_K|}\right)^2\left[2\left\{\sum_{j=\Lambda,\Sigma^0}\frac{f_A(j)^2}{M_j-M_N}-\frac{f_A(Y_0^*)^2}{M_{Y_0^*}+M_N}\right\}(\omega E+q^2x)+\frac{g_A(Y_1^*)^2}{M_{Y_1^*}-M_N}\right.$$

$$\left.\left\{q^2(1-x)+\frac{2}{3}(\omega E+q^2x)+\frac{2}{3M_{Y_1^*}^2}(\omega E+q^2x)q^2(1-x)\right\}\right]+O(k^2)\;,$$

$$B^R = \left(\frac{m_K{}^2}{|a_K|}\right)^2 \left[2M_N \left\{ \sum_{j=\Lambda,\Sigma^o} \frac{f_A(j)^2}{M_j - M_N} - \frac{f_A(Y_0^*)^2}{M_{Y_0^*} + M_N} \right\} + \frac{g_A(Y_1^*)^2}{3(M_{Y_1^*} - M_N)} \right.$$

$$\left. \left\{ 2M_N - \frac{q^2(1-x)}{M_{Y_1^*}} \right\} \right] + O(k^2).$$

Let us now substitute these in (3). Then

$$\left(\frac{d^2 f_0^C}{dq^2}\right)_{q=0} = - \frac{m_K{}^3}{2\pi |a_K|^2} \frac{1}{1 + \frac{m_K}{M_N}} \left\{ 1 - \frac{m_K{}^2}{2M_N{}^2} + \frac{\bar{\mu}_p}{2} \frac{m_K}{M_N} (1 - \frac{m_K}{M_N}) \right\} ,$$

and $\left(\frac{df_0^C}{dq}\right)_{q=0} = 0.$

Moreover,

$$\left(\frac{d^2 f_0^R}{dq^2}\right)_{q=0} = - \frac{m_K{}^2}{2\pi |a_K|^2} \frac{1}{1 + \frac{m_K}{M_N}} \left\{ \sum_{j=\Lambda,\Sigma^o} \frac{f_A(j)^2}{M_j - M_N} m_K(1 - \frac{m_K}{M_N}) + \frac{f_A(Y_0^*)^2}{M_{Y_0^*} + M_N} m_K(1 - \frac{m_K}{M_N}) \right.$$

$$\left. - \frac{2}{3} \frac{g_A(Y_1^*)^2}{M_{Y_1^*} - M_N} (1 + \frac{m_K}{2M_N} + \frac{m_K M_N}{M_{Y_1^*}{}^2} + \frac{m_K}{6M_N}) \right\}, \quad \text{and}$$

$$\left(\frac{df_0{}^R}{dq}\right)_{q=0} = 0.$$

Thus[12] $$r^C = 2\pi(1 + \frac{m_K}{M_N}) \frac{f_K{}^2}{m_K} \left\{ 1 - \frac{m_K{}^2}{2M_N{}^2} + \frac{\mu_p}{2M_N} (1 - \frac{m_K}{M_N}) \right\}$$

and $$r^R = 2\pi(1 + \frac{m_K}{M_N}) \frac{f_K{}^2}{m_K} \left\{ \sum_{j=\Lambda,\Sigma^o} \frac{f_A(j)^2}{M_j - M_N} m_K(1 - \frac{m_K}{M_N}) \right.$$

$$\left. - \frac{f_A(Y_0^*)^2}{M_{Y_0^*} + M_N} m_K(1 - \frac{m_K}{M_N}) - \frac{2}{3} \frac{g_A(Y_1^*)^2}{M_{Y_1^*} - M_N} (1 + \frac{m_K}{2M_N} + \frac{m_K M_N}{M_{Y_1^*}{}^2} + \frac{m_K}{6M_N}) \right\}.$$

V. COMPARISON WITH EXPERIMENT

To compute a_1 and r from the theoretical expressions, we first need to know f_K. From $K \rightarrow \mu\nu$ decay, one knows that $f_K \sin\theta_A = .070$. In this we substitute the experimental value of θ_V[13], as obtained from $K_{\ell 3}$ decay[1,14] which gives $\sin\theta_V = .221 \pm .006$, noting that the vector coupling constants are free from symmetry-breaking effects to first order by the Ademollo-Gatto theorem.[15] Thus, $f_K = .317 \pm .008$.

In order to compute r^R we need $f_A(j)$ for $j=\Lambda,\Sigma^0$, $f_A(Y_0^*)$ and $g_A(Y_1^*)$. We now use the value $f_A(\Lambda) = .68 \pm .07$[17] predicted from Cabibbo theory with available experimental numbers for F and D. Using the estimate of $\frac{f_A(\Sigma^0)}{f_A(\Lambda)} = \frac{1}{\sqrt{2}}\,\frac{.103 \pm .022}{.213 \pm .007}$ made by Brene et al.[3], on the basis of Cabibbo theory, we obtain $f_A(\Sigma^0) = .23 \pm .08$. Y_0^* decays physically into $\Sigma\pi$ and from the width one obtains[16] that $g^2_{Y_0^*\Sigma^0\pi^0}/4\pi = 0.045 \pm 0.007$. If we write $g^2_{Y_0^*PK^-} = \alpha\, g^2_{Y_0^*\Sigma^0\pi^0}$, then according to Weil[16], α can be shown to be between 4.8 and 13.8 depending on the model he chooses. Now using the Goldberger-Treiman relation for the axial coupling between Y_0^* and P, one can obtain $f_A(Y_0^*)$. The contribution of the state Y_0^* to r^R is then seen to be absolutely negligible for $\alpha \sim 10$. For Y_1^*, we calculate its rate for going into $\Lambda\pi^0$ using PDDAC and obtain from the experimental width that $g_A(Y_1^*\Lambda\pi^0)^2 = .67 \pm .05$. To compute $g_A(Y_1^*PK^-)$, for reasons explained below, we use the exact SU(3) relation

$$g_A(Y_1^*PK^-)^2 = \frac{2}{3}\, g_A(Y_1^*\Lambda\pi^0)^2 ,$$

hence $g_A(Y_1^*PK)^2 = .45 \pm .04$.

We have three reasons for using the exact SU(3) relation for $g_A(Y_1^*)$

and for selecting the estimate of $f_A(\Lambda)$ made from F, D values:

1. There are no other estimates of $f_A(\Lambda)$ and $g_A(Y_1^*)$ that are free from large experimental errors or model uncertainties.

2. We are considering a correction that is already small because the contributions from Λ and Y_1^* have opposite signs. On the basis of what we know about mass-splittings in the baryon octet and decuplet, the effect of symmetry-breaking (which is partly taken care of by our PCAC constant a_K) on this difference is not expected to affect our conclusions significantly.

3. We know that in the leptonic decay of Λ, the use of SU(3) (Cabibbo theory) is in reasonable agreement with experiment.

Putting in all these numbers, we finally obtain $a_{I=1}$, r^C and r^R. Table I shows the results in comparison with experiment. In view of the small magnitude of r^R, we expect the current algebra value for r^C to be close to the experimental value.

VI. CONCLUSION

We claim that the disagreement between a_1 and $a_{K^+P}^{exp}$ is mainly due to quadratic and higher order corrections in the extrapolation of the kaon four momentum. The fact that the current-algebra result for r gives a fairly good value in comparison with experiment illustrates two points:

1. The "weak amplitude" term T^R can only contribute through the exchange pole here, and not through the direct pole (whose effects are more dominant) as in πN scattering.[10] Even in the crossed channel pole contribution to the first order term in k, the intermediate Λ and Y_1^* tend to cancel each other's effects.

2. Since $r = -\frac{1}{a^2}\left(\frac{d^2 f_0}{dq^2}\right)_{q=0}$, the quadratic corrections to a^2 and to

$\left(\frac{d^2 f_0}{dq^2}\right)_{q=0}$ seem to be compensating each other.

Finally, our results indicate that, despite the large extrapolations involved, PDDAC for the K-meson appears to be in fairly good standing.

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5. Our approach is thus contrary to that of Reference 3 where renormalized angles are used, so that $\theta_V{}^R \neq \theta_A{}^R$, and the form factors are taken to be the same for vertices that are similar from an SU(3) standpoint.

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9. See, for example, S. Gasiorowicz, Elementary Particle Physics (John Wiley and Sons), p. 371.

10. H. J. Schnitzer, Current Algebra Determination of Low Energy Pion Nucleon Scattering, Brandeis University preprint. The part of the scattering

amplitude that contributes to the S-wave effective range goes as q^2 at threshold (q = relative momentum in CM frame). Schnitzer shows, starting with the general Rarita-Schwinger projection operator, that the contributions from all intermediate states with $J > \frac{3}{2}$ corresponding to the normal axial vector transition (i.e. with $J = \frac{5}{2}^-, \frac{7}{2}^+, \ldots\ldots$) go as $q^2 \times$ [contribution from $J = \frac{3}{2}^+$] and the bracketted quantity vanishes at threshold. The contributions from intermediate baryons corresponding to the abnormal axial vector transition (i.e. with $J = \frac{3}{2}^-, \frac{5}{2}^+, \ldots$) are shown to be of order higher than q^2. Thus baryons with $J = \frac{3}{2}^-$ and $J > \frac{3}{2}$ do not contribute to the S-wave effective range at threshold.

11. In fact, $A_\mu^{K^+}(Y_1^*) = \bar{\Psi}_\sigma^{Y_1^*}(P) \Big\{ g_1(k^2)k^\sigma(k^2 p_\mu - k_\mu p\cdot k) + ig_2(k^2)k^\sigma\gamma^\alpha\epsilon_{\alpha\rho\mu}p^\rho k^\lambda\gamma_5 + ig_3(k^2)\epsilon^{\sigma\alpha\beta\gamma}\epsilon_{\gamma\rho\tau\mu}P_\alpha p_\beta k^\tau + ig_A(k^2)\delta_\mu^\sigma \Big\}\psi(p)$. See Reference 10.

12. In applying PDDAC and evaluating the matrix-element by taking only the single-particle contributions in the "weak amplitude", we have reduced it to something real and hence nonunitary. In using this to calculate r, we are assuming that there is little contribution to r from the unitarity branch cut through the non-linear term $2/f_0^3(df_0/dq)^2$ at $q = 0$. This assumption of a "weak singularity" in the unitarity branch-point has been verified in detail for $\pi\pi$ scattering by N. N. Khuri, Phys. Rev. 153, 1477 (1967).

13. In order to use an experimental value of $\sin\theta_A$ directly, one would have to take into account the renormalization of $m_K f_K/m_\pi f_\pi$ in $\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)}$ due to symmetry-breaking. A. Sirlin, (Phys. Rev. Letters 16, 872 (1966)) has done this to first order and obtains $m_K f_K/m_\pi f_\pi = 1.4 \pm 0.3$, whereas if one substitutes $\theta_V = \theta_A$ in the ratio of the decay rates mentioned above, one finds $m_K f_K/m_\pi f_\pi = 1.21 \pm 0.04$.

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TABLE I

S-WAVE KN SCATTERING	SCATTERING LENGTHS a_1 (fermis)	a_0 (fermis)	EFFECTIVE RANGE r^{K^+P} (fermis)
EXPERIMENTAL	$-0.29 \pm .02$ (18) $-0.31 \pm .01$ (19)	$.04 \pm .04$ (19)	0.5 ± 0.15 (18)
THEORETICAL	$-0.41 \pm .02$	0	$r^C = 0.40 \pm .02$ $r^{correction} \equiv r^R = \pm 0.09$

(The errors quoted in the theoretical values of a_1 and r^C are due to errors in sin θ.)

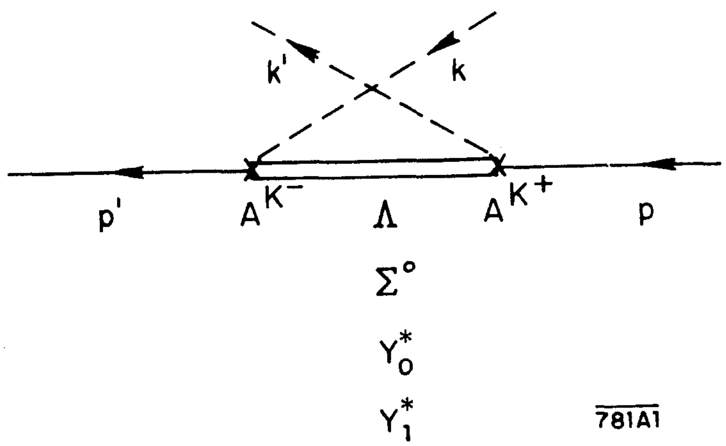

FIG. 1 - Exchange Poles in the "Weak Amplitude".