SLAC-PUB-313
June 1967

DETERMINATION OF THE QUALITY OF MULTIPOLE MAGNETS*

J. K. Cobb, A. W. Burfine, and D. R. Jensen
Stanford Linear Accelerator Center
Stanford, California

(To be printed in Proceedings of the Second International Conference on Magnet Technology, Oxford, England, July 1967.)

*Work supported by the U.S. Atomic Energy Commission.

DETERMINATION OF THE QUALITY OF MULTIPOLE MAGNETS

Introduction

The techniques for analyzing the quality of multipole fields by spectroscopy have previously[1] been described. It has been shown that one can use the results to express the vector magnetic induction of a multipole magnet in closed analytical form. As a practical matter, as the techniques for fabrication of multipole magnets have been improved with the inclusion of hyperbolic pole shapes, one must more than ever consider the quality of the multipole as a whole, taking into consideration the perturbation from the ideal field caused by end effects of the element. This quantity is the linearity of the gradient field integrated over the length of the element. In general, the deviation of the gradient over the length can be considerably different than the deviation at one longitudinal position. This condition is caused by the vector sum of the higher harmonics, and by differences in phase along the longitudinal axis which changes the magnitude and phase of each integrated harmonic.

We have analyzed a great number of multipole magnets and have developed methods of final representation that give in functional form

$$\int_{-\infty}^{+\infty} \frac{\delta B\, dZ}{B} = f(r, \theta) \quad \text{and} \quad \int_{-\infty}^{+\infty} \frac{\delta G\, dZ}{G_o} = g(r, \theta) \quad \text{for the elements,}$$

where $\delta B = B - G_0 r$ and G_0 is the gradient at the center. We have written a computer program that uses as input data the directly read harmonic amplitudes at many discrete longitudinal positions and then computes the total integrated perturbation at various positions across the aperture. Output from the computer is in the form of tables and graphs of the above functions which allow easy analysis of quality and can be used in beam transport calculations directly.

Mathematical Basis

Using the results of multipole spectroscopy it has been shown that the vector magnetic induction may be expressed in closed analytical form as:

$$B_r = -\mu_o \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \nu_{mn} \left(\frac{k_{mn}}{k_{22}} (2 K_{22}) \right) r^{n-1} \sin (n\theta - \alpha_{mn})$$

$$B_\theta = -\mu_o \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \nu_{mn} \left(\frac{k_{mn}}{k_{22}} (2 K_{22}) \right) r^{n-1} \cos (n\theta - \alpha_{mn})$$

where

$$\nu_{mn} = \begin{cases} 1 & \text{for } n=m,\ n=3m,\ n=5n,\ n=7m, \ldots \text{ etc.} \\ 0 & \text{for all other } n \end{cases}$$

k_{mn} = voltage amplitude for harmonic mn

α_{mn} = orientation angle for harmonic mn

---

[1] J. K. Cobb and R. Cole, Proceedings of the International Symposium on Magnet Technology, 1965, p. 431.

For the coordinate systems used, refer to Fig. 1.

One usually expresses the quality of a quadrupole lens graphically as the deviations of field B and gradient G in the following manner: For $\theta = 0, \pi/2, \pi, 3\pi/2$,

$$\frac{B_Y - G_o X}{G_o X} = f_o(X) ; \quad \frac{B_X - G_o Y}{G_o Y} = g_o(Y) \quad \text{for field deviations}$$

and

$$\frac{G_X - G_o}{G_o} = f_1(X) ; \quad \frac{G_Y - G_o}{G_o} = g_1(Y) \quad \text{for gradient deviations}$$

Also for $\theta = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

$$\frac{B_x - G_o x}{G_o x} = h_o(x) ; \quad \frac{B_y - G_o y}{G_o y} = j_o(y)$$

and

$$\frac{G_x - G_o}{G_o} = h_1(x) ; \quad \frac{G_y - G_o}{G_o} = j_1(y) .$$

Since along $\theta = 0$ we are interested in the Y component of fields and gradients, then B_Y is the expression for B_θ when $\theta = 0$. So

$$B_Y = -\mu_o \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \nu_{mn} \left(\frac{k_{mn}}{k_{22}} (2K_{22}) \right) X^{n-1} \cos(\alpha_{mn})$$

$$G_X = \frac{\partial B_Y}{\partial X} = -\mu_o \sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \nu_{mn} \left(\frac{k_{mn}}{k_{22}} (2K_{22}) \right) (n-1) X^{n-2} \cos(\alpha_{mn})$$

at $X = 0$

$$G_o = -2\mu_o \nu_{22} K_{22}$$

Then

$$\left.\frac{\delta B}{B}\right|_{\theta=0} = \left.\frac{B_Y - G_o X}{G_o X}\right|_{\theta=0} = \frac{-\mu_o \sum_{n=2}^{\infty}\sum_{m=2}^{\infty} \nu_{mn}\left(\frac{k_{mn}}{k_{22}}(2K_{22})\right) X^{n-1}\cos(\alpha_{mn}) - 2\mu_o \nu_{22} K_{22} X}{-2\mu_o \nu_{22} K_{22} X}$$

$$= \sum_{3}^{\infty} \sum_{3}^{\infty} \frac{\nu_{mn}}{\nu_{22}} \left(\frac{k_{mn}}{k_{22}} \right) X^{n-2} \cos(\alpha_{mn})$$

$$= \sum_{3}^{\infty} \sum_{3}^{\infty} \frac{k_{mn}}{k_{22}} X^{n-2} \cos(\alpha_{mn}) \quad \text{for all allowed values of m and n} \tag{1}$$

Now $G_X = \dfrac{\partial B_Y}{\partial X}$ so similar treatment yields

$$\left.\frac{\delta G}{G}\right|_{\theta=0} = \left.\frac{G_X - G_o}{G_o}\right|_{\theta=0} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} (n-1) X^{n-2} \cos(\alpha_{mn}) \tag{2}$$

On the axis $\theta = \pi/2$

$$\left.\frac{\delta B}{B}\right|_{\theta=\pi/2} = \left.\frac{B_X - G_o X}{G_o X}\right|_{\theta=\pi/2} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} Y^{n-2} \cos(n\,\pi/2 - \alpha_{mn}) \tag{3}$$

$$\left.\frac{\delta G}{G}\right|_{\theta=\pi/2} = \left.\frac{G_Y - G_o}{G_o}\right|_{\theta=\pi/2} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} (n-1) Y^{n-2} \cos(n\pi/2 - \alpha_{mn}) \tag{4}$$

For the minor axes where $\theta = \pi/4$ and $3\pi/4$ we are interested in the B_r component and we can derive

$$\left.\frac{\delta B}{B}\right|_{\theta=\pi/4} = \left.\frac{B_x - G_o x}{G_o x}\right|_{\theta=\pi/4} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} x^{n-2} \sin(n\pi/4 - \alpha_{mn}) \tag{5}$$

$$\left.\frac{\delta G}{G}\right|_{\theta=\pi/4} = \left.\frac{G_x - G_o}{G_o}\right|_{\theta=\pi/4} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} (n-1) x^{n-2} \sin(n\pi/4 - \alpha_{mn}) \tag{6}$$

and

$$\left.\frac{\delta B}{B}\right|_{\theta=3\pi/4} = \left.\frac{B_y - G_o y}{G_o y}\right|_{\theta=3\pi/4} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} y^{n-2} \sin(n\,3\pi/4 - \alpha_{mn}) \tag{7}$$

$$\left.\frac{\delta G}{G}\right|_{\theta=3\pi/4} = \left.\frac{G_y - G_o}{G_o}\right|_{\theta=3\pi/4} = \sum_3^\infty \sum_3^\infty \frac{k_{mn}}{k_{22}} (n-1) y^{n-2} \sin(n\,3\pi/4 - \alpha_{mn}) \tag{8}$$

In this analysis we have not considered the r component of field at $\theta = 0$ and $\theta = \pi/2$, nor have we considered the θ component at $\theta = \pi/4$ and $\theta = 3\pi/4$.

Experimental Technique

Ideally, using a long coil that extends completely through the magnet one could measure directly the integral of the field and gradient deviations by measuring the components k_{mn}/k_{22} and α_{mn}.

Another possibility is to use a coil of small longitudinal extent and measure the components k_{mn}/k_{22} and α_{mn} at a great number of longitudinal positions. By making a numerical integration one may form the quantities

$$\int_{-\infty}^{\infty} \frac{\delta B}{B}\,dz \quad \text{and} \quad \int_{-\infty}^{\infty} \frac{\delta G}{G}\,dz$$

at least along the major and minor axes. For our work we prefer to use a coil of small longitudinal extent, typically 1.0 inches but with as large radius of rotation as is possible in the quadrupole to be tested. The large radius allows us to have maximum sensitivity for the harmonics which we pick up. Figure 2 shows the coil shape that is used in the analysis. As the coil is rotated in the magnet the voltage induced in the coil is fed into an audio-frequency wave analyzer and the absolute amplitude of each harmonic as well as the fundamental (quadrupole induced) is recorded for that longitudinal position. The output from the wave analyzer is fed into an oscilloscope where the phase angle of the particular harmonic is found by measurement of its time-distance from the quadrupole fundamental sinusoidal wave form.* Since the quadrupole fundamental sinusoidal wave form goes through two complete cycles per coil bundle revolution, a magnetic pulse trigger is simultaneously fed into the oscilloscope which triggers the oscilloscope once each coil revolution. Thus for each longitudinal position z the amplitudes k_{mn} and k_{22} and the phase angle α_{mn} are measured.

Computer Handling of Data

To find the integrated values of k_{mn} and α_{mn}, an iteration through the individual points is made which resembles the trapzoidal rule:

Let

$$AX = \text{Area in } Xz \text{ plane}$$

and

$$AY = \text{Area in } Yz \text{ plane}$$

$$\text{Sum}_i = i^{th} \text{ iteration of the area}$$

$$\theta_i = i^{th} \text{ iteration of the angle}$$

$$AY = k'_{mn_i} \sin \alpha_{mn_i} (z_{i+1} - z_{i-1})/2 + \text{Sum}_{i-1} \sin \theta_{i-1}$$

$$AY = k'_{mn_i} \cos \alpha_{mn_i} (z_{i+1} - z_{i-1})/2 + \text{Sum}_{i-1} \cos \theta_{i-1}$$

$$\text{Sum}_i = \sqrt{AY^2 + AX^2}$$

$$\theta_i = \text{Arc Tan } \frac{AY}{AX}$$

This process is repeated for each at the harmonics. From these integrated harmonics

$$\int_{-\infty}^{+\infty} \frac{\delta B_\theta}{B_\theta} \, dz \quad \text{and} \quad \int_{-\infty}^{+\infty} \frac{\delta G_\theta}{G_o} \, dz$$

are calculated for 0 and $\pi/2$ at various points across the aperture. Similarly for

$$\int_{-\infty}^{+\infty} \frac{\delta B_r}{B_r} \, dz \quad \text{and} \quad \int_{-\infty}^{+\infty} \frac{\delta G_r}{G_o} \, dz$$

at $\pi/4$ and $3\pi/4$. These values are then plotted for final representation. Figures 5-11 show a typical computer output.

This technique of analysis can also be used for multipoles other than quadrupoles and we have analyzed sextupoles in an altogether analogous manner at this laboratory. The emphasis has been placed on analysis of quadrupoles for the simple reason that the quadrupole is by far the most often used multipole configuration.

*See Figs. 3 and 4.

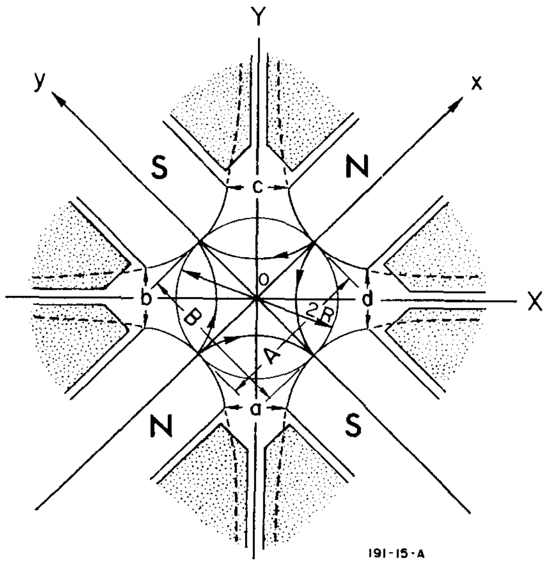


FIG. 1--Coordinate system

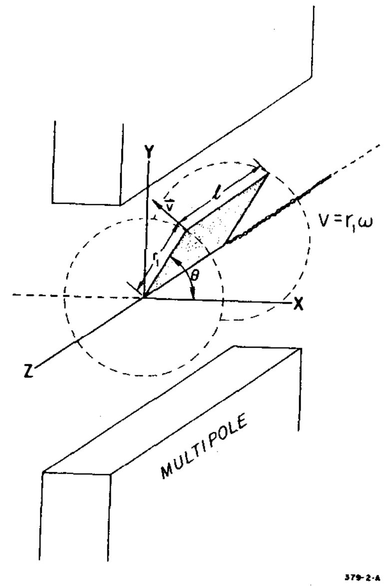


FIG. 2--Schematic of coil

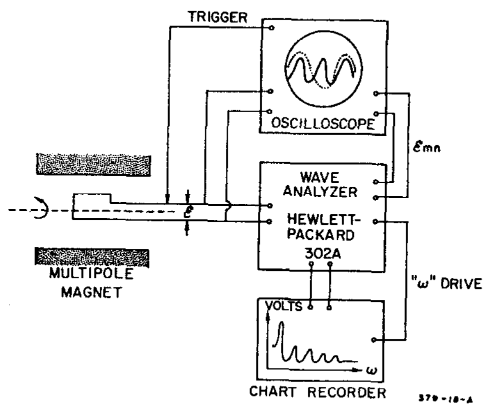


FIG. 3--Block diagram of harmonic analyzer system

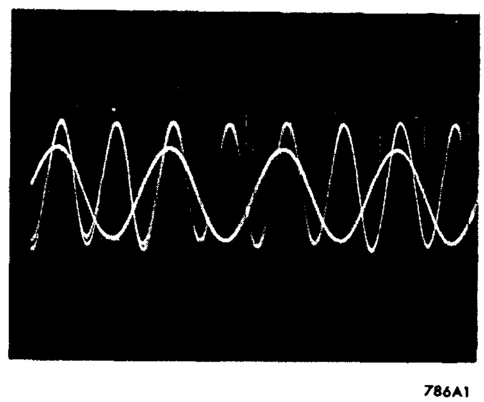


FIG. 4--Oscillograph of two superposed sinusoidal waves (Quadrupole and Decapole)

Q-201 QUADRUPOLE INTEGRATED HARMONIC ANALYSIS DATE 121-1967

FULL MAGNET

APERTURE RADIUS = 5.500
COIL RADIUS = 5.000
CURRENT = 1000.00

THE DATA POINTS ARE

(IN MILLIVOLTS)

POLE Z	2	3	4	5	6	7	8	9	10	14
0.0	2850@0	0.00	0.00	2.50@-090	2.50@+000	0.80@+000	0.40@+090	0.40@-090	1.40@+180	1.50@+180
10.0	2850@0	0.00	0.00	2.50@-090	2.50@+000	0.80@+000	0.40@+090	0.40@-090	1.40@+180	1.50@+180
14.0	2850@0	0.00	0.00	2.50@-090	2.60@+000	0.80@+000	0.40@+090	0.40@-090	1.40@+180	1.50@+180
18.0	2900@0	0.00	0.00	2.90@-090	2.00@+000	0.60@+000	0.40@+090	0.60@-090	1.30@+180	1.50@+180
22.0	2900@0	0.00	0.00	2.80@-090	1.80@+000	0.70@+000	0.40@+090	0.50@-090	1.50@+180	1.50@+180
26.0	2900@0	0.00	0.00	2.60@-090	1.80@+000	0.70@+000	0.60@+090	0.50@-090	1.40@+180	1.50@+180
29.0	2900@0	0.00	0.00	2.60@-090	1.50@+000	0.50@+000	0.40@+090	0.40@-090	1.30@+180	1.40@+180
31.0	2870@0	0.00	0.00	2.50@-090	2.70@+000	0.50@+000	0.30@+090	0.30@-090	1.10@+180	1.40@+180
32.0	2860@0	0.00	0.00	2.50@-090	9.10@+180	0.60@+000	0.30@+090	0.30@-090	0.70@+180	1.40@+180
33.0	2810@0	0.00	0.00	2.20@-090	22.00@+180	0.70@+000	0.40@+090	0.40@-090	2.00@+000	1.60@+180
33.5	2790@0	0.00	0.00	2.30@-090	26.50@+180	0.50@+000	0.40@+090	0.40@-090	4.30@+000	2.20@+180
34.0	2740@0	0.00	0.00	2.50@-090	25.50@+180	0.30@+000	0.50@+090	0.30@-090	5.00@+000	2.30@+180
34.5	2680@0	0.00	0.00	2.80@-090	18.00@+180	0.40@+000	0.50@+090	0.00	3.50@+000	1.90@+180
35.0	2600@0	0.00	0.00	2.70@-090	11.00@+180	0.40@+000	0.50@+090	0.00	2.40@+000	1.70@+180
35.5	2530@0	6.00@-045	0.00	2.50@-090	4.00@+180	0.00	0.60@+000	0.50@-090	2.30@+000	1.70@+180
36.0	2460@0	7.00@-045	0.00	2.40@-090	1.80@+180	0.00	0.60@+000	0.00	2.80@+000	1.80@+180
36.5	2390@0	7.50@-045	0.00	2.40@-090	1.80@-090	0.00	0.70@+000	0.50@-090	3.80@+000	1.90@+180
37.0	2290@0	8.00@-045	0.00	2.40@-090	1.70@-090	0.00	0.70@+000	0.40@-090	5.10@+000	1.90@+180
37.5	2190@0	9.00@-045	0.00	2.50@-090	1.70@-090	0.30@-090	0.70@+000	0.40@-090	6.50@+000	2.00@+180
38.0	2020@0	10.00@-045	1.20@+000	2.50@-090	3.80@+000	0.30@-090	0.70@+000	0.40@-090	7.40@+000	2.10@+180
38.5	1900@0	11.50@-045	1.30@+000	2.50@-090	11.50@+000	0.90@+180	0.70@+000	0.40@-090	7.00@+000	2.10@+180
39.0	1800@0	12.00@-045	1.40@+000	2.80@+180	22.00@+000	0.80@+135	0.90@+000	0.30@+000	5.50@+000	1.70@+180
39.5	1650@0	13.00@-045	1.50@+000	2.20@+180	32.00@+000	1.00@+135	0.90@+000	0.30@+000	4.60@+000	1.70@+180
40.0	1500@0	14.00@-045	1.60@+000	2.40@+180	40.00@+000	0.80@+135	1.00@+000	0.40@+000	3.60@+000	1.40@+180
40.5	1300@0	14.00@-045	2.00@+000	2.70@+180	44.00@+000	1.10@+135	1.10@+000	0.40@+000	2.70@+000	1.10@+180
41.0	1180@0	14.00@-045	2.20@+000	2.20@+180	42.00@+000	0.80@+135	1.20@+000	0.40@+000	1.80@+000	1.00@+180
41.5	1050@0	14.00@-045	2.40@+000	1.70@+180	37.00@+000	1.10@+135	1.30@+000	0.40@+000	0.80@+000	1.00@+180
42.0	890@0	13.50@-045	2.40@+000	1.00@+180	29.00@+000	1.20@+135	1.30@+000	0.40@+000	0.60@+180	1.20@+180
42.5	770@0	12.50@-045	2.40@+000	0.40@+180	20.50@+000	1.40@+135	1.20@+000	0.00	1.40@+180	1.30@+180
43.0	650@0	11.50@-045	1.80@+000	0.00	12.00@+000	1.20@+135	1.00@+000	0.30@-045	1.70@+180	1.00@+180
43.5	550@0	10.50@-045	1.50@+000	0.00	6.70@+000	1.00@+135	0.80@+000	0.30@-045	1.40@+180	0.50@+180
44.0	460@0	9.50@-045	1.20@+000	0.00	3.00@+000	0.60@+135	0.50@+000	0.20@-045	0.90@+180	0.50@+180
44.5	380@0	8.20@-045	1.00@+000	0.20@-090	1.00@+000	0.30@+135	0.20@+000	0.20@-045	0.40@+180	0.40@+180
45.0	300@0	7.00@-045	0.90@+000	0.30@-090	0.30@+000	0.00	0.00	0.00	0.50@+180	0.20@+180
45.5	250@0	6.10@-045	0.80@+000	0.00	0.00	0.00	0.00	0.00	0.00	0.00
46.0	190@0	5.20@-045	0.80@+000	0.00	0.00	0.00	0.00	0.00	0.00	0.00
46.5	140@0	4.00@-045	0.90@+000	0.00	0.20@+000	0.00	0.00	0.00	0.00	0.00
47.0	100@0	3.00@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
47.5	80@0	2.40@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
48.0	56@0	1.80@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
48.5	39@0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

FIG. 5--Input data

Q-201 QUADRUPOLE INTEGRATED HARMONIC ANALYSIS DATE 121-1967

THE DATA POINTS ARE

(NORMALIZED TO APERTURE RADIUS)

(IN PERCENT)

POLE Z	2	3	4	5	6	7	8	9	10	14
0.0	100@0	0.00	0.00	0.12@-090	0.13@+000	0.05@+000	0.02@+090	0.03@-090	0.11@+180	0.17@+180
10.0	100@0	0.00	0.00	0.12@-090	0.13@+000	0.05@+000	0.02@+090	0.03@-090	0.11@+180	0.17@+180
14.0	100@0	0.00	0.00	0.12@-090	0.13@+000	0.05@+000	0.02@+090	0.03@-090	0.11@+180	0.17@+180
18.0	100@0	0.00	0.00	0.13@-090	0.10@+000	0.03@+000	0.02@+090	0.04@-090	0.10@+180	0.16@+180
22.0	100@0	0.00	0.00	0.13@-090	0.09@+000	0.04@+000	0.02@+090	0.03@-090	0.11@+180	0.16@+180
26.0	100@0	0.00	0.00	0.12@-090	0.09@+000	0.04@+000	0.04@+090	0.03@-090	0.10@+180	0.16@+180
29.0	100@0	0.00	0.00	0.12@-090	0.08@+000	0.03@+000	0.02@+090	0.03@-090	0.10@+180	0.15@+180
31.0	100@0	0.00	0.00	0.12@-090	0.14@+000	0.03@+000	0.02@+090	0.02@-090	0.08@+180	0.15@+180
32.0	100@0	0.00	0.00	0.12@-090	0.47@+180	0.03@+000	0.02@+090	0.02@-090	0.05@+180	0.15@+180
33.0	100@0	0.00	0.00	0.10@-090	1.15@+180	0.04@+000	0.03@+090	0.03@-090	0.15@+000	0.18@+180
33.5	100@0	0.00	0.00	0.11@-090	1.39@+180	0.03@+000	0.03@+090	0.03@-090	0.33@+000	0.25@+180
34.0	100@0	0.00	0.00	0.12@-090	1.36@+180	0.02@+000	0.03@+090	0.02@-090	0.39@+000	0.26@+180
34.5	100@0	0.00	0.00	0.14@-090	0.98@+180	0.02@+000	0.03@+090	0.00	0.28@+000	0.22@+180
35.0	100@0	0.00	0.00	0.14@-090	0.62@+180	0.02@+000	0.03@+090	0.00	0.20@+000	0.21@+180
35.5	100@0	0.26@-045	0.00	0.13@-090	0.23@+180	0.00	0.04@+000	0.04@-090	0.19@+000	0.21@+180
36.0	100@0	0.31@-045	0.00	0.13@-090	0.11@+180	0.00	0.04@+000	0.00	0.24@+000	0.23@+180
36.5	100@0	0.35@-045	0.00	0.13@-090	0.11@-090	0.00	0.05@+000	0.04@-090	0.34@+000	0.25@+180
37.0	100@0	0.36@-045	0.00	0.14@-090	0.11@-090	0.00	0.05@+000	0.03@-090	0.48@+000	0.26@+180
37.5	100@0	0.45@-045	0.00	0.15@-090	0.11@-090	0.02@-090	0.06@+000	0.04@-090	0.64@+000	0.29@+180
38.0	100@0	0.54@-045	0.07@+000	0.16@-090	0.26@+000	0.02@-090	0.06@+000	0.04@-090	0.79@+000	0.33@+180
38.5	100@0	0.67@-045	0.08@+000	0.18@-090	0.80@+000	0.08@+180	0.07@+000	0.04@-090	0.79@+000	0.35@+180
39.0	100@0	0.73@-045	0.09@+000	0.21@+180	1.70@+000	0.07@+135	0.09@+000	0.03@+000	0.65@+000	0.30@+180
39.5	100@0	0.87@-045	0.11@+000	0.18@+180	2.84@+000	0.10@+135	0.10@+000	0.04@+000	0.60@+000	0.32@+180
40.0	100@0	1.03@-045	0.13@+000	0.21@+180	3.90@+000	0.09@+135	0.12@+000	0.05@+000	0.51@+000	0.29@+180
40.5	100@0	1.18@-045	0.19@+000	0.28@+180	4.96@+000	0.14@+135	0.15@+000	0.06@+000	0.45@+000	0.27@+180
41.0	100@0	1.31@-045	0.23@+000	0.25@+180	5.21@+000	0.11@+135	0.18@+000	0.07@+000	0.33@+000	0.27@+180
41.5	100@0	1.47@-045	0.26@+000	0.22@+180	5.16@+000	0.17@+135	0.22@+000	0.07@+000	0.16@+000	0.30@+180
42.0	100@0	1.67@-045	0.33@+000	0.15@+180	4.77@+000	0.22@+135	0.26@+000	0.09@+000	0.14@+180	0.42@+180
42.5	100@0	1.79@-045	0.38@+000	0.07@+180	3.90@+000	0.29@+135	0.28@+000	0.00	0.39@+180	0.53@+180
43.0	100@0	1.95@-045	0.34@+000	0.00	2.70@+000	0.30@+135	0.27@+000	0.09@-045	0.56@+180	0.48@+180
43.5	100@0	2.10@-045	0.33@+000	0.00	1.78@+000	0.29@+135	0.26@+000	0.11@-045	0.55@+180	0.29@+180
44.0	100@0	2.27@-045	0.32@+000	0.00	0.95@+000	0.21@+135	0.19@+000	0.08@-045	0.42@+180	0.34@+180
44.5	100@0	2.37@-045	0.32@+000	0.07@-090	0.39@+000	0.13@+135	0.09@+000	0.10@-045	0.23@+180	0.33@+180
45.0	100@0	2.57@-045	0.36@+000	0.13@-090	0.15@+000	0.00	0.00	0.00	0.36@+180	0.21@+180
45.5	100@0	2.68@-045	0.39@+000	0.00	0.00	0.00	0.00	0.00	0.00	0.00
46.0	100@0	3.01@-045	0.51@+000	0.00	0.00	0.00	0.00	0.00	0.00	0.00
46.5	100@0	3.14@-045	0.78@+000	0.00	0.21@+000	0.00	0.00	0.00	0.00	0.00
47.0	100@0	3.30@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
47.5	100@0	3.30@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
48.0	100@0	3.54@-045	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
48.5	100@0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

THE INTEGRATED VALUES ARE

	2	3	4	5	6	7	8	9	10	14
(IN MILLIVOLTS)	254@0	284.59@-045	36.34@+000	294.56@-094	493.53@-001	71.82@+011	65.32@+061	71.53@-084	54.23@+180	460.88@+180
(IN PERCENT)	100@0	0.11@-045	0.01@+000	0.12@-094	0.19@-001	0.03@+011	0.03@+061	0.03@-084	0.02@+180	0.18@+180

786A2

FIG. 6--Data points normalized to aperture radius and length integrated values

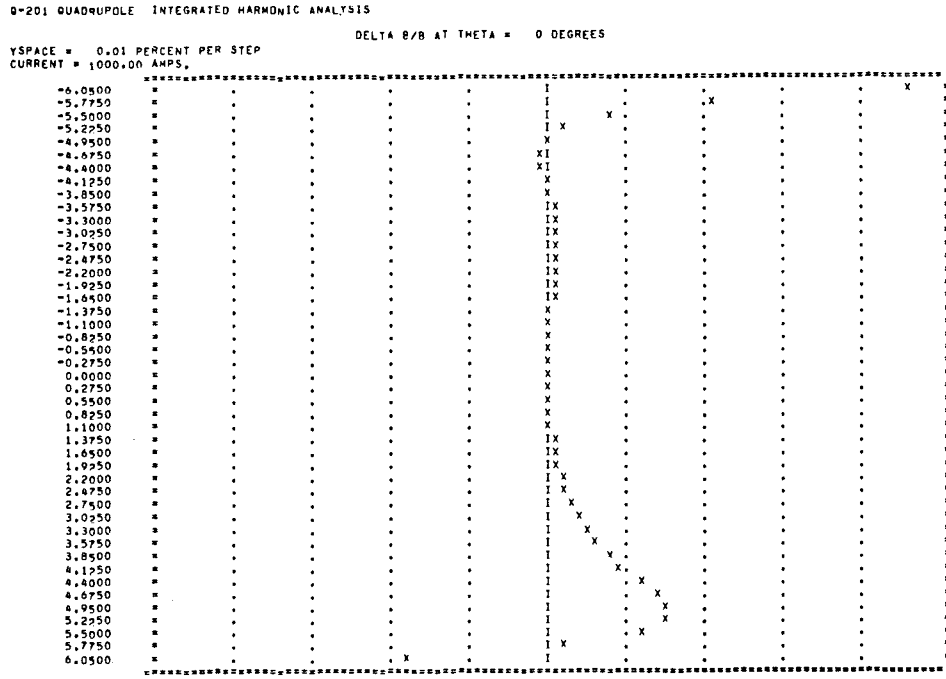
Q-201 QUADRUPOLE INTEGRATED HARMONIC ANALYSIS DATE 121-1967

THE CALCULATED POINTS ARE

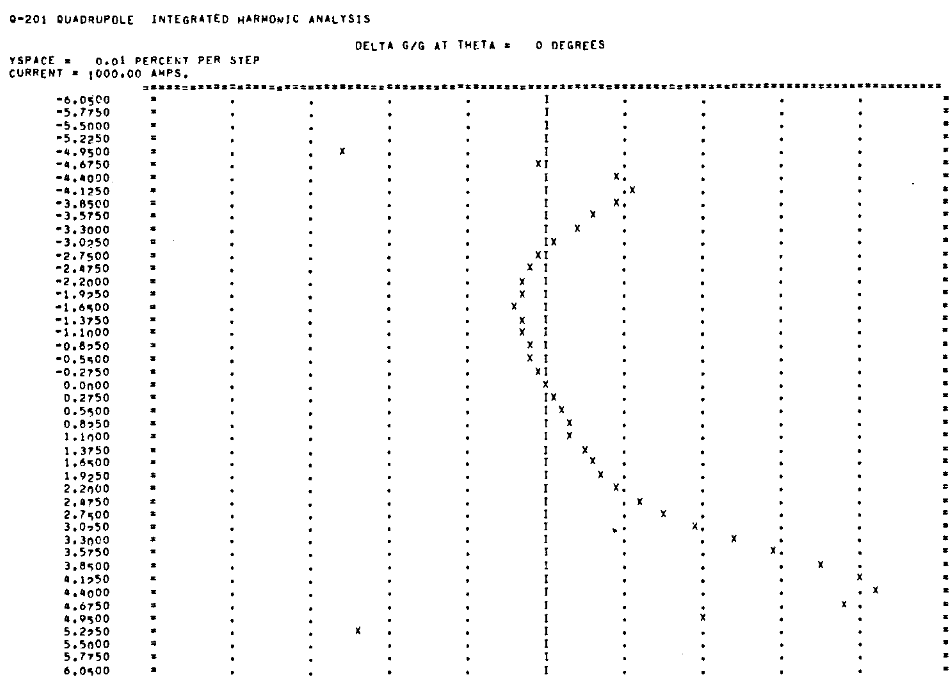
R	THETA = 0 DEGREES DELTAB/B	DELTAG/G	THETA = 45 DEGREES DELTAB/B	DELTAG/G	THETA = 90 DEGREES DELTAB/B	DELTAG/G	THETA = 135 DEGREES DELTAB/B	DELTAG/G
-6.0500	0.00459	-0.06628			-0.00550	0.07528		
-5.7750	0.00211	-0.03547			-0.00263	0.04228		
-5.5000	0.00083	-0.01763	-0.00020	0.01305	-0.00105	0.02269	0.00155	-0.01691
-5.2250	0.00022	-0.00774	0.00022	0.00485	-0.00022	0.01140	0.00094	-0.00813
-4.9500	-0.00002	-0.00257	0.00035	0.00082	0.00017	0.00513	0.00066	-0.00364
-4.6750	-0.00008	-0.00011	0.00034	-0.00092	0.00034	0.00179	0.00054	-0.00154
-4.4000	-0.00006	0.00086	0.00028	-0.00146	0.00038	0.00014	0.00049	-0.00072
-4.1250	-0.00001	0.00107	0.00021	-0.00143	0.00037	-0.00060	0.00046	-0.00051
-3.8500	0.00004	0.00092	0.00014	-0.00118	0.00033	-0.00087	0.00043	-0.00057
-3.5750	0.00008	0.00065	0.00009	-0.00087	0.00028	-0.00091	0.00040	-0.00070
-3.3000	0.00011	0.00036	0.00005	-0.00060	0.00024	-0.00085	0.00036	-0.00082
-3.0250	0.00012	0.00011	0.00003	-0.00038	0.00020	-0.00075	0.00032	-0.00091
-2.7500	0.00012	-0.00009	0.00001	-0.00023	0.00017	-0.00066	0.00027	-0.00094
-2.4750	0.00011	-0.00022	0.00000	-0.00012	0.00013	-0.00058	0.00023	-0.00092
-2.2000	0.00010	-0.00031	0.00000	-0.00005	0.00011	-0.00051	0.00018	-0.00087
-1.9250	0.00008	-0.00035	0.00000	-0.00001	0.00008	-0.00044	0.00014	-0.00079
-1.6500	0.00006	-0.00035	0.00000	0.00000	0.00006	-0.00039	0.00010	-0.00069
-1.3750	0.00005	-0.00033	0.00000	0.00001	0.00004	-0.00033	0.00007	-0.00058
-1.1000	0.00003	-0.00028	0.00000	0.00001	0.00003	-0.00028	0.00005	-0.00046
-0.8250	0.00002	-0.00022	0.00000	0.00001	0.00002	-0.00022	0.00003	-0.00034
-0.5500	0.00001	-0.00015	0.00000	0.00000	0.00001	-0.00015	0.00001	-0.00023
-0.2750	0.00000	-0.00008	0.00000	0.00000	0.00000	-0.00008	0.00000	-0.00011
0.0000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.2750	0.00000	0.00008	0.00000	0.00000	0.00000	0.00008	0.00000	0.00011
0.5500	0.00001	0.00014	0.00000	0.00000	0.00001	0.00016	0.00001	0.00023
0.8250	0.00002	0.00025	0.00000	-0.00002	0.00002	0.00023	0.00003	0.00035
1.1000	0.00003	0.00035	0.00000	-0.00004	0.00003	0.00028	0.00005	0.00049
1.3750	0.00005	0.00046	0.00000	-0.00009	0.00005	0.00031	0.00008	0.00065
1.6500	0.00008	0.00059	-0.00001	-0.00016	0.00006	0.00031	0.00011	0.00084
1.9250	0.00011	0.00075	-0.00002	-0.00028	0.00008	0.00026	0.00016	0.00107
2.2000	0.00015	0.00095	-0.00004	-0.00046	0.00009	0.00016	0.00022	0.00135
2.4750	0.00021	0.00120	-0.00007	-0.00071	0.00009	-0.00002	0.00030	0.00169
2.7500	0.00028	0.00152	-0.00011	-0.00104	0.00008	-0.00029	0.00039	0.00210
3.0250	0.00036	0.00191	-0.00018	-0.00148	0.00006	-0.00065	0.00051	0.00259
3.3000	0.00047	0.00238	-0.00026	-0.00203	0.00002	-0.00112	0.00065	0.00316
3.5750	0.00060	0.00292	-0.00038	-0.00269	-0.00005	-0.00168	0.00083	0.00379
3.8500	0.00076	0.00349	-0.00053	-0.00343	-0.00015	-0.00230	0.00103	0.00443
4.1250	0.00095	0.00399	-0.00072	-0.00417	-0.00028	-0.00287	0.00127	0.00499
4.4000	0.00116	0.00422	-0.00095	-0.00473	-0.00044	-0.00321	0.00152	0.00525
4.6750	0.00136	0.00377	-0.00119	-0.00475	-0.00059	-0.00291	0.00178	0.00483
4.9500	0.00151	0.00195	-0.00140	-0.00357	-0.00070	-0.00128	0.00198	0.00303
5.2250	0.00151	-0.00244	-0.00151	-0.00005	-0.00068	0.00287	0.00204	-0.00131
5.5000	0.00119	-0.01138	-0.00134	0.00770	-0.00034	0.01150	0.00178	-0.01016
5.7750	0.00025	-0.02807			0.00060	0.02779		
6.0500	-0.00183	-0.05750			0.00264	0.05671		

786A3

FIG. 7--Calculated points of integrated field and gradient deviations



$$\text{FIG. 8--}\int_{-\infty}^{\infty} \frac{\delta B_Y}{B_Y}\, dz \Big|_{\theta=0} \text{ vs } X$$



786A4

$$\text{FIG. 9--}\int_{-\infty}^{\infty} \frac{\delta G_X}{G_X}\, dz \Big|_{\theta=0} \text{ vs } X$$

Q-201 QUADRUPOLE INTEGRATED HARMONIC ANALYSIS

DELTA B/B AT THETA = 45 DEGREES

YSPACE = 0.01 PERCENT PER STEP
CURRENT = 1000.00 AMPS.

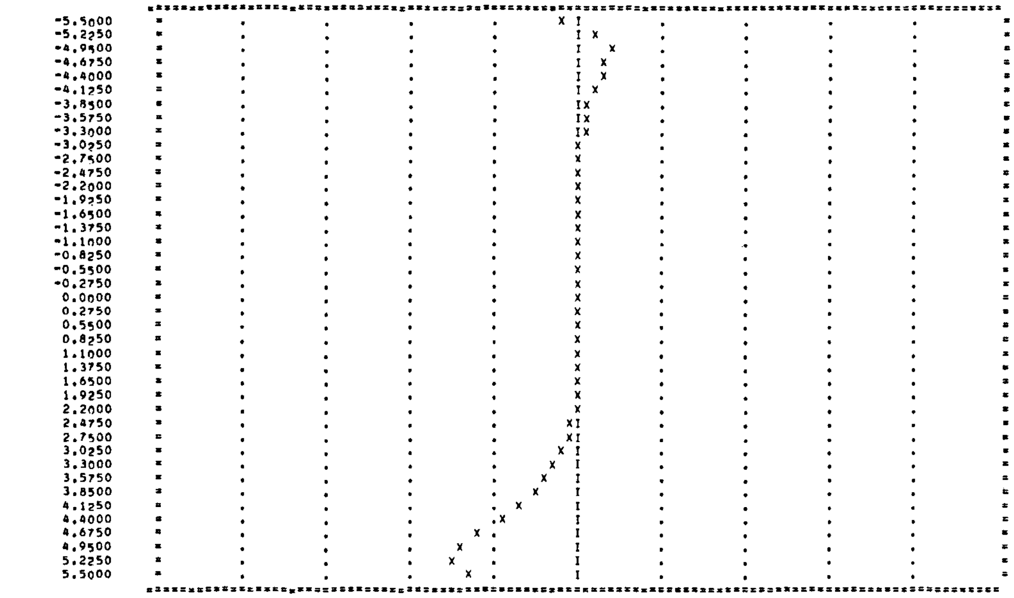
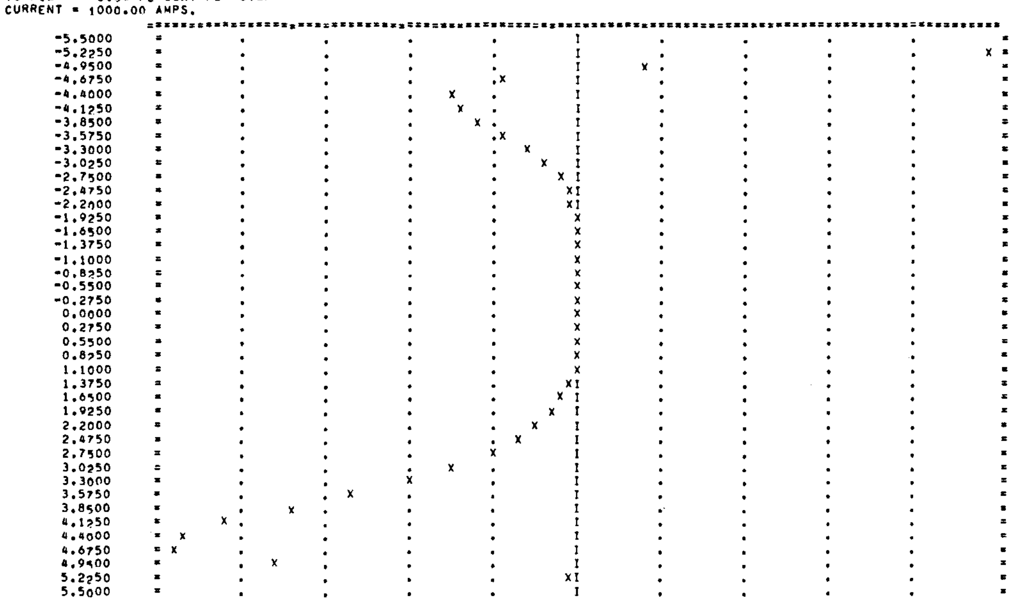


FIG. 10-- $\int_{-\infty}^{\infty} \frac{\delta B_x}{B_x} dz \Big|_{\theta=\pi/4}$ vs x

Q-201 QUADRUPOLE INTEGRATED HARMONIC ANALYSIS

DELTA G/G AT THETA = 45 DEGREES

YSPACE = 0.01 PERCENT PER STEP
CURRENT = 1000.00 AMPS.



786A5

FIG. 11-- $\int_{-\infty}^{\infty} \frac{\delta G_x}{G_x} dz \Big|_{\theta=\pi/4}$ vs x