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THRESHOLD KN^* PRODUCTION AND CURRENT ALGEBRA*

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Recent experiments by Bland[1] et al. have shown that the reaction $K^+p \to K^oN^{*++}(1236)$ has a rather large cross-section which rises rapidly from threshold. In this note we show that many of the threshold features of this experiment, i.e., the value of the cross-section, the slope with respect to energy, the C.O.M. angular distribution and the N^* decay angular distribution can be reasonably well accounted for by the same current algebra and meson mass extrapolation arguments which have given good agreement for the S-wave pion nucleon scattering lengths.[2][3]

Although extrapolating in the K-meson mass is a priori a more questionable procedure than the equivalent extrapolation in pion mass we may test the K-meson extrapolation procedure by comparing the experimental S-wave K^+p scattering length with current algebra prediction. The experimental value of $a_{K^+p}(\text{Exp})^{(4)} = \frac{-0.65 \pm 0.05}{m_K}$ agrees within 30% of the current algebra prediction of

$$a_{K^+p}(\text{Theo})^{(5)} = \left(\frac{-1}{2\pi}\right)\left(\frac{1}{f_K^2 m_K}\right) \frac{1}{1 + m_K/m_p} = (-1.03 \pm 0.06)\, m_K^{-1}$$

based on the Chiral $SU_3 \times SU_3$ algebra of charges proposed by Gell-Mann.[6] Furthermore, P. Roy[7] has shown that the K^+p effective range predicted from the pure current algebra result is also within 30% agreement of the experimental number.[4]

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Encouraged by these comparisons we give the cross-section for the process $K^+p \to K^+N^{*++}(1236)$ by extrapolating in the K-meson mass and again using the Chiral $SU_3 \times SU_3$ algebra of charges.[6] Whereas the matrix element for the S-wave K^+p scattering is given by[2]

$$\left(\frac{1}{m_K f_K}\right)^2 2K_{1\mu} < P\left|[V_\mu^3 + 3Y_\mu/2]\right|P >$$

the matrix element for the threshold $K^+p \to K^oN^{*++}$ is given by

$$M_{K^+p \to K^oN^{*++}} \equiv \left(\frac{1}{m_K f_K}\right)^2 K_{1\mu} < N^{*++}\left|V_\mu^+\right|P > \qquad (1)$$

where V_μ is the isovector current, Y_μ is the hypercharge current, f_K is the usual constant describing the decay $K \to \mu\nu$ and $K_{1\mu}$ is the incident K-meson four momentum.

The main difference between the elastic and inelastic case is that the matrix element $< N^*\left|V_\mu\right|P >$ vanishes in the limit of zero momentum transfer between P and N^* since they are in different isospin multiplets. In the physical inelastic process the momentum transfer between N^* and p is space-like and of order the K-mass while the derivation which connects the matrix element $< N^*\left|V_\mu\right|P >$ with the process $K^+p \to K^oN^{*++}$ requires this momentum transfer to be time-like and equal to the K-meson mass. The required extrapolation in the elastic case is from time-like momentum transfers of magnitude the K-meson mass to of order zero while in the inelastic case the extrapolation has to be continued further into the space-like region to of order the K-meson mass. A rough idea of how good this extrapolation might be can be estimated by looking at the Borm terms which yield a correction of order $m_K/(M_p + M_\Sigma)$ or about 25%.

Assuming that the extrapolation can be made, the $K^+p \to K^oN^*$ cross-

section can be estimated by determining the matrix element $< N^{*} | V_{\mu} | P >$ from the experimentally measured photoproduction cross-section.[8] This induces some uncertainty in the evaluation of the matrix element which arises from the amount the photoproduction cross-section which is alloted to background. We take as the matrix element

$$< N^{++} | V_{\mu}^{+} | P > = \sqrt{3} < N^{+} | V_{\mu}^{3} | P >$$
$$= \sqrt{3}\, 6 [\bar{u}_{\mu}(p^{*})\gamma_{5} u(p) - \frac{(p^{*} - p)_{\alpha}}{M+M^{*}} \bar{u}_{\alpha}(p^{*})\gamma_{\mu}\gamma_{5} u(p)] \tag{2}$$

where the factor $\sqrt{3}\, 6$ is approximate coming from the photoproduction data. The resultant cross-section is

$$\frac{d\sigma}{dt} = \left(\frac{9}{8\pi}\right)\left(\frac{1}{m_K f_K}\right)^4 \frac{1}{|\vec{K}_1|^2 S} \left[\frac{1}{(M+M^{*})^2} \left\{ S |\vec{K}_1|^2 |\vec{K}_2|^2 \sin^2\theta \left(\frac{3M^2 + M^{*2} - t}{M^{*2}}\right) + \frac{t}{4}(t - 4m_K^2)[(M-M^{*})^2 - t] \right\} \right] \tag{3}$$

where $\sqrt{S}$ is the C. O.M. energy, $|K_1||K_2|$ are the initial and find K-meson momentum in the C.O.M., θ is the C.O.M. scattering angle and t is the momentum transfer. The numerical value of the theoretical cross-section of 1.8 mb can be compared with the experimental value of (2.3 ± 0.3) mb at incident K momentum of 960 MeV/c. Equation (3) also predicts the N^{*} production angular distribution, the slope with respect to incident energy, and the N^{*} desnity matrix which are all in fairly good agreement with experiment.[1]

It is our opinion that the similarity between the expression for the $K^{+}p \rightarrow K^{o}N^{*}$ matrix element here and the vector meson exchange model with the ρ-γ analogy of Sakurai and Stoldolsky[9] is fortuitous. There appears

to be no reason to think that perturbation theory at low energies and relatively large momentum transfers should be applied to the K^+p reactions and that the vector meson nucleon vertex should be of a very special form. On the other hand knowing, after the fact, that the matrix element for $K^+p \to K^oN^*$ is proportional to the matrix element of the vector current one can introduce vector meson dominance and relate this matrix element to ρ meson couplings.[10]

In addition the cross-section for the background process $K^+p \to \pi^+p\,K^o$ may be calculated by evaluating the matrix element $< \pi^+p|V_\mu^+|P >$ which can be computed to first order in q and $K_1 - K_2$ by using PCAC for the pion and taking advantage of the conservation of the isovector current.[11] Neglecting interference between background and resonance we find for the non-resonant three body state the cross-section

$$\frac{d\sigma}{d\Omega_a dE_a d\Omega_b dE_b} = \left(\frac{1}{4\pi m_K f_K}\right)^4 \left(\frac{g^2_{\pi NN}}{16\pi}\right) \frac{1}{\sqrt{S}\,|\vec{K}_1|}$$

$$\times \left\{ \frac{(K\cdot q)(K\cdot p_1)}{(p_1p_2-M^2)(p_1\cdot q)} [p_1\cdot q + p_2\cdot(K_2-K_1)] - \frac{K^2}{2(p_1\cdot q)^2}(p_1\cdot q)(p_2\cdot q) \right\}$$

where the four-vectors K_1, p_1 refer to the incident K-meson and proton and K_2, p_2, q refer to the final K-meson, proton and pion respectively. The vector $K = K_1 + K_2$ while the subscripts a,b on the differentials refer to any two of the final three particles. In general the background matrix element will interfere with the resonant two body state for finite width of the $N^*(1236)$. Comparison with the data neglecting the interference indicates that the above expression is in approximate agreement. A detailed

study including the interference is presently under investigation.[12]

The observed rapid fall of the inelastic cross-section shortly after the threshold for $K^+p \rightarrow K^oN^{*++}$ cannot be explained by current algebras and low energy theorems. However, we conjecture that the rapid fall may be due to the onset of the p-wave unitarity limit which the theoretical expression reaches just at the peak of the experimental cross-section.

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