# MAGNETIC SLIT ${ }^{*}$ 

by

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ABSTRACT<br>The design of a magnetic slit used for momentum definition and beam separation for high energy particle beams is discussed.

(To be submitted to Review of Scientific Instruments)

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## I. INTRODUCTION

Momentum definition by means of magnetic slits for high intensity and high energy beams was considered first by Chu and Ballam. ${ }^{1,2}$ For this purpose, sharply defined magnetic fields as illustrated in Fig. 1 were required. The aim was to establish a field-free region of width $x= \pm a$ and height $y= \pm h$ in a gap of 2 g . For all values of $\mathrm{x}>\mathrm{a}$ the field should have an amplitude of $B_{y}=+B_{m}$ and for $x<-a$ an amplitude of $B_{y}=-B_{m}$.

If particles of a desired momentum range enter the center part of the slit at $x= \pm a$, where the magnetic field vanishes, the particles pass without being deflected. Particles of momentum $p_{o} \pm \delta p$ enter the slit at opposite sides of the field-free region and pass through the region of constant transverse field $B_{y}= \pm B_{m}$. The unwanted particles are separated further while passing through the slit. Also, if a beam of momentum $\mathrm{p} \pm \delta \mathrm{p}$ is deflected at a certain angle prior to entering the magnetic slit, in such a way that it will pass the region of constant $\mathrm{B}_{\mathrm{m}}$, it will be further bent and guided to a predetermined direction, or it will pass through the field-free region without being deflected. Thus, the beams can be guided to three experimental areas.

In the ideal slit, where $B_{y}$ changes discontinuously with $x$ at the median plane, $y=0$ and $x= \pm a$. Infinite current sheets located at $x= \pm a$ were proposed, with the current flowing in the z direction only. Practically, the sharp rise of field at $x= \pm a$ can only be achieved if the current-conducting planes are extended into the gap, and thus the useful slit region would not be free of obstructions for the passage of the beam. The current sheets would intercept part of the wanted and unwanted beams and subsequently would give rise to heating due to the impact of beam particles, secondary particles, and scattering.

In order to avoid this difficulty, the slit region should be free of any obstructing material, which means that the field inside the gap does not have the idealized shape of Fig. 1 but is an analytic function of the coordinates $x$ and $y$. Hence, the field will not vanish in any region inside the gap. However, the field can be made small enough in the regions $\mathrm{x}= \pm \mathrm{a}$, and thus does not lead to second-order effects on the beam due to field nonuniformities.

The transverse magnetic field in the momentum slit has the following symmetry conditions:

$$
\begin{align*}
& \mathrm{B}_{\mathrm{x}}(\mathrm{x}, \mathrm{y})=\left\{\begin{array}{l}
\text { even in } \mathrm{x} \text { direction } \\
\text { odd in } \mathrm{y} \text { direction }
\end{array}\right.  \tag{1}\\
& \mathrm{B}_{\mathrm{y}}(\mathrm{x}, \mathrm{y})=\left\{\begin{array}{l}
\text { odd in } \mathrm{x} \text { direction } \\
\text { even in } \mathrm{y} \text { direction }
\end{array}\right. \tag{2}
\end{align*}
$$

These two conditions require that the magnetic vector potential should obey the conditions:

$$
\mathrm{A}(\mathrm{x}, \mathrm{y})=\left\{\begin{array}{l}
\text { odd in } \mathrm{x} \text { direction }  \tag{3}\\
\text { odd in } \mathrm{y} \text { direction }
\end{array}\right.
$$

If one considers the field near the center of the slit, it is easier to use cylindrical coordinates ( $\mathrm{r}, \phi, \mathrm{z}$ ) and express the magnetic vector potential in terms of a polynomial:

$$
\begin{equation*}
A(r, \phi)=\sum_{n=1}^{\infty} C_{2 n} r^{2 n} \sin (2 n \phi) \tag{4}
\end{equation*}
$$

which, by differentiating $\mathrm{A}(\mathrm{r}, \phi)$ with respect to r and $\phi$, delivers the two field components:

$$
\begin{align*}
& \mathrm{B}_{\mathrm{x}}=-\mu_{\mathrm{o}} \sum_{\mathrm{n}=1}^{\infty} \mathrm{C}_{2 \mathrm{n}} \cdot 2 \mathrm{n} \mathrm{r}^{2 \mathrm{n}-1} \sin \left[\begin{array}{ll}
(2 \mathrm{n}-1) & \phi
\end{array}\right]  \tag{5}\\
& \mathrm{B}_{\mathrm{y}}=-\mu_{\mathrm{o}} \sum_{\mathrm{n}=1}^{\infty} \mathrm{C}_{2 \mathrm{n}} \cdot 2 \mathrm{n} \mathrm{r}^{2 \mathrm{n}-1} \cos \left[\begin{array}{lll}
(2 \mathrm{n}-1) & \phi
\end{array}\right] \tag{6}
\end{align*}
$$

where $C_{2 n}$ are constant coefficients.
The first term is a quadrupole with nonvanishing derivatives $\partial \overrightarrow{\mathrm{B}} / \partial \mathrm{r}$ independent of r . The other terms are higher-order multipoles. By superposing quadrupoles it is possible to eliminate the quadrupole terms and the resultant potential obtained would be an octupole. Again by superposition the octupole may be suppressed and a 16 -pole is obtained, and the octupole vanishes.

The basic slit design used ferromagnetic blocks with current-carrying conductors or sheets placed in slots, according to Fig. 2. The field calculation in the proximity of the iron faces confining the gap is complicated by the fact that not only the line currents, but also the iron magnetization affects the field in the slit aperture. Further, the magnetization varies with the currents. The $\mathrm{B}_{\mathrm{y}} \mathrm{com}-$ ponent of the field did correspond to prediction at the median plane, but deviated due to the saturation effect in iron in the proximity of iron.

The slit geometry of Fig. 2 needs, for high energy, slit current densities in the current-carrying sheets which are difficult to achieve without extensive water cooling. Thus the useful slit aperture and solid angle are reduced.

Therefore, the current sheets and the configuration according to Fig. 2 were dropped in favor of a magnetic slit, based on an original idea by Hedin, ${ }^{3}$ which was proposed for use with the $20-\mathrm{GeV} / \mathrm{c}$ electron beams at SLAC. The basic configuration of the slit, which makes use of auxiliary poles and shimming, is illustrated schematically in Fig. 3.

The slit is essentially a combination of two C-type bending magnets with additional shims separated by a vertical gap. By proper shaping of the auxiliary poles such that $2 \mathrm{~b} \ll 2 \mathrm{~g}$ and $2 \mathrm{a}_{1}>2 \mathrm{~g}$, it was possible to achieve at the region $\mathrm{x}= \pm \mathrm{a}, \mathrm{y}= \pm \mathrm{g}$ the field components $\mathrm{B}_{\mathrm{x}} \cong 0, \mathrm{~B}_{\mathrm{y}} \ll \mathrm{B}_{\mathrm{m}}$.

At $x \geqq \pm a$ the field $B_{y} \cong \pm B_{m}$. As shown in the next section, the field at $\mathrm{x}= \pm \mathrm{a}$ is a quadrupole, where all higher-order terms can be eliminated by proper choice of the parameters $b$ and $a_{1}>a$. The slope of the quadrupole field may also be varied within practical limits at $x= \pm a$ if $a_{1}>a \quad$ is selected; however, $a_{1} \gg a \quad$ would result in a large and impractical magnetic slit. The maximum value of $\pm B_{y}$ at $x= \pm a$ is essentially determined by the beam energy and the secondary beam aberration. For our practical case, $\mathrm{B}_{\mathrm{y}}= \pm 10^{-3} \mathrm{Vs} / \mathrm{m}^{2}$ was given as the maximum tolerable value.

## II. FIELD REPRESENTATION

With reference to Fig. 3, which illustrates the basic concept of the magnetic slit configuration, for $2 \mathrm{~b} \ll 2 \mathrm{~g}$ the potential function is given by 4 :

$$
\begin{align*}
W=U+j V=- & 4 \mathrm{I}\left\{\ln \left[\cosh \frac{\pi}{2 g}\left(x+a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)\right]\right. \\
& \left.+\ln \left[\cosh \frac{\pi}{2 g}\left(x-a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)\right]\right\} \tag{7}
\end{align*}
$$

By differentiating Eq. (7) with respect to $x$ and $y$ we get the two field components:

$$
\begin{align*}
& B_{y}=-\frac{d V}{d x}=+\frac{2 \pi I}{g}\left[\frac{\sinh \frac{\pi}{2 g}\left(x+a_{1}\right)}{\cosh \frac{\pi}{2 g}\left(x+a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)}\right. \\
&\left.+\frac{\sinh \frac{\pi}{2 g}\left(x-a_{1}\right)}{\cosh \frac{\pi}{2 g}\left(x-a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)}\right]  \tag{8}\\
& B_{x}=\frac{d V}{d y}=-\frac{2 \pi I}{g}\left[\frac{\sin \frac{\pi}{2 g}(y+g)+\sin \frac{\pi}{2 g}(y-g)}{\cosh \frac{\pi}{2 g}\left(x+a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)}\right. \\
&\left.+\frac{\sin \frac{\pi}{2 g}(y+g)+\sin \frac{\pi}{2 g}(y-g)}{\cosh \frac{\pi}{2 g}\left(x-a_{1}\right)-\cos \frac{\pi}{2 g}(y+g)-\cos \frac{\pi}{2 g}(y-g)}\right] \tag{9}
\end{align*}
$$

At the median plane $y=0$ the field components become:

$$
\begin{align*}
& \mathrm{B}_{\mathrm{y}}=\frac{2 \pi \mathrm{I}}{\mathrm{~g}}\left[\tanh \frac{\pi}{2 \mathrm{~g}}\left(\mathrm{x}+\mathrm{a}_{1}\right)+\tanh \frac{\pi}{2 \mathrm{~g}}\left(\mathrm{x}-\mathrm{a}_{1}\right)\right]  \tag{10}\\
& \mathrm{B}_{\mathrm{x}}=0 \tag{11}
\end{align*}
$$

The magnetic slit, neglecting saturation effects, is represented by a set of four parallel conductors buried in the surface of the iron, carrying the same current $I$ in the direction of positive $z$ and separated in the $x$ direction by:

$$
2 a_{1}=2 a+2 b+2 c
$$

and in the $y$ direction by the gap width 2 g . Equation (10) expressed in terms of
the maximum field in the right or left pole inside the gap is written in the form:

$$
\begin{align*}
& \mathrm{B}_{\mathrm{y}}=\frac{\mathrm{B}_{\mathrm{m}}}{2}\left[\tanh \frac{\pi}{2 \mathrm{~g}}\left(\mathrm{x}+\mathrm{a}_{1}\right)+\tanh \frac{\pi}{2 \mathrm{~g}}\left(\mathrm{x}-\mathrm{a}_{1}\right)\right]  \tag{12}\\
& \mathrm{B}_{\mathrm{x}}=0 \tag{13}
\end{align*}
$$

As can be seen from Eqs. (8) and (9), the symmetry conditions 1 and 2 are fulfilled for this design.

Equation (5) covers the full range of the $\mathrm{B}_{\mathrm{y}}$ field component at the median plane from the homogeneous region at the left pole to the field of opposite sign at the right pole. In the region $x \leq \pm a$, the $B_{y}$ component is linear and zero at $\mathrm{x}=\mathrm{y}=0$.

## III. 2.5-METER MAGNETIC SLIT

As a practical application of the magnetic slit, it was proposed that an electron beam of momentum $p_{0}=20 \mathrm{GeV} / \mathrm{c}$ with a dispersion of $\delta \mathrm{p} / \mathrm{p}=3 \%$ and the input data

$$
\begin{aligned}
& \delta \mathrm{x}=\delta \mathrm{y}=0.3 \mathrm{~cm} \\
& \delta \phi=\delta \Theta=10^{-4} \text { radians } \\
& \delta \mathrm{z}=0.3 \mathrm{~cm}
\end{aligned}
$$

be bent $\pm 0.2^{\circ}$ by a pulsed bending magnet with a repetition rate of 360 pps and be deflected further $\pm 1.8^{\circ}$ when passing through the magnetic slit. The beam transport layout is shown schematically in Fig. 4. The second moments of the distribution were calculated to be:
Quadrupole Pulsed Magnet Magnetic Slit Target

Pulsed Magnet Off: $\delta x=1.5 \mathrm{~cm}$
1.5 cm
0.6 cm 0.4 cm
$\delta \mathrm{y}=0.9 \mathrm{~cm}$
1.0 cm
0.5 cm
0.2 cm

Pulsed Magnet On: $\quad \delta x=1.5 \mathrm{~cm}$
1.5 cm
0.7 cm
1.5 cm
$\delta \mathrm{y}=0.9 \mathrm{~cm}$
1.0 cm
0.5 cm
0.2 cm

If we draw these second moment envelopes and allow ample space for vacuum chamber, etc. within the gaps, we arrive at a gap height of $2 \mathrm{~g}=5 \mathrm{~cm}$ and a distance of $\ell=10 \mathrm{~cm}$ between the center of the slit and the homogeneous field region between the slit poles. This distance defines the center of the undeflected and deflected beams and dictates the length $\mathrm{a}_{1}<\ell$ at the slit entrance. The maximum field in the gaps is calculated from the relation:

$$
\int \mathrm{Bd} \ell=\frac{\mathrm{p} \Theta}{3} \cdot 10^{-1}=2.095 \mathrm{~Wb} \cdot \mathrm{~m}^{-1}
$$

with p the particle momentum in $\mathrm{GeV} / \mathrm{c}$ and $\Theta$ the deflection angle in radians. The maximum field was chosen to be 8.5 kG and the effective slit length, 2.5 m . With the design parameters of Fig. 5, the calculated field component $B_{y}$ is illustrated in Fig. 6.

In order to reduce the field gradient inside the region $\mathrm{x}= \pm \mathrm{a}$, the two slit halves are opened to $\pm 1^{\circ}$, which is permissible due to the curvature of the beam inside the left and right gaps, and thus values of $6.2 \mathrm{~cm} \leqq \mathrm{a}_{1} \leqq 10.44 \mathrm{~cm}$ could be achieved. The field calculations have been performed for three $a_{1}$ values according to the entrance, middle, and exit of the magnetic slit.

Although the shims are saturated, the slit effect has been preserved. Over the region $\mathrm{x}= \pm \mathrm{a}= \pm 1.0 \mathrm{~cm}$, the maximum field value calculated for $\mathrm{a}_{1}=8.32 \mathrm{~cm}$ is

$$
\left.{ }^{B}\right|_{x= \pm 1.0 \mathrm{~cm}}= \pm 3.1 \text { gauss }
$$

which leads to $\int \mathrm{Bd} \ell=7.75 \mathrm{G} \cdot \mathrm{m}$ and results in a second-order spread of $2 \times 10^{-10}$ radians, which is negligible.

Special attention is required for the vertical and horizontal alignment of the auxiliary gaps 2 b with respect to each other and for the location of the shims,
which had been within $\Delta x \leqq \pm 0.01 \mathrm{~cm}, \Delta \mathrm{y}=+0.05 \mathrm{~cm}$; the corresponding pole alignment is within 0.0025 cm , location of the slits $\Delta \mathrm{b} / \mathrm{b}=+0.005 \mathrm{~cm}$.

## IV. MEASUREMENTS

To obtain the magnetic characteristics of the slit, two types of measurements were performed:

1. Point measurements where the field components $B_{x}(x, y, z)$ and $B_{y}(x, y, z)$ were measured at the entrance, exit and the central plane of the magnetic slit (Figs. 7 and 8).
2. Long coil measurements to obtain $\int_{-\infty}^{+\infty} \mathrm{B}_{\mathrm{y}} \mathrm{dz}$ (Fig. 9). If the origin of the Cartesian coordinate system is assumed to be at the center of the slit, the measurements performed were in the planes $y=0$ and $y= \pm 1.25 \mathrm{~cm}$ and $z= \pm 100 \mathrm{~cm}$ for various $x$ values. As seen from the figures, the $B_{x}$ component is zero at the plane $y=0$ for all values of $x$, but $B_{y}$ has a quadrupole shape over $x= \pm a_{1}$. Measurements were performed at various currents, up to $I= \pm 800 \mathrm{amps}$, which produced a field of $B_{m}=9.5 \mathrm{kG}$. The gradient $\partial B_{y} / \partial \mathrm{x}=3 \mathrm{G} / \mathrm{cm}$ at the highest measured excitation current of 800 amps . Figure 10 shows the measured and calculated profiles of $B_{y} / B_{m}$ versus $x$ for values of $\mathrm{B}_{\mathrm{m}}=5 \mathrm{kG}$ and $\mathrm{B}_{\mathrm{m}}=9.5 \mathrm{kG}$.

Figure 11 shows the field homogeneity $\mathrm{B}_{\mathrm{y}} / \mathrm{B}_{\mathrm{m}}$ between the poles for two excitation currents. Figure 12 is a photograph of the magnetic slit taken prior to its installation in the SLAC B-Beam area.

The slit effect with a field-free region is destroyed if the magnet stand or other structural materials close to the slit are ferromagnetic. Part of the fringing field is short-circuited over these materials, and thus the median plane and the symmetry conditions [Eqs. (1) and (2)] are destroyed.

## REFERENCES

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Machines (Dover Publication, 1962); p. 172 ff .

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| :---: | :---: | :---: |
| 2. $\mathrm{z}=-100 \mathrm{~cm}$ | $\mathrm{y}=-1.25 \mathrm{~cm}$ |  |
| 3. $z^{-}+100 \mathrm{~cm}$ | $y=+1.25 \mathrm{~cm}$ | Exit |
| 4. $\mathrm{z}=+100 \mathrm{~cm}$ | $\mathrm{y}=-1.25 \mathrm{~cm}$ |  |
| 5. $\mathrm{z}= \pm 100 \mathrm{~cm}$ | $\mathrm{y}=0$ |  |

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9. $\mathrm{z}=+100 \mathrm{~cm} \quad \mathrm{y}=+1.25 \mathrm{~cm})$
$\begin{array}{ll}\text { 2. } \mathrm{z}=+100 \mathrm{~cm} \\ 3 . & \mathrm{z}=+100 \mathrm{~cm}\end{array} \quad \begin{aligned} & \mathrm{y}=-1.25 \mathrm{~cm} \\ & \text { 3 }=0\end{aligned} \quad$ Exit
$y=0$
10. $\mathrm{z}=-100 \mathrm{~cm} \quad \mathrm{y}=+1.25 \mathrm{~cm}$
11. $\mathrm{z}=-100 \mathrm{~cm} \quad \mathrm{y}=-1.25 \mathrm{~cm}$

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1. Field free region
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4. Return yoke
5. Auxiliary yokes



Fig. 2



Fig. 4


Fig. 5


Fig. 6


Fig. 7



Fig. 9


Fig. 10


Fig. 11


Fig. 12


[^0]:    *Work supported by U. S. Atomic Energy Commission

