INFORMATION ABOUT THE TWO AND THREE NUCLEON SYSTEMS OBTAINED AND OBTAINABIE FROM THE USE OF FOIARIZED TARGETS AND BEAMS ${ }^{+*}$

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ABSTRACT
Review of recent experiments and theory makes it clear that the spin structure of the three-nucleon system is too complicated to allow a direct determination of the scattering matrix in the near future. In order to provide more readily applicable tools for experimental analysis, the rigorous three-body theory provided by the Faddeev equations should be articulated to provide a low energy n-d theory comparable to effective range theory, a unitary parameterization for the coupled elastic scattering and breakup problems comparable to phase shift analysis, and a theory of high angular momentum states based on the deuteron wave function analagous to one-pion-exchange for the $\mathbb{N}-\mathbb{N}$ system. As physical input we also require both better nucleon-nucleon scattering measurements, particularly in the $n-p$ system, and a method for extending these results to the energy-momentum regions allowed by the uncertainty principle in the threenucleon system. Whether we attempt to derive this extension from electromagnetic measurements of the two-nucleon wave functions, or directly from two-nucleon scattering, we find that this extension requires understanding of the coupling of the nucleon-nucleon system to other elementary particles.

[^0]Thanks to the high precision of recent nucleon-nucleon scattering measurements provided by polarization techniques, we find that all the characteristic features of the two-nucleon interaction can in fact be interpreted as arising from the exchange of known bosons and boson resonances, and hence that we can hope to make this extension in a physically reasonable way. We conclude that the three-nucleon problem should be approached both phenomenologically and from the point of view of elementary particle physics, and that the coupling of these two approaches offers great promise, if kept in close contact with experiment, and if full use is made of the new polarized target and polarized beam techniques.

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## I. INTRODUCTION

It has been customary for a number of years in nuclear physics conferences to discuss the three-nucleon system together with 4,5,...n ( $n$ small) nucleon systems, and to consider the two-nucleon system, if at all, either as an isolated topic, or in conjunction with elementary particle physics. I believe the time has come to change this pattern. In fact, one of the main things I hope to convince you of in this talk is that the three-nucleon system will have to be treated as a problem in elementary particle physics, if we are to reach any fundamental understanding of its characteristic features, and that the theoretical and experimental techniques for such a treatment have reached a promising stage of development. A second reason for treating these two topics together is that the work on the two-nucleon system has reached a certain stage of completion (although there is much important work still to be done) and it may well prove profitable to aim future experiments at those features of the two-nucleon system which we will need to know better for three-nucleon calculations; in a very real sense, I believe the frontier of the two-nucleon problem has become the three-nucleon system. A third point $I$ wish to emphasize is that there is more than a

[^1]superficial similarity between the current state of development of work on the three-nucleon system and the problems which faced us ten years ago in trying to understand nucleon-nucleon scattering. We should, therefore, be able to profit, if we are wise, from the lessons learned in that arduous and difficult development. In particular, I have learned to my cost that every complexity allowed by the conservation laws is in fact present in the two-nucleon system, and that many mistakes were made by ignoring this possibility. The three-nucleon system is bound to be at least as complicated, and we should be correspondingly cautious about introducing simplifying assumptions into the analysis before we have a firm grasp on the dynamics. This line of thought leads me to discuss the work on the three-nucleon system first; the recent experimental work on the two-nculeon system can then be described in what $I$ believe to be the appropriate context.

At first sight, the problem of experimentally determining the threenucleon scattering matrix for $n-d$ scattering looks formidable. If we write the independent terms in this matrix as coefficients of rotationally invariant tensors, we find 12 invariant amplitudes for elastic scattering and 24 for inelastic scattering. A priori we therefore expect that we need 72 different experiments at each energy and angle to determine these 36 complex numbers; this is an underestimate, since the observables are bilinear combinations of the amplitudes, and further experiments are required to resolve sign ambiguities. If full angular distributions are measured, or enough information obtained for a phase shift analysis, unitarity should bring the total number of experiments needed somewhat below 72 , which is small comfort. Since there are 648 non-zero elastic scattering observables, and 2304 inelastic ones, there are plenty of
experiments to choose from; it is also clear that care should be exercised in the choice so that (except for purposes of increasing precision) experiments are not picked which simply give the same bilinear combinations of amplitudes already determined by earlier experiments.

With this large number of possibilities available, it is not surprizing that so far only a few types of observables have been measured. These are: differential and total cross sections (both elastic and inelastic), polarization of either protons or neutrons, vector and some of the tensor polarization coefficients of the deuteron, and one measurement of $D$ and $R^{\prime}$. It is clear from what has just been said that progress will be much more rapid once a comprehensive theory of the process exists, and we have some idea of which amplitudes are the most dynamical significance. Unfortunately this theory is only just beginning to be worked out.

## II. THREE-NUCLEON POLARIZATION EXPFRIMENTS

Since this subject was reviewed by Barschall at Karlsruhe, I have attempted only to survey the literature published this year very cursorily, and have contacted groups at Berkeley, Dubna, Ios Alamos, Rutherford, Saclay, and Wisconsin about recent work. It is very likely that I have overlooked significant papers and preprints, and wish to state explicitly that any omissions are unintentional, and to apologize for them. It should also be noted that, because of the specialized nature of this conference, I have made no attempt to survey differential cross section, total cross section, or breakup data.

Al illustration of how far we have to go in exploring the threenucleon system is the fact that prior to this year we had no experimental
way of choosing between the two alternative choices for the doublet and quartet $n-d$ scattering lengths allowed by the total cross section measurements. The corresponding ambiguity between singlet and triplet scattering lengths in the $n-p$ system is easy to resolve theoretically, since we know from effective range theory that the smaller of the two scattering lengths must go with the system that has the bound state, and as the deuteron has a quadrupole moment, we know that this must be the triplet state; of course the ambiguity was also resolved experimentally long ago. Since we still lack a rigorous low-energy theory for the $n-\mathrm{d}$ system, the theoretical situation here is less clear. Many theoretical arguments have been advanced favoring the set in which the doublet scattering length, $a_{2}$, is smaller than the quartet scattering length, $a_{4}$, but it was still possible last year to argue for the alternative choice ${ }^{(1)}$. However, by measuring the transmission of polarized neutrons through a polarized deuteron target, Alfimenkov, Luschikov, Nikolenko, Taran, and Shapiro (2). have shown conclusively that $a_{2}$ is less than $a_{4}$, as will be discussed in the paper to be presented this afternoon by Luschikov.

Granted this choice, the values of $a_{2}$ and $a_{4}$ which have been accepted for a number of years are ${ }^{(3)} a_{2}=0.7 \pm 0.3 \mathrm{~F}, a_{4}=6.38 \pm 0.06 \mathrm{~F}$. However, in a paper to be presented at the Stanford Meeting of the American Physical society at the end of this month, seagrave (4) provides a re-evaluation of recent measurements of the coherent and incoherent scattering of "cold" neutrons, including corrections due to residual binding effects of as much as $7 \%$, and reaches the preliminary conclusion that $\mathrm{a}_{2}=0.1 \pm 0.2 \mathrm{~F}$, $a_{4}=6.2 \pm 0.1 \mathrm{~F}$, assuming a free cross section of $3.2 \pm 0.1 \mathrm{~b}$, and the value of the incoherent cross section obtained by $W$. Gissler ${ }^{(5)}$ of $2.25 \pm 0.04 \mathrm{~b}$. If one accepts the unpublished value of the coherent
scattering length of $6.17 \pm 0.06 \mathrm{~F}$ obtained by R. E. Donaldson ${ }^{(6)}$, one finds $a_{2}=0.11 \pm 0.07 \mathrm{~F}, a_{4}=6.14 \pm 0.06 \mathrm{~F}$ and predicts $\sigma_{\text {free }}=$ $3.14 \pm 0.06 \mathrm{~b}$, which is consistent with this analysis. It is perhaps significant that, according to A. C. Phillips ${ }^{(7)}$, it is easier to achieve consistency between these nev values and the binding energy of the triton in the separable approximation to the three-body problem to be discussed below, than to fit the older values together with $\epsilon_{H} 3$.

I would like to emphasize that what is needed here for efficient analysis of the experiments is a model-independent theory of low energy n-d scattering, analagous to effective-range theory for the two-nucleon system, taking full account both of the spin-structure of the threenucleon system, and of the low-lying inelastic threshold. Working out the complications due to coulomb interactions will be still more difficult but again is needed if we are to exploit the high experimental precision available in low energy p-d measurements.

Early work on p-d, polarization at low energy has been summarized by Chalmers, Cox, seth, and Strait ${ }^{(8)}$ in comparison with their results at $1.50,2.02,2.52$ and 4.1 MeV , and more recent results by Grübler, Haeberli, and Extermann presented at Karlsruhe have since been published ${ }^{(9)}$. Still more accurate results from Wisconsin at $4.0,6.0,8.0$, and 10.0 MeV covering the angular range from $30^{\circ}$ to $150^{\circ}$ were presented by H. B. Clegg at the Washington meeting of the American Physical Society this spring ${ }^{(10)}$, and unpublished results at 12.0 MeV from the same author were submitted to me for this conference. Clegg's results are everywhere positive in this angular range, and show a maximum at slightly below $120^{\circ}$ at 4 MeV which moves to slightly above $120^{\circ}$ as the energy increases. The value of the polarization at the peak is $0.072 \pm 0.012$ at 4.0 MeV , rising to
$0.140 \pm 0.010$ at 10.0 MeV ; the 12.0 MeV data continue this trend. There is a partial overlap between these experiments and a measurement at 10.5 MeV over the range $30-100^{\circ}$ made by McKee, Clark, Slobodrian and Tivol (11), and the same authors have also made available to me unpublished results at $12.5,16.5$ and 19.5 MeV . By 16.5 MeV the peak has moved out to about $145^{\circ}$ and risen to a value of about 0.18 , and at 19.5 MeV a minimum around $100^{\circ}$ is also clearly shown. In the region of overlap, there is generally speaking reasonable agreement with Clegg's data, except that at 10.5 MeV , the Berkeley group find two negative values for the polarization below $40^{\circ}$, and that the $30^{\circ}$ point at 12.5 MeV is also slightly negative.

The general trend of the polarization data is illustrated in Figure 1 , which gives my own free-hand curves through Clegg's data at 12 MeV , the data of Conzett, Igo and Knox ${ }^{(12)}$ at 22 MeV , of Conzett, Goldberg, Shield, Slobodrian, and Yamabe ${ }^{(13)}$ at 40 MeV , of Hall, Johnston, and Griffiths (14) at 30 MeV , and of Johnston, Gibson, Megaw, Griffiths, and Eisberg ${ }^{(15)}$ and Johnston, Gibson, McClatchie, Megaw and Griffiths (16) at 50 MeV . By making use of a double focusing magnetic spectrometer, Gibson, Johnston, McClatchie, Megaw, and Griffiths ${ }^{(17)}$ have recently extended the angular range of the 30 and 50 MeV measurements to both smaller and larger angles, and this nèw, unpublished data has also been drawn on in constructing my free hand curves.

It is clear that the polarization exhibits much interesting structure: the backward peak which develops at very low energy and gradually moves outward with angle as the energy increases, but which seems to saturate at about +0.2 , the very deep negative minimum which appears already below 20 MeV as noted above, and goes to large negative values at higher energy, and the small forward positive peak. The impulse approximation
gives no indication of this structure, even when off-shell scattering corrections are included ${ }^{(18)}$. Huifner and de-Shalit ${ }^{(19)}$ have shown that this type of polarization structure can be correlated with the differential cross section in a diffraction model, and obtain a qualitative fit to both at 40 MeV using a single parameter, but this hardly provides a dynamical explanation. It appears that the region around 50 MeV will provide an interesting challenge to detailed three-nucleon theories.

Since low energy $n-d$ polarization results were reviewed by Barschall at Karisruhe ${ }^{(20)}$, I will mention only the 22.7 MeV measurement of Malanify, Simmons, Perkins, and Walter ${ }^{(21)}$ which has now been published. You will note in Figure $I$ that $I$ have indicated that the large-angle p-d polarizations measured at 22 and 40 MeV apparently go to much higher values than the value of 0.2 , which now appears to be accurately measured at 30 and 50 MeV . If we compare the 22 MeV experiment with the $\mathrm{n}-\mathrm{p}$ results, in Figure 2, we see that the $n-d$ experiment favors the smaller value found at 30 and 50 MeV . Since the proton experiment was at the end of its angular range, and the statistical errors, particularly at 40 MeV are large, while the neutron experiment is most accurate in this angular range ${ }^{(22)}$, it would appear that in this instance the neutron work is more reliable. Of course, at these large angles we expect charge-independence to be directly applicable, and hence that $n-d$ and $p-d$ experiments should give the nearly same result, but until we have a rigorous three-body theory including coulomb interactions, we cannot be absolutely sure that this is true.

The only triple scattering experiments of which I am aware are measurements of $D$ and $R^{\prime}$ at 135 MeV by Poulet, Michalowicz, Kuroda,

Cronenberger, and Coignet $(23)$. These show very qualitative agreement with an impulse approximation calculation using known nucleon-nucleon phase-shifts, but detailed theoretical analysis is again lacking.

The existence of tensor polarization components $\mathrm{T}_{20}, \mathrm{~T}_{21}$, and $\mathrm{T}_{22}$ for elastic p-d scattering in the energy range of $3-10 \mathrm{MeV}$ for the deuteron has been demonstrated by Young and Ivanovich ${ }^{(24)}$ and Young, Ivanovich, and Olsen (25). In a recent preprint from Wisconsin, P. Exter$\operatorname{mann}(26)$ gives the energy variation of the vector polarization of the deuteron $\mathrm{T}_{11}$ at angles between $107^{\circ}$ and $120^{\circ}$, and finds this roughly linear between 4 and 12 MeV . He also gives angular distributions of the vector polarization and of the combination $T_{22}+0.41 T_{20}$ at 8 and 11 MeV. Comparable preliminary results by Arvieux, Beurty, Goddergues, Lechazinski, Mayer, Mikumo, Papeneau, and Thirion were presented at Karksruhe (27) last year, and are new being prepared for publication (28). Since the experimental techniques differ considerable, and even the definitions used for the tensor components are not identical, it is suggested that the authors be contacted for details.

I have gained the impression from this cursory survey of the experimental material already available that rapid progress is being made in achieving results of reasonable precision, resolving experimental discrepancies, determining accurate parameters for a low energy theory, and that on the experimental level the possibjlity of much more sophisticated spin-dependent experiments already exists. But is also will become clear in a moment that the number of experimental possibilities is so rich that it is hopeless for the time being to think of a direct empirical determination of the complete scattering matrix for many years to come, and that theoretical guidance will be needed to select crucial experiments.

I will now attempt to indicate what has been done theoretically, and what I believe is still needed in order for progress to be made.

## III. THEORETICAL STRUCTURE OF THE THREP-NUCLEON SYSTEM

Since we start experimantally with an $n-d$ or $p-d$ system, and the density matrix for the spin-1/2 particle requires 4 numbers for complete specification, while the density matrix for spin - 1 requires 9 numbers, there are 36 possible independent initial states. For elastic scattering there are 36 independent final states, and hence $36 \times 36=1296$ possible independent elastic scattering experiments. Since we can form no pseudoscalar from the initial and final momenta, and the scattering matrix is linear in the (pseudoscalar) spins, parity conservation requires half of the rotational invariants we can form to vanish identically leaving only 648 non-zero elastic scattering experiments, which is not much help. If the interaction leaves three free nucleons in the final state, the final density matrix has $4 \times 4 \times 4=64$ independent elements, so there are $36 \times 64=\dot{2} 304$ independent breakup experiments. If the three final momenta are coplanar (in the c.m. system), we can again form no pseudoscalar, and roughly half of these amplitudes will vanish in that plane, but not out of it. Since the determination that the momenta are non-coplanar necessarily requires coincidence measurements, we can conclude immediately that the angular distribution of coincidence measurements and not just of single-particle energy spectra will be essential in exploring the full structure of the three-nucleon system. It is by no means premature to start thinking about the experimental techniques needed for such measurements.

If we now add the requirement of total angular momentum conservation
the number of independent amplitudes is drastically reduced. This is illustrated in Table 1. For elastic scattering $(1 / 2+1 \rightarrow 1 / 2+1)$, we see that there are 18 possible transitions, but that time-reversal invariance reduces these to 12. For breakup, all 18 transitions are allowed, and in addition we have 6 possible transitions of the type $1 / 2+1 \rightarrow$ $1 / 2+0$, making 24 inelastic amplitudes. At this stage we, therefore, have $12+24=36$ complex functions of energy and angle to be determined, less one overall phase, or 71 numbers in all. As we know from familiar analyses of elastic scattering with open inelastic channels, the 12 elastic amplitudes can be parameterized in terms of 12 real phase parameters, and 12 absorption parameters lying between $O$ and $I$. In principle, these 12 absorption parameters can be determined in terms of the 24 inelastic amplitudes by means of unitarity, giving a modest reduction to 60 phase parameters for each value of J. However, so far as I know, no one has yet worked out the unique parameterization which automatically guarantees unitarity in this case, comparable to the phase shift analysis in twoparticle réactions. Since no real progress in unscrambling the two-nucleon dynamics was made prior to the development of that formalism, it is clear that this task should have high priority for theorists interested in the three-nucleon problem.

The general non-dynamical structure for the cases $1 / 2+1 \rightarrow 1 / 2+1$, and for $1 / 2+1 \rightarrow 1 / 2+0$ have been published in a paper by Csonka, Moravcsik, and Scadron $(29)$, together with the machinery needed to reduce this to a phase-shift parameterization for two-particle final states, but the generalization to three-particle breakup is not transparent. Comparable formulae have been sent me by $D$. Fick (30), together with the evaluation of many possible observables in terms of the invariant amplitudes.

The next step along this line is to work out which classes of observables lead to the same combinations of invariant amplitudes; the usefulness of such an analysis is illustrated for the simpler case of $1 / 2+1 \rightarrow 1 / 2+0$ in a recent paper by Csonka, Moravcsik and Scadron ${ }^{(3 I)}$ and applied to the reaction $\mathrm{He}^{3}(d, p) \mathrm{He}^{4}$. I have decided not to reproduce the formulae for $T$ in terms of invariant amplitudes here, since they do not yet include the parameterization mentioned in the last paragraph, and since they also do not give the composition of these invariant amplitudes in terms of the Faddeev subchannels, the necessity for which I will discuss below.

Of more direct relevance to this Colloquium is a communication from Raynal and Arvieux which is reproduced as the Appendix to this talk (q.v.). They discuss all possible polarized-target polarized-beam elastic scattering experiments starting with $n-d$ (or $p-d$ ), and show these can give 18 independent numbers. If there are no doublet-quartet transitions, there are 8 relations among these numbers, and a specific test of one of these relations is proposed: I would like to inject a work of caution at this point about making the assumption that only 10 experiments are needed even if this first test succeeds. We know both that the deuteron is a very loose structure, and that there are strong exchange and spin-flip forces between two nucleons. Consequently, I would personally be quite surprized if, even at quite low energy, we do not find double-quartet transitions in the n-d system, and would accept the simplified analysis only in energy regions where all 8 restrictions have been shown to hold experimentally to reasonable precision. This is just the type of simplifying assumption which got us into trouble in the early days of analysing the two-nucleon system.

In designing spin-dependent experiments it is important to keep in
mind a point that Raynal has made before, but which cannot be repeated too often. This is that if one starts from a polarized-target polarizedbeam system, important information is contained in the azimuthal variation other than that given by a left-right measurement in one plane, and an appropriate counter arrangement can greatly increase the information obtainable with no increase in running time. A good illustration is the Saclay measurements of $A_{x x}$ and $A_{y y}$, where by having counters in the planes both perpendicular and parallel to the plane containing the beam and target polarizations, both the accuracy and the usefulness of the experiment was greatly increased. When one adds the possibility of tensor polarization components, the azimuthal variation is still more complicated, and it is very important to make sure all appropriate azimuthal ranges are covered.

As we saw above, the non-dynamical structure of the three-nucleon system is quite complicated, and it will be a long time before we can hope to see a direct experimental determination of the transition matrix at any energy; it is therefore clear that it will be essential to make use of a dynamical theory of this system before we can hope for much progress. Let me remind you that in the much simpler case of $p-p$ scattering, where only five invariant amplitudes need be determined, and the phase shift parameterization was completely understood, the direct empirical approach failed. It was only after the dynamical assumption that the highest partial waves could be computed from one-pion-exchange (ope) $(32,33)$ was included in the data analysis that unique phase-shift analyses became possible, and the detailed transition matrix I will discuss below emerged from the experiments.

Thanks to the work of Fadueev (34), a well-defined mathematical theory
of the non-relativistic three-body quantum mechanical problem now exists, and I am convinced it is only a matter of time, and a lot of hard work, before it can be made into a practical tool for the analysis of threenucleon experiments. The basic difficulty with earlier approaches to the three-body problem was that, starting from the Schroedinger equation, no one knew how to formulate the scattering boundary conditions in a way that allowed solutions to be defined, while starting from the Lippmann-Schwinger equation $T=V+V G_{o} T$, again ambiguous and infinite terms were obtained. The physical origin of these difficulties is that even though one of the particles always separates from the other two if we wait long enough, the other two can continue to interact, either as a bound state or as a correlated continuum state of the two-particle subsystem, long after the threeparticle interaction has ceased, and that this can happen in three different ways which must be correctly connected in order to preserve unitarity. The Faddeev approach is to split the three-body transition matrix into these three subchannels, in. which the $i^{\text {th }}$ particle is asymptotically free, that is $T=T^{1}+T^{2}+T^{3}$. It is then shown that the se three amplitudes satisfy the coupled equations

$$
T^{i}(Z)=t^{i} \delta+\sum_{s=j, k} t^{i} \delta G_{0}(Z) T^{s}(Z)(i, j, k \text { cyclic })
$$

Here $t^{i}$ is the fully off-shell lwo-body transition matrix for the $j k$ pair expressed as an operator in the three-body Hilbert space, the $\delta$ function insures that the $i^{\text {th }}$ particle retains the energy not available to this $j k$ interacting pair, and $G_{0}(Z)$ is the Green's function for the three-body system having extended energy $Z$.

If we now represent these operator equations as integral equations, the operators $\mathrm{T}^{i}$ become functions of (for example) nine variables
representing the nine momentum components of the three particles in the final state, nine parameters determined by the momentum components in the initial state, and the extended energy $Z$. The physical transition matrix is then obtained by solving these equations and taking the limit $Z \rightarrow E+i O, E$ being the total energy of the three-body system; that is $E=\omega_{1}+\omega_{2}+\omega_{3}=\omega_{1}^{\prime}+\omega_{2}^{\prime}+\omega_{3}^{\prime}$ where $\omega_{i}\left(\omega_{i}{ }^{\prime}\right)$ is the energy of the $i^{\text {th }}$ particle in the initial (final) state. Faddeev has shown that if these equations are iterated a sufficient number of times, the apparent singularities from the $\delta$ functions and the Green's function disappear leaving Fredholm equations posing a well-defined mathematical problem, although not exactly an easy one to solve. By taking out the total momentum, and taking as variables the three final energies $\omega_{i}^{\prime}$ ', together with a magnetic quantum number representing the projection of the total angular momentum on an axis fixed in the plane of the triangle determined by the three c.m. momenta (i.e. a body-fixed axis), Omnes (35) has reduced this system to coupled integral equations in three continuous variables with $3 \times(2 J+1)$ components:

If one makes the further assumption, which is appropriate to the threenucleon problem if one ignores the coulomb complication in p-d scattering (which may prove troublesome at a later stage), that the interactions are of short range, and hence that only interactions for orbital angular momenta less than I in the two-body subsystems need be included, Ahmezadeh and Tjon (36), and independently Osborn and $I(37)$, have shown that this system can be still further reduced to coupled integral equations in only two continuous variables for functions with $3 \times(I+1) \times \min (2 J+1,2 L+1)$ components. Since these two variables are the total three-body energy $E$ defined above, and (in each Faddeev subchannel) the energy $\dot{\omega}_{i}^{\prime \prime}$ of the
particle which is asymptotically free, bound-state and resonance singularities of the three-body system occur only in $E$, and are cleanly separated from the bound-state and resonance singularities of the two-body subsystems which are reflected in the variables $\omega_{i}{ }^{\prime}$. This suggests that a powerful phenomenology for three-particle final states capable of testing the assumption that only pairwise interactions of the two-body subsystems are important without detailed dynamical assumptions will be possible, but this has yet to be worked out. Specific numerical solutions of these two-variable equations have yet to be obtained, but Osborn has by now pushed the development of practical computer techniques for this purpose to the point where I am confident that these will soom become available.

The remaining question to ask is whether we know enough about the two-nucleon interactions which provide the driving terms in the Faddeev equations to believe the solutions (a) well enough to call agreement with experiment significant, or (b) if there is disagreement with experiment, to believe that this is evidence for actual three-body forces in the threenucleon system. The quantities we need for the Faddeev equations are now the two-nucleon partial-wave amplitudes $t_{l}(q, p ; z)$ (with $0 \leq l \leq L$ ) for the scattering from a state of relative c.m. momentum $p$ to a state of relative c.m. momentum $q$ at an extended energy $z$. If we examine the kinematics of the three-particle system, we find that the values needed are for $p^{2} / 2 \mu_{i}=$ $E-r_{i} \omega_{i}, q^{2} / 2 \mu_{i}=E^{\prime}-r_{i} \omega_{i}^{\prime}, z=Z-\omega_{i}^{\prime} ;$ here $\mu_{i}=m_{j} m_{k} /\left(m_{j}+m_{k}\right)$, and $r_{i}=\left(m_{j}+m_{k}\right) / m_{i}$. The ranges of $E$ and $\omega_{i}$ are such that the values of $p^{2}$ and $q^{2}$ so defined are always positive or zero; this is fortunate since $p$ and $q$ are radial variables in the Schroedinger equation for the two-particle system, and we would have difficulty in giving a physical interpretation to
them outside this range. However, since in the Faddeev equations, $\omega_{i}$ ranges up to $+\infty$, we find that we must be able to interpret $t_{l}$ when the energy, value in the Schroedinger equation ranges from $-\infty$ to $Z$. A furthur difficulty is that, from two-nucleon scattering experiments we can only directly determine $t_{l}\left(k, k ; k^{2} / \mu_{i}\right) \equiv \tau_{l}(k) \equiv e^{i \delta_{l}} \sin \delta_{l} / k$, while the uncertainty principle allows this connection between free-particle momentum and energy to be broken independently in all three variables in the three-body dynamical equations.

The solution to this problem can be made in two steps. As has been shown elsewhere ${ }^{(38)}$, it is possible to factor the half-off-shell transition matrix $t_{l}\left(p, k ; k^{2} / 2 \mu_{i}\right)=\tau_{l}(k) f_{k}(p)$ with $f_{k}(k)=1$; as noted above $\tau_{\ell}(k)$ is directly determinable from two-nucleon scattering experiments. Further, it was shown that the function $f_{k}(p)$ is simply the representation in momentum space of the difference between the exact wave function in configuration space and the usual asymptotic form $n_{\ell}(k r)-c t n \delta_{\ell} j \ell(k r)$. It is, therefore, smooth and finite, the structure in energy as one moves off the energy shell occurring only over regions of order $k R$, where $R$ is some average range of forces; rapid variations in energy are confined to the bound states and resonances as reflected in the experimentally knowable ${ }^{\tau} l(k)$. If we know the off-shell born approximation for the potential in momentum space, $V_{l}(q, p)$, then it was shown $(38,39)$ that $f_{k}(p)$ can easily ( be computed from a non-singular Fredholm integral equation. On the other hand, if we know $f_{k}(p)$ experimentally from, for example, e-d scattering, photodisintegration of the deuteron, $p-p$ bremstrahlung, etc.., $V_{l}(p, q)$ can be computed from the relation

$$
\begin{aligned}
& \text { from the relation } \\
& \begin{aligned}
2 \mu_{i} V_{l}(p, q) & =\frac{2}{\pi} \int_{0}^{\infty} d k \frac{\sin ^{2} \delta_{\ell}(k)\left[f_{k}(p) f_{k}(q)-f_{q}(p)\right]}{q^{2}-k^{2}} \\
& =\frac{2}{\pi} \int_{0}^{\infty} \frac{\sin ^{2} \delta_{\ell}(k)\left[f_{k}(p) f_{k}(q)-f_{p}(q)\right]}{p^{2}-k^{2}}
\end{aligned}
\end{aligned}
$$

The two different forms come from the time-reversal invariance requirement, $V_{l}(p, q)=V_{l}(q, p)$ and are the only a-priori restrictions on the off-shell extension function $f_{k}(p)$. Of course the interaction so constructed will in general be non-local as well as l-dependent, a point I will return to below.

The full off-shell extension required for the Faddeev equations cen then be constructed from the Lippmann-Schwinger equation $T(z)=V+V G_{0}(z) T(z)$ as follows: Recall that if the full Hamiltonian $H=H_{0}+V$, the free Green's. function $G_{0}^{-1}=-H_{0}$, the exact Green's function $G=Z-H$, and $G_{0} t=G V$, or $T(z)=V+V(z) V$. If we now introduce the plane-wave states $|K\rangle$, the exact scattering states $\mid \Psi^{+}(k)>$, and the usual result that $T_{l}\left(p, k ; k^{2} / 2 \mu_{i}\right)=\langle p| V\left|\psi^{+}(k)\right\rangle$, the completeness relation

$$
\left.1=\frac{2}{\pi} \int_{0}^{\infty} k^{2} d k|k><k|=\frac{2}{\pi} \int_{0}^{\infty} k^{2} d k\left|\psi^{+}(k)><\psi^{+}(k)\right|+\sum_{b}|b\rangle<b \right\rvert\, .
$$

gives us immediately that $<q\left|t_{l}(z)\right| p>=-2 \mu_{i} V_{l}(q, p)-$

$$
-\sum_{b} \frac{\gamma_{b}(q) \gamma_{b}(p)}{z+\epsilon_{b}}+\frac{2}{\pi} \int_{0}^{\infty} k^{2} d k \frac{<q\left|t_{l}\right|^{+}(k)><\psi^{+}(k)\left|t_{l}\right| p>}{k^{2}-2 \mu_{i} z-i \epsilon}
$$

or

$$
t_{l}(q, p ; z)=-2 \mu_{i} V_{l}(q, p)-\sum_{b} \frac{\gamma_{b}(q) \gamma_{b}(p)}{z+\epsilon_{b}}+\frac{2}{\pi} \int_{0}^{\infty} \frac{d k \sin ^{2} \delta_{l} f_{k}(p) f_{k}(q)}{k^{2}-2 \mu_{i} z-i \epsilon}
$$

As we noted above, the off-shell potential can be computed from $f_{k}(p)$, and the residues of the bound state terms $\gamma_{b}$ are also in principle experimental quantities (reduced widths). The current extent of our knowledge of $\sin ^{2} \delta_{\ell}$ and $f_{k}(p)$ for nucleon-nucleon interactions will be discussed below.

So far, we have ignored spin in the above equations. A priori, we would expect the number of coupled equations to increase by a factor of

36 when we include spin, but this is much too pessimistic. For instance, including ${ }^{I_{S}},^{3}{ }^{3} S_{1}$, and ${ }^{3} D_{1}$ states with both tensor and central forces, Sitenko and Karchenko (41) needed only three coupled equations for three functions in order to compute the triton binding energy and the doublet scattering length, if they also assume the interaction separable. Also Aaron, Amado and $Y a m$ (42) found that even though values of $J$ up to 10 or so are needed to fit $14 \mathrm{MeV} \mathrm{n}-\mathrm{d}$ scattering, only the $\mathrm{J}=0$ and 1 states required any sophisticated techniques for the solution of the integral equations ${ }^{(43)}$; for higher values of $J$, iterative solution of the equations converged rapidly, and this poses few problems for modern computers. This also suggests that it may be possible to exploit the loose structure of the deuteron to compute the higher $J$ states in the $n-d$ system in terms of the deuteron wave function in a reasonably model independent way, and hence reduce drastically the number of parameters which need be determined from experiment at low energy, in much the same way that the known OPE inter- • action simplifies the analysis of nucleon-nucleon scattering.

The calculations mentioned above $(41,42)$ as well as the earlier work of Mitra and Bashin ${ }^{(44)}$, and the comparable work of A. C. Phillips (45), all make the apparently drastic assumption that the interaction is separable (i.e. $V(p, q)=F(p) F(q)$ ), which reduces the Faddeev equations to coupled equations in a single variable. Until we have exact solutions of the two-variable equations for comparison, the physical justification of this approximation will remain dubious, but in the meantime this gives an interesting phenomenology. As is to be expected in calculations which include only pure attraction in $S$ states, these calculations overbind the triton. Since, in the quartet state the exclusion principle keeps the neutron in the long-range region, the quartet scattering length comes out
about right; the doublet scattering length is sensitive to the details of the calculation, but of the right order of magnitude. Differential cross sections for elastic $n-d$ scattering and total cross sections for breakup are reasonable well represented up to 14 MeV . The triton electromagnetic form factors are not so well represented. $(46,47)$

I believe several theoretical tasks should be vigorously attacked in order to allow for the maximum fruitful interaction between theory and experiment: (1) complete representation of the invariant amplitudes of the three-nucleon system in terms of (a) a unique unitary parameterization, and (b) the contribution to each of these from the Faddeev subchannels (two-nucleon amplitudes). This will allow calculation of observables, tests of simplifying assumptions, and guides to interesting experiments. (2) Complete formal development of the Faddeev equations for this system including spin, and the simplifications at (a) low energy, and (b) high J. This will allow the development of the analogs to effective range theory and OPE in the two-nucleon system, and hence cut down the number of parameters which need be measured experimentally. (3) Detailed investigation of the region of validity of various separable approximations so that they can be used with confidence in the regions where they are justifiable.

## IV. TWO-NUCIEON POLARIZATION EXPERIMENTS

A preliminary analysis of the $A_{x x}$ abd $A_{y y}$ Saclay experiments on p-p scattering near 25 MeV , together with the $n-p C_{n n}$ measurement from Ios Alamos at a similar energy was presented at Karlsruhe (48). Following a suggestion of Catillon's, it has proved possible to give an absolute normalization (49) to the Saclay experiments (50), and these, together with existing work in this energy region now define the $25 \mathrm{MeV} \mathrm{p}-\mathrm{p}$ phase shifts
to high precision. The Los Alamos $n-p C_{n n}$ experiment, which originally gave a value only at $180^{\circ}$, has been pushed to smaller angles (51), and a new analysis completed by Arndt and MacGregor (52). This analysis differs in important respects from an earlier work presented by the Dubna group (53), but the discrepancies have been resolved as due to differences in data selection, energy dependence assumed for the phase shifts or observables measured at different energies, and the complications due to a genuine solution ambiguity in the $n-p$ analysis. The ambiguity allows two values of ${ }^{3} S_{1}$, one greater than $90^{\circ}$, and the other less. Since we know from Levinson's theorem and effective range theory that ${ }^{3} S_{1}$ starts from $180^{\circ}$ at zero energy and falls monatonically, passing through $90^{\circ}$ below 20 MeV , there is no doubt that the latter is the physically ${ }^{*}$ correct solution, but the existence of the spurious possibility makes the error analysis unreliable. However, one can use the correlated errors to compute observables and their uncertainties in order to determine which additional experiments will be most useful for increasing the accuracy of the analysis. This is illustrated for $C_{n n}$ in Figure 3. We see that extending the measurements to still smaller angles will not remove the solution ambiguity between $C$ and $C^{\prime}$, but if comparable precision to the existing experiments is achieved, will reduce the uncertainties in the phase shift determinations. On the other hand, $D$ and $D_{T}$, as shown in Figures 4 and 5 , if measured to even modest accuracy at the right angle, would eliminate the spurious solution. I think it is important to realize that this type of analysis can always be carried through whenever a theoretically reliable parameterization of the experiments exists, and should always be carried out in advance of designing new experiments, both to insure maximum usefulness of the results and to avoid spending a couple of years on an experiment that, even if
successful, will not give any essentially new information.
I have spent this much time discussing the 24 MeV experiments because it is important to finish up this job in order to obtain a reliable experimental value for the ${ }^{3} S_{1}-{ }^{3} D_{1}$ coupling parameter $\epsilon_{1}$. Because of the strong OPE tensor force in this state, and the loose structure of the deuteron, recent models of the deuteron obtain most of the binding from this tensor force, leaving room for only a rather weak central force. Blatt (54) claims, on the basis of variational calculations, that these models then cannot give the observed 8.49 MeV binding energy for the triton, but only 4 or 5 MeV binding. As noted above, separable models with purely attractive central, $S$-wave interactions overbind the triton by about 3 MeV , but when the Yamaguchi tensor force is added, the triton is overbound by only about $1 \mathrm{MeV}{ }^{(41)}$. However, the Yamaguchi tensor force corresponds to a $4 \%$ D-state probability rather than the usually accepted $7 \%$, and also does not have the OPE range, so the physics of this calculation is not clear. Further, when the calculations are extended to include short-range repulsion, it is quite possible that the binding will turn out too small, in agreement with the variational calculations. It is, therefore, important to know experimentally that the OPE-tensor force is actually present in the ${ }^{3} \mathrm{~S}_{1}-3^{3} \mathrm{D}_{1}$ state, and one way to do this is to improve the precision of the 25 MeV analysis to the point where a clean test of Wong's prediction ${ }^{(55)}$ of $\epsilon_{1}$ from OPE becomes possible. I should also note that Perring ${ }^{(56)}$ has also carried through a new analysis of the 25 MeV data, and differs on some points of detail with Arndt and MacGregor.

Now that a differential cross section at 50 MeV will soon be available (57) it is to be hoped that a similar analysis at that energy can be completed, and the $n-p$ experiments needed to complete the picture to reasonable
precision both pin-pointed and performed. This would again give interesting information about $\epsilon_{1}$. So far as work at other energies in concerned, I believe that $n-p$ experiments are the most important and should be given priority, simply because of the much greater uncertainties which still exist in the $n-p$ phase shift analyses. If it can be shown in advance that a p-p measurement will resolve an uncertainty, or actually improve the precision to which some set of parameters is measured, then it is obviously worth doing, but my feeling is that the time is approaching when p-p experiments in the elastic scattering region should be attempted primarily when needed to supplement some $n-p$ measurement or when it can be shown that the $p-p$ scattering matrix at that energy is needed to higher precision for some specific purpose.

I will not attempt to review experiments above meson production threshold ( 280 MeV ), since this brings in a new three-body problem (NNT), : which not only has all the complications discussed above, but also lands us squarely in the middle of the still unsolved problem of connecting relativity and quantum mechanics in a theory containing only a finite number of particles, or some other approach which one is willing to follow into the battlegrounds of elementary particle theory.
V. THEORETICAL STRUCTURE OF THE TWO-NUCLEON INLERACTION

We have seen in our discussion of the theory of the three-body problem that what we require as input, assuming only pairwise interactions, is (a) the on-shell amplitude $\tau(k)$ which we can get immediately from a phase-shift analysis of nucleon-nucleon scattering, and (b) the half offshell extension function $f_{k}(p)$ which requires additional theoretical or experimental information to obtain. p-p phase shifts are now known with
considerable precision over the entire elastic scattering region, and $n-p$ phase shifts are also falling into place. The first question is, therefore, whether we can obtain the function $f_{k}(p)$ from this information by a theoretical argument. Sticking for the moment to non-relativistic quantum mechanics (which means in practical terms that we assume the details of how $\tau(k)$ and $f_{k}(p)$ go to zero as $k$ goes to infinity will not significantly affect three-nucleon calculations at. low energy), the Gelfand-Ievitan theorem tells us that if we know the phase shift for a single partial wave at all energies, and there are no bound states in that partial wave, we can construct uniquely the corresponding static, local potential; the function $f_{k}(p)$ and all other partial waves can then be calculated theoretically. Including spin, we must do this for five amplitudes for each of the two isp-spin states, and for the ${ }^{3} S_{1}-{ }^{3} D_{1}$ state must also know the asymptotic normaljzation of the $S$ and the $D$ wave functions, but these two additional. parameters are also in principle experimentally determinable.

We consider first the singlet state with I-spin one. The ${ }^{l_{S}}$ low energy behavior is determined to a good approximation by the scattering length and effective range, and it has been shown (58) that the small deviations from this behavior are accounted for by the same one-pionexchange (OPE) interaction which fits the highest partial waves. A third parameter is provided by the change in sign of the ${ }^{l_{S}}{ }_{O}$ phase shift near 250 MeV , which shows that (in a static, local model) there is also shortrange repulsion. Since we know, both experimentally and theoretically that the longest range part of the interaction is given by OPE, we must adjust two parameters in the intermediate range attraction to fit the scattering length and effective range and, as the effective radius of the short-range repulsion is fixed by the energy at which the phase shift
changes sign, about the only freedom left in the model is how we treat the short-range repulsion. One extreme assumption is that this is due to an infinitely repulsive hard core, which gives an essential singularity to $\tau(k)$ as $k$ goes to infinity, and an oscillating phase shift; a more physical assumption is to postulate a repulsive yukawa potential with the $\omega$-meson mass, which gives a phase shift that falls smoothly to zero at high energy; the truth should lie in between. Because the attraction must have a range less than 2 pion compton wavelengths, and the repulsion be still shorter, protons with wavelengths corresponding to energies up to 300 MeV cannot explore the details of this structure, and both models give reasonable agreement with observed ${ }^{l_{S}}{ }_{0}$ phase shifts. The crucial test is then to see whether both models give the same ${ }^{I_{D}}{ }_{2}$ and ${ }^{l_{G}}$ phase shifts over the same energy range, and is shown in Figure 6. In fact the two predictions lie on top of each other, so are labeled "IOCAL", indicating that they are the unique prediction of a static, local potential fitted to the ${ }^{l_{S}}$ phase under the above assumptions. We also see from the experimental points that the predictions are too high by several standard deviations, demonstrating conclusively that the singlet nucleonnucleon interaction is non-local, and hence that the off-shell extension function $f_{k}(p)$ cannot be reliably computed from this assumption. Since we cannot compute $f_{k}(p)$ from the local potential assumption, the next question is whether we can obtain it from some new type of experiment. We saw above that $f_{k}(p)$ is directly related to the twonucleon wave function, so what is required is a measurement of this wave function inside the range of forces. Since the electromagnetic structure of the proton and neutron have been accurately measured by electron scattering, if we are willing to make the assumption that the electromagnetic
charge and current distribution in the two-nucleon system follows the motion of the proton and neutron, we could do this from experiments such as e-d scattering, photodisintegration of the deuteron, $p-p$ and $n-p$ bremstrahlung, etc. However, this also fails. For example, it is a straightforward matter to calculate the capture of epithermal neutrons by protons via the magnetic dipole process $n+p \rightarrow \gamma+d$, and to show that we know enough about nuclear forces to make this calculation to high precision (59). The calculation fails by $10 \%$, with a theoretical limit of uncertainty of only $1.2 \%$, showing conclusively that even for zero relative energy between the two particles, there are sources of current in the two-nucleon system other than those due to the motion of the proton and the neutron. Presumably these are due to meson currents, so we find that in order to obtain the information needed for a physical solution of the three-nucleon problem, even in the non-relativistic region, we are forced to understand the coupling of the neutron and the proton to other elementary particles, and cannot simply treat them as non-relativistic mass-points interacting via a phenomenologically determinable potential.

Fortunately, the devoted experimental work of the last ten years on the spin structure of the two-nucleon system, using first triple-scattering and spin-correlation techniques, and now the powerful combination of polarized targets with polarized beams, has given us sufficiently detailed information so that we can make the interpretation of the two-nucleon interaction as due to the coupling to the nucleons to known bosons and boson resonances with some confidence. Turning first to the singlet $p-p$ interaction, the phase shifts shown in Figure 7 give us a great deal of information. As already noted, the longest-range interaction is correctly predicted by OPE, but this is both too long range and too wèak to explain
quantitatively the observed ${ }^{l_{S}}{ }_{0}$ scattering length and effective range, so there is in addition an intermediate range attractive interaction. To get some idea of the spin and parity of this intermediate mass boson, we look first at the central interaction in the triplet-odd state, which is roughly measured by $\left({ }^{3} \mathrm{P}_{0}+3^{3} \mathrm{P}_{1}+5{ }^{3} \mathrm{P}_{2}\right) / 9$. At 25 MeV and above, this is strongly positive, but below 3 MeV it is slightly negative, and an order of magnitude less than the weak negative repulsion predicted by OPE in this state ${ }^{(58)}$. Taking account of centrifugal shielding, this shows that the intermediate range attraction we found in the singlet-even states (confirmed by the large values of ${ }^{1} D_{2}$ and ${ }^{l_{G}}{ }_{4}$ compared to OPE) can be explained as due to a boson of zero spin and positive parity. 'Turning to the deuteron, we find a quadrupole moment roughly accounted for by OPE, but also the need for an attractive central interaction of intermediate range, showing that this boson also has zero isospin. Whether or not there is a $\sigma$-meson with $I=0, J^{P}=0^{+}$, we have reasonable confidence that the $\pi-\pi$ state with these quantum numbers is attractive, and the consequent correlation of two-pion exchange in the nuclear force would produce the effect we have just identified.

In order to get more information about the short-range repulsion we found in the ${ }^{l_{S}}{ }_{0}$ state, we turn to the triplet-odd $P$-waves as shown in Figure 8. We see the +-+ signature of the ${ }^{3} P_{0,1,2}$ phases at low energy to be expected from the long-range $O P E$ tensor force, but also find that ${ }^{3} P_{0}$ changes sign about 210 MeV , giving the -+ signature characteristic of an $I \cdot S$ interaction at higher energies. The short range of this $I \cdot S$ interaction is confirmed by the fact that the ${ }^{3} F_{2,3,4}$ phases retain the OPE signature up to 300 MeV . Recalling that the exchange of electromagnetic quanta produces repulsion between like charges, and an $L \cdot S$ (Ihomas)
term, it is no surprize that both features can be explained by the exchange of a heavy quantum with $J^{P}=I^{-}$, that is, a massive vector meson. This identification is further confirmed by the prediction that between unlike (nuclear) charges, this strong repulsion will change to strong attraction, which accounts for the very large $\bar{p} p$ and $\bar{n} p$ annihilation cross sections in the multi-BeV region. The $I=0, J^{P}=I^{-} \omega$ meson is ready to hand to explain all these features, but the data is not sufficiently precise to show how the effect should be split between the $a$ and the still heavier $\phi$ with the same quantum numbers, or whether they have tensor as well as vector coupling to the nucleons. On the basis of $\mathrm{SU}_{3}$, it appears likely that the $\mathrm{I}=1 \rho$ makes a much smaller contribution to the nuclear force, but it will require more precise $n-p$ data than those now available before this prediction can be checked in detail. The $I^{\circ}=0, J^{P}=0^{-} \eta$-meson is also expected to give a small contribution, but it is possible to use the forward nucleon-nucleon dispersion relations to give an indication that it is in fact there in the nucleon-nucleon scattering data. We should also note that the failure of the static, local assumption to fit the singlet state is in at least qualitative agreement with a velocity-dependent effect to be expected in vector-meson exchange. Finally, if one takes the $\pi, " \sigma ", \eta, \rho, \omega$ and $\varnothing$ as given, Ball, Scotti and Wong (60) have shown that one can not only get semi-quantitative fit to nucleon-nucleon scattering with only four adjustable parameters, but also that when one "crosses" this fit to the nucleon-antinucleon system, these, and only these, bosons show up as bound states. Of course, a great deal is left out in this description of the nucleon-antinucleon system, so close agreement with the experimental masses is not obtained or expected, but it does show that we are beginning to understand how the strong
interactions hang together and support each other.
This excursion into elementary particle theory was undertaken in order to show how rich and detailed a description of the basic two-nucleon interactions has been made possible by detailed experimental work. Because the theory is confirmed in so much detail, it would seem reasonable that it can be trusted, if the proper techniques can be developed, to allow us to take the two-nucleon $T$ matrix off-shell in a physically consistent way, and hence compute $f_{k}(p)$ with some confidence. It is to be hoped that, for the low-energy three-nucleon problem, only a rough calculation of $f_{k}(p)$ will suffice, because of the generally smooth structure of this function adduced in Section III. The same theory will presumably generate genuine three-body forces as well, but I feel we should first push the pairwise interaction calculations far enough to demonstrate the existence of three-body forces experimentally before tackling the more difficult problem of deriving them from elementary particle theory.

## VI. CONCLUSION

We have seen that the three-nucleon problem offers more technical difficulties at the phenomenological level, both experimentally and theoretically, than the two-nucleon problem, but that on the experimental side the polarized-target polarized-beam technique, and on the theoretical side the Faddeev equations, give us the opportunity of attacking this problem with a reasonable hope of achieving successes in ten to fifteen years comparable to those achieved in the two-nucleon problem over a corresponding period of time. The immediate theoretical needs for making efficient use of existing experiments and planning new experimental programs are (a) a low-energy theory comparable to effective range theory,
(b) an explicitly unitary parameterization of three-body reactions comparable to two-particle phase shifts and inelasticity parameters, (c) a complete transcription of the Faddeev two-particle subchannels into the three-particle invariant amplitudes and the unitary three-particle parameterization, (d) a model-independent high-J analysis for $n-d$ scattering and breakup which exploits the loose structure of the deuteron to provide the analog to the OPE calculation in the two-nucleon system. To some extent, the separable interaction approximation to the Faddeev equations gives a phenomenological framework for answering these questions, but the exact theories still have to be worked out in order to determine where this approximation can be safely applied. At a deeper level, we have also seen that we have to go beyond the phenomenological analysis of nucleonnucleon scattering into elementary particle theory in order to make physically reasonable three-nucleon calculation, but that the physical picture needed for that extension is reasonably well understood.
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Elastic scattering and breakup transitions for nd．scattering which con－ serve total angular momentum $J$ and parity in the $J$ ，$L$ ，$S$ representation． Solid arrows（ $\rightarrow$ ）indicate elastic scattering transitions；dotted arrows $(--\rightarrow)$ transitions which are independent in breakup，but known in terms of the solid arrow transitions from time－reversal invariance in elastic scattering；wavy arrows（ $\sim$ ）indicate transitions which occur only in breakup．Each entry gives the value of total orbital angular momentum $I(=J \pm I / 2$ or $J \pm 3 / 2)$ ；final states where two of the nucleon spins add to zero are designated by $\sigma=0$ ．

Final State
Spin flipped
［Spin broken］ Spin not flipped ［Spin broken］

$$
\begin{aligned}
& s=3 / 2 \\
& \mathrm{~J}-1 / 2 \longleftarrow \mathrm{~J}=1 / 2 \\
& J+3 / 2 \longleftarrow[J-I / 2, \sigma=0] \\
& J+1 / 2 \leftarrow \\
& S=1 / 2 \\
& J+I / 2 \\
& \begin{array}{r}
J+I / 2 \\
{[J+I / 2, \sigma=0]}
\end{array} \\
& \text { 下ミー- - - - - - } \\
& s=3 / 2 \\
& \underline{s}=3 / 2 \\
& \underline{S}=1 / 2 \\
& s=1 / 2 \\
& J-1 / 2 \\
& {[J-1 / 2, \sigma=0]} \\
& J+1 / 2 \\
& {[J+1 / 2, \sigma=0]} \\
& \underline{S}=3 / 2 \\
& J-3 / 2
\end{aligned}
$$

$$
\begin{aligned}
& {[J-1 / 2, \sigma=0]}
\end{aligned}
$$

4
[12]

8
［12］

Hence number of elastic amplitudes $\quad=12$
number of amplitudes for $1 / 2+1 \rightarrow 1 / 2+1$
number of amplitudes for breakup $=[24]$

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APPENDIX - d-p OBSERVABLES WITH POLARIZED BEAM AND TARGET

## J. Raymal ${ }^{\dagger}$

If the $6 \times 6$ elastic scattering matrix $M$ is expressed in the helicity formalism, incorporating the restrictions due to parity conservation, we find

$$
M^{+} M=\left(\begin{array}{cccccc}
a & b & c & d & e & f \\
b^{*} & g & h & i & j & e^{*} \\
c^{*} & h^{*} & k & l & i^{*} & -d^{*} \\
d^{*} & i^{*} & -l & k & -h^{*} & c^{*} \\
e^{*} & -j & i & -h & g & -b^{*} \\
-f & e & -d & c & -b & a
\end{array}\right\}
$$

where $a, g, k$ are real
f, $j, \ell$ are imaginary
b, c, d, e, h, i are complex
making 18 numbers in all.
If there are no doublet-quartet transitions, we have a spin $1 / 2$ scattering ( 2 expressions) and $\operatorname{spin} 3 / 2\left(\sigma, P, 3 T_{2}\right.$ and $3 T_{3}: 8$ expressions $)$. This hypothesis introduces 8 relations between the polarizations and the correlation of spin parameters. They are

$$
\begin{array}{ll}
d=(1 / \sqrt{2}) b ; \quad e=\sqrt{2} c & \text { (4 relations) } \\
j+l=(1 / \sqrt{2}) h ; h \text { imaginary } & \text { (2 relations) } \\
i=\sqrt{2}(g-k) & (2 \text { relations })
\end{array}
$$

This discussion of the possible observables in p-d elastic scattering was submitted to the conference by J. Raynal. The algebra has been checked independently by $W$. Ross (Stanford) and J. Arvieux (Saclay), and is believed correct.

For this class of experiments, we can choose the axis of quantization along the beam direction and express the density matrix of the initial state in the usual tensor notation as $\mathrm{T}_{\mathrm{s}_{1} \mu_{1}}^{(1 / 2)} \otimes \mathrm{T}_{\mathrm{s}_{2} \mu_{2}}^{(1)}$. The possible observables are then

$$
A_{s_{1} \mu_{1} s_{2} \mu_{2}}=\operatorname{Trace}\left(M^{\dagger} M_{s_{1} \mu_{1}}(1 / 2) \otimes T_{s_{2} \mu_{2}}^{(1)}\right)
$$

The complete results, and those for no doublet-quartet transitions, are given in Table A-1.

Table A-l. Polarized-beam polarized-target observables for elastic p-d scattering in the helicity notation. "Simplified" means that no doublet-quartet transitions are allowed.

| complete | simplified |
| :---: | :---: |
| $\mathrm{A}_{0000}=2(\mathrm{a}+\mathrm{k}+\mathrm{g})$ | $=2(\mathrm{a}+\mathrm{k}+\mathrm{g})$ |
| $A_{1010}=\sqrt{6}(\mathrm{a}-\mathrm{k})$ | $=\sqrt{6}(\mathrm{a}-\mathrm{k})$ |
| $A_{0020}=\sqrt{2}(\mathrm{a}+\mathrm{k}-2 \mathrm{~g})$ | $=\sqrt{2}(a+k-2 g)$ |
| $A_{0022}=A_{002-2}=\sqrt{3}\left(c+c^{*}\right)$ | $=\sqrt{3}\left(c+c^{*}\right)$ |
| $A_{1022}=-A_{102-2}=\sqrt{3}\left(c^{*}-c\right)$ | $=\sqrt{3}\left(c^{*}-c\right)$ |
| $A_{0011}=\hat{A}_{001-1}=\sqrt{3 / 2}\left(b+h-b^{*}-h^{*}\right)$ | $=\sqrt{3 / 2}\left(b-b^{*}+2 h\right)$ |
| $A_{0021}=-A_{002-1}=\sqrt{3 / 2}\left(h+h^{*}-b-b^{*}\right)$ | $=-\sqrt{3 / 2}\left(b+b^{*}\right)$ |
| $A_{1011}=-A_{101-1}=-\sqrt{3 / 2}\left(b+b^{*}+h+h^{*}\right)$ | $=-\sqrt{3 / 2}\left(b+b^{*}\right)$ |
| $A_{1021}=A_{102-1}=\sqrt{3 / 2}\left(b-b^{*}+h^{*}-h\right)$ | $=\sqrt{3 / 2}\left(b-b^{*}-2 h\right)$ |
| $A_{1100}=A_{1-100}=\sqrt{2}\left(j+d-d^{*}\right)$ | $=h-\sqrt{2} l+b-b^{*}$ |
| $A_{1110}=A_{1-100}=-\sqrt{3}\left(d+d^{*}\right)$ | $=-\sqrt{3 / 2}\left(b+b^{*}\right)$ |
| $A_{1120}=A_{1-120}=d-d^{*}-2 j$ | $=-\sqrt{2} h+2 l+(1 / \sqrt{2})\left(b-b^{*}\right)$ |
| $A_{111-1}=A_{1-111}=-\sqrt{3}\left(i+i^{*}\right)$ | $=-2 \sqrt{6}(\mathrm{~g}-\mathrm{k})$ |
| $A_{1111}=A_{1-11-1}=\sqrt{3}\left(e+e^{*}\right)$ | $=\sqrt{6}\left(c+c^{*}\right)$ |
| $A_{1121}=-A_{1-12-1}=\sqrt{3}\left(e-e^{*}\right)$ | $=\sqrt{6}\left(c^{*}-c\right)$ |
| $A_{112-1}=-A_{1-121}=\sqrt{3}\left(i-i^{*}\right)$ | $=0$ |
| $A_{1122}=A_{1-12-2}=\sqrt{6} \mathrm{f}$ | $=\sqrt{6} \mathrm{f}$ |
| $A_{112-2}=A_{1-122}=\sqrt{6} l$ | $=\sqrt{6} l$ |

The cross section is (1/6) A $\mathrm{A}_{0000}$, the polarization of the proton is $\mathrm{A}_{1100} / \mathrm{A}_{0000^{\circ}}{ }^{(+)}$Experiments done at Saclay with a polarized beam of 22 MeV deuterons gave ${ }^{(*)} \mathrm{A}_{0011}, \mathrm{~A}_{0020}$, and $\mathrm{A}_{0022}$. Thus we have

$$
\begin{aligned}
& \quad 2(a+k+g) \\
& \sqrt{3 / 2}\left(b-b^{*}\right)+\sqrt{3 / 2} h-\sqrt{3} \cdot l, \sqrt{3 / 2}\left(b-b^{*}+2 h\right) \\
& \sqrt{2}(a+k-2 g) \\
& \sqrt{3}\left(c+c^{*}\right)
\end{aligned}
$$

The last measurement is interesting because one can use it to check the requirement that $A_{1111}=\sqrt{2} A_{0022}$ if there are no doublet-quartet transitions. This coefficient is the same as $A_{x x}-A_{y y}$. There are two possible ways of obtaining it:
(1) The same as in proton-proton scattering; one can eliminatc the single polarization effects by flipping the polarization of the beam and the target.
(2) It is more difficult to obtain it directly because the polarization of the beam is turned and there is a vertical deviation of the beam by the magnet of the target. If we have two transverse polarizations with an angle of $\pi / 2$ between them, the correlation is $A_{1111} \sin 2 \phi$ and can bo measured at $\pi / 4$ (see sketch).


Other experiments might be used to test the absence of presence of doublet-quartet transitions. We note that $A_{112-1}$ is predicted to be zero, but it is very difficult to obtain a component $T_{21}$. in a deuteron beam. It could be obtained by using a polarized deuteron target in which the vector polarization could be flipped independently of the tensor one, and the target could be inclined. A fuller discussion will be published elsewhere.
$\dagger$ See References 9, 10, 11 above.

* See References 26, 27, 28 above.


## FIGURE CAPTIONS

Figure 1 - General behavior of proton polarization in p-d scattering up to 50 MeV .

Figure 2 - Comparison of 22.7 Mev neutron polarization in $n-d$ scattering with proton polarization in p-d scattering at 22 MeV , as given in Reference 21.

Figures 3, 4, 5 - Predictions of $C_{n n}, D$, and $D_{T}$ for $25 \mathrm{MeV} \mathrm{n-p}$ scattering as given in Reference 52. Error bands indicate the precision needed to improve the precision of the phase parameter determination; the two bands show which measurements would resolve the solution ambiguity discussed in the text.

Figure 6 (from Reference 48) $-{ }^{l_{S}}$ curves compare the prediction from a hard-core model (SYH) with the shape-independent approximation (SI), low energy dispersion theory including one-pion-exchange (CFS), and boundary condition model (BC). The three yukawa potential model using $\pi$, " $\sigma$ ", and $\omega$ masses agrees with SYH at low energy and differs by less than $2^{\circ}$ at all energies shown. Both the SYH (hard core) and the three yukawa models predict the same ${ }^{I_{D_{2}}}$ and ${ }^{l_{G}}$ phase shifts shown by the "IOCAL" curve. Experimental points are from R. A. Arndt and M. H. MacGregor, Phys. Rev. 141, 873 (1966).

Figure 7 - Singlet $p-p$ phase shifts from R. A. Arndt and M. H. MacGregor, Phys. Rev. 141, 873 (1966).

Figure 8 - Triplet $l=1 \mathrm{p}-\mathrm{p}$ phase parameters from R. A. Arndt and M. H. MacGregor, Phys. Rev. 141, 873 (1966).


PROTON POLARIZATION IN PROTON - DEUTERON ELASTIC SCATTERING


Figure 2


Figure 3


Figure 4


Figure 5


Figure 6


Figure 7





[^0]:    $\dagger$ Paper presented at the International Colloquium on Polarized Targets and Beams, C.E.N. Saclay, December 5-9, 1966.

    * Work performed under the auspices of the U. S. Atomic Energy Commission

[^1]:    ${ }^{\dagger}$ Work performed under the auspices of the U. S. Atomic Energy Commission.

