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NEW SUM RULES FOR COMPTON SCATTERING*

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ABSTRACT

Assuming that all $I = 2$ meson Regge trajectories have $\alpha(0) < 0$ we derive a new set of sum rules for Compton scattering on $I \geq 1$ hadrons. We compare our results with experiment and with other theoretical ideas.

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1. Assuming that the high energy behavior of the forward amplitude for Compton scattering on hadrons is correctly described by Regge pole theory, we have derived a new family of sum rules. Our basic simple idea is to consider certain combinations of non-spin-flip Compton amplitudes which are predicted to decrease rapidly with energy, and to assume that they obey unsubtracted dispersion relations. Using the well known Thomson limit for the amplitude at zero frequency, we can then derive a set of sum rules which, as far as we can tell, is not inconsistent with experiment. In addition, we find some interesting relations between our sum rules and previously derived results of current algebras and quark models.

It is well known that the forward, non-spin-flip amplitude for Compton scattering of real photons on any given hadron <u>cannot</u> satisfy an unsubtracted dispersion relation. Such a relation, when evaluated for zero photon energy, would read:

$$-\frac{\alpha}{m}Q^2 = f(0) = \frac{2}{\pi}\int_0^\infty \mathrm{Im}\, f(\nu)\,\frac{d\nu}{\nu} = \frac{1}{2\pi^2}\int_0^\infty \sigma(\nu)d\nu \tag{1}$$

Where $\alpha = \frac{1}{137}$, m is the hadron mass, Q - its electric charge (in units of electron charge), ν - the laboratory photon energy and σ - the total photoabsorption cross-section. For any <u>single</u> hadron, Eq. (1) cannot be true since it implies that a positive definite integral is equal to a non-positive constant. Furthermore, if the total cross-section σ is constant at high energies (or even if it falls off like $\frac{1}{\nu}$) the integral in Eq. (1) will diverge. On the other hand, if $f(\nu)$ is the <u>difference</u> between two forward non-spin-flip Compton amplitudes on two different hadrons, it may obey Eq. (1), provided that at high energies $\mathrm{Im}\, f(\nu) \to 0$. Both

Regge theory and the experimental meson-baryon and baryon-baryon total cross-sections suggest the convenient parametrization $f(\nu) \propto \nu^{\alpha(0)}$ where, in Regge theory, $\alpha(0)$ is the $t = 0$ intercept of the leading trajectory having the quantum numbers of the t-channel. Using this form we conclude that Eq. (1) will be satisfied whenever $\alpha(0) < 0$.

It was recently suggested by de-Alfaro, Fubini, Furlan and Rossetti[1] that all $I = 2$ meson trajectories have $\alpha(0) < 0$. This assumption has already yielded a number of interesting results[2] and no contradiction with experiment. By applying it to Compton scattering we now find that the relation (1) will be obeyed by all $I = 2$ t-channel amplitudes. In particular, we obtain the following interesting new sum rule for $\gamma\pi$ scattering:

$$-\frac{\alpha}{m_\pi} = \frac{1}{2\pi^2}\int_0^\infty \left[\sigma_{\gamma\pi^+}(\nu) - \sigma_{\gamma\pi^0}(\nu)\right] d\nu \qquad (2)$$

The convergence of the integral is guaranteed if $\alpha_{I=2}(0) < 0$. It is extremely plausible to assume that the sum rule (2) converges rapidly and that it is dominated by s-channel resonances up to, say, 2 BeV. This will be the case if the couplings of the $I = 2$ trajectory are small and the $t = 0$ intercept is actually far below zero.[2][3]

2. Since we do not have any direct data on $\gamma\pi$ scattering we can only try to estimate the various contributions to the sum rule (2). It is easy to see[4] that the only s-channel resonances contributing to Eq. (2) are the $G = -1$ states. The most important of these is the ω which is the lowest lying $G = -1$ state and which has a relatively large partial width to $\pi\gamma$. Other candidates are the φ (which has a very small decay rate into π-ρ and presumably even a smaller rate to $\pi\gamma$)[5], the A_1 (if it exists) and the A_2. The η, ρ, X^0, f^0, f^* and σ do not contribute to (2). A rough

estimate for the sign and magnitude of the dispersion integral in (2) can be obtained by assuming that it is dominated by the contribution of the ω. Using the narrow resonance approximation and neglecting terms of order $(\frac{m_\pi}{m_\omega})^2$ we find:[6]

$$\frac{\alpha}{6} = \frac{\Gamma(\omega \to \pi\gamma)}{m_\omega} \tag{3}$$

leading to $\Gamma(\omega \to \pi\gamma) = 0.9$ MeV. Experimentally[7]: $\Gamma(\omega \to \pi\gamma) = 1.2 \pm 0.3$. This remarkable result is probably accidental to a certain extent, since the vector meson dominance assumption for s-channel dispersion relations does not work so well, in most other cases.[8] We do consider it, however, as an indication that the sum rule (2) may be true, and as a convenient rough approximation. We also notice that the contribution of the next important state, the A_2 will shift $\Gamma(\omega \to \pi\gamma)$ in the right direction. The A_2 and A_1 contributions can be very crudely estimated by using the experimental $A_{1,2} \to \pi + \rho$ decay rates and the ρ-dominance model for the isovector part of the electromagnetic current. This model predicts $\Gamma(A_1^+ \to \pi^+\gamma) = 0.4 \pm 0.2$ MeV, $\Gamma(A_2^+ \to \pi^+\gamma) = 1 \pm 0.4$ MeV, where the quoted errors reflect the uncertainties in the masses and widths of the resonances[7] and the ρ coupling constant[9] but not the error introduced by the ρ-dominance assumption. The same model predicts $\Gamma(\varphi \to \pi\gamma) \sim 0.01$ MeV. It is impossible to draw any decisive conclusions from such estimates. If we insert our values for the φ, A_1 and A_2 contributions in Eq. (2) we obtain: $\Gamma(\omega \to \pi\gamma) = 2 \pm 0.5$ MeV. Only a direct experimental determination of $\Gamma(A_2 \to \pi\gamma)$ will enable us to reach stronger conclusions, since the A_2 contribution is the most important after that of the ω.[10]

3. The only other sum rule which has been derived, so far, for Compton scattering on pions, is that of Cabibbo and Radicati who found[11]:

$$\frac{2}{3} < r_\pi^2 > = \frac{1}{2\pi^2\alpha} \int_0^\infty \left[\sigma^V(\gamma+\pi^0 \to I=0)+\sigma^V(\gamma+\pi^+ \to I=1) - \frac{5}{4}\,\sigma^V(\gamma+\pi^0 \to I=2)\right] \frac{d\nu}{\nu} \,. \qquad (4)$$

Where r_π is the charge radius of the pion and σ^V is the total absorption cross-section for isovector photons in a given s-channel isospin. The resonances which contribute to (4) are the same as those contributing to our sum rule (2). In both cases, the ω and A_2 are, presumably, the most important states. Approximating the integrals in both (2) and (4) by ω and A_2 alone we can solve for the A_2 width and the pion radius. We find:

$$\Gamma(A_2 \to \pi\gamma) = 0.3 \pm 0.3 \text{ MeV} \qquad (5)$$

$$r_\pi = 0.55 \pm 0.1 \text{ f} \qquad (6)$$

Equation (6) should be compared with $r_\pi = 0.63$ f (predicted by ρ dominance), $r_\pi = 0.7 \pm 0.2$ f (the only available experimental[12] number) and $r_p = 0.82$ f (the experimental charge radius of the proton). If we include the A_1 contribution as an additional unknown quantity in Eqs. (2) and (4) we can still derive (6) but not (5).

A surprising relation is obtained when we assume ω-dominance for the sum rules (2) and (4), and eliminate $\Gamma(\omega \to \pi\gamma)$. We find:

$$\frac{< r_\pi^2 >}{6} = \frac{1}{2m_\omega^2} \qquad (7)$$

This result differs by a factor 2 from the prediction of the vector meson dominance model for the pion form factor, which gives:

$$\frac{< r_\pi^2 >}{6} = \frac{1}{m_\rho^2} \qquad (8)$$

We suspect, although we have not been able to prove it, that this factor of 2 is related to the similar factor which we find when we compare the Cabibbo-Radicati sum rule to the Dashen-Gell-Mann-Lee[13] sum rule.[14]

4. Another interesting relation follows when we relate Eq. (3) to the well known relation:

$$\Gamma(\omega \rightarrow \pi\gamma) = \frac{\alpha\, m_\omega^3\, \mu_p^2}{24\, m_N^2} \tag{9}$$

where μ_p and m_N are, respectively, the total magnetic moment and the mass of the proton. Eq. (9) was first derived on the basis of the quark model[15] and was recently rederived[16] using saturation and pole dominance assumptions on sum rules obtained from the algebra of currents at infinite momentum. Eqs. (3) and (9) lead to a familiar relation which was previously derived either from $\tilde{U}(12)$-type theories or from the nonchiral $U(6) \times U(6)$ current algebra between states at rest[13]:

$$\mu_p = \frac{2m_N}{m_\omega} \tag{10}$$

Eq. (10) gives $\mu_p = 2.4$.

5. Using the model suggested by Gell-Mann, Sharp and Wagner[17] for the $\omega \rightarrow \pi + \gamma$ decay we can write:

$$\Gamma(\omega \rightarrow \pi\gamma) = \frac{\alpha m_\omega^3}{24} \frac{f_{\rho\omega\pi}^2}{f_\rho^2} \tag{11}$$

where f_ρ is the direct γ-ρ coupling constant. Assuming $f_\rho = f_{\rho\pi\pi}$ and inserting Eq. (11) into (3) we find:

$$m_\omega^2\, f_{\rho\omega\pi}^2 = 4\, f_{\rho\pi\pi}^2 \tag{12}$$

This result is identical (apart from replacing m_ω by m_ρ) to the prediction derived by Fubini and Segre[18] while saturating their superconvergent dispersion relation for π-ρ scattering by the π and ω intermediate states. Notice, however, that their result is derived from a spin-flip, I = 1 t-channel amplitude which has nothing to do with our non-spin-slip I = 2 amplitude.

6. In addition to the sum rule (2) for Compton scattering on pions, we can derive similar relations for any I $\geq$ 1 hadron. The simplest of these is a relation for $\gamma\Sigma$ scattering:

$$-\frac{2\alpha}{m_\Sigma} = \frac{1}{2\pi^2}\int_0^\infty \left[\sigma_{\gamma\Sigma^+}(\nu) + \sigma_{\gamma\Sigma^-}(\nu) - 2\sigma_{\gamma\Sigma^0}(\nu)\right] d\nu \tag{13}$$

where, again, the integral converges if $\alpha_{I=2}(0) < 0$. We find it very hard to compare (13) with experiment in view of our ignorance about $Y^*\Sigma\gamma$ or even $Y^*\Sigma\rho$ vertices. In particular, we cannot use SU(3) for calculating the $Y_0^*(1405)$ and $Y_0^*(1520)$ contributions since both are, presumably, in SU(3) singlets and are not related to any N^*. We notice, however, that the largest single contributor to (13) is probably the Λ and that its contribution has the right sign. The next states are the Y_0^*'s at 1405, 1520, 1670($\Lambda\eta$), 1700, 1815 ... and the Y_1^*'s at 1385, 1660, 1765 ... Since all Y_0^*'s have negative contributions to (13) while the Y_1^*'s contribute positively, we believe that the sign, at least, does not change and that the sum rule may be satisfied. This will be tested, of course, only when the Σ^0 lifetime and the Y^* radiative decay widths are measured.

7. We can generalize our approach to SU(3) and assume that meson trajectories in the $\underline{27}$ representation have $\alpha_{27}(0) < 0$. This immediately leads to many new sum rules which we can now write not only for I = 2 t-channel amplitudes but also for I = 0 and I = 1, provided that they belong to the $\underline{27}$. From a long list of sum rules which can be derived in

this way, we mention here only two examples which deal with the $I = 0$ part of the $\underline{27}$:

$$\frac{2\alpha}{m_\Sigma} - \frac{3\alpha}{m_N} - \frac{3\alpha}{m_\Xi} = \frac{1}{2\pi^2}\int_0^\infty \left[3\sigma(p)+3\sigma(n)+3\sigma(\Xi^0)+3\sigma(\Xi^-)-9\sigma(\Lambda)-\sigma(\Sigma^+)-\sigma(\Sigma^0)-\sigma(\Sigma^-)\right]d\nu \quad (14)$$

$$-\frac{6\alpha}{m_K} + \frac{2\alpha}{m_\pi} = \frac{1}{2\pi^2}\int_0^\infty \left[6\sigma(K^+)+6\sigma(K^0)-9\sigma(\eta)-2\sigma(\pi^+)-\sigma(\pi^0)\right]d\nu \quad (15)$$

where $\sigma(x) \equiv \sigma_{\gamma x}(\nu)$. If $\alpha_{27}(0) < 0$ for <u>all</u> components of the $\underline{27}$, the integrals will converge and the sum rules may be obeyed. One should remember, however, that <u>small</u> deviations from SU(3) symmetry may follow a pattern that will prevent (14) and (15) from converging. We should, therefore, regard these sum rules as highly speculative. Needless to say, we cannot do too much about comparing these sum rules with experiment.

8. Our new sum rules may be regarded as complementary to those derived from current commutation relations, PCAC and the superconvergent dispersion relations. Taken together, they may lead to a better understanding of the algebraic structure of the complete set of sum rules with or without saturation. They may also enable us to determine various strong and electromagnetic coupling strengths, by solving large sets of coupling constant equations which are derived from a combination of principles such as the ones mentioned here. A particularly good illustration of this point is provided by the π-ρ system for which we now have <u>9 independent sum rules</u>[19] for <u>forward</u> π-ρ scattering alone. It would be extremely interesting to study the self-consistency of these sum rules and the interrelations among them.

After deriving the sum rule (2) we have learned from Dr. H. Pagels that he had independently derived the same relation as well as other relations by using quark model assumptions for high energy scattering.

Footnotes and References

1. V. de Alfaro, S. Fubini, G. Furlan and G. Rossetti, Phys. Letters 21, 576 (1966).
2. See e.g. H. Harari, Phys. Rev. Letters 17, 1303 (1966), B. Sakita and K. C. Wali, Phys. Rev. Letters 18, 31 (1967), P. Babu, F. J. Gilman and M. Suzuki, Phys. Letters, to be published.
3. The only directly measurable process which allows $I = 2$ in the t-channel is $\pi + N \rightarrow \pi + N^*(1238)$. The experimental data above 4 BeV (which still have very large errors) are consistent with no $I = 2$ contribution. Another indication for the absence of $I = 2$ contributions is the success of the so-called "weak" Johnson-Treiman relation which implies that the full 27 representation of SU(3) is absent in the t-channel.
4. The $I = 2$ t-channel amplitude involves only isovector $(G = +1)$ photons. Such photons, when scattered on pions, can form only $G = -1$ intermediate states.
5. A discussion of the different models which suppress $\varphi \rightarrow \pi\gamma$ is given e.g. in H. Harari, SLAC-PUB-239, to be published in Phys. Rev.
6. We estimate the error introduced by the narrow resonance approximation in this case as smaller than 10% in view of the small width of the ω and its distance from the $\pi\gamma$ threshold.
7. A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz and M. Roos, UCRL8030, August 1965, corrected April 1966.
8. Examples can be found in S. Adler, Phys. Rev. 140, B739 (1965), G. Segre and J. D. Walecka, Annals of Physics, to be published.
9. We have used $f_\rho^2/4\pi = 2.5 \pm 0.4$ as given by J. J. Sakurai, Phys. Rev. Letters 17, 1021 (1966).

10. The partial width $\Gamma(A_2 \to \pi\gamma)$ is also important if we want to Reggeize the photoproduction of charged pions. The A_2 and ρ will be the leading trajectories.

11. N. Cabibbo and L. Radicati, Phys. Letters 19, 697 (1966).

12. C. W. Akerlof et al, Phys. Rev. Letters 16, 147 (1966).

13. R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 145 (1965), B. W. Lee, Phys. Rev. Letters 14, 676 (1965).

14. Another mysterious factor of 2 distinguishes between the result obtained from ρ-dominance of the Adler sum rule for π-π scattering and the result which follows from ρ-dominance of the $a_1 - a_3$ πN s-wave scattering length. See J. J. Sakurai, Phys. Rev. Letters 17, 552 (1966); M. Ademollo, to be published.

15. C. Becchi and R. Morpurgo, Phys. Rev. 140, B687 (1965), W. E. Thirring, Phys. Letters 16, 335 (1965).

16. Segre and Walecka, reference 8. In equation (9), and throughout our paper, we neglect terms of order $(\frac{m_\pi}{m_\omega})^2$.

17. M. Gell-Mann, D. Sharp and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

18. G. Segre and S. Fubini, Nuovo Cimento 45, 641 (1966).

19. These are the two superconvergence relations of reference 1; an additional superconvergence relation of the type $\int \nu \, \mathrm{Im}\, f(\nu)\, d\nu = 0$; two independent Adler-Weisberger sum rules; two independent forward dispersion relations for the $I = 2$ non-spin-flip amplitude supplemented by the PCAC and current algebra calculation of the s-wave scattering lengths; the Cabibbo-Radicati sum rule (4) and our Eq. (2) supplemented by ρ-dominance of the electromagnetic current.