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W BOSON CONTRIBUTION TO THE ANOMALOUS MAGNETIC MOMENT OF THE MUON*

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ABSTRACT

The contribution of the intermediate vector boson to the muon anomalous moment is calculated in the ξ-limiting formalism. We find

$$\Delta\kappa_\mu = \Delta\left(\frac{g_\mu - 2}{2}\right) = \left(g_W^2/8\pi^2\right)\left(M_\mu/M_W\right)^2\left[2\left(1-\kappa_W\right)\ln\xi + 7/3\right]$$

where $\sqrt{2}\left(g_W/M_W\right)^2 = G_F$ is the Fermi weak coupling constant and κ_W is the W anomalous moment. For $\kappa_W = 0$, and ξ chosen according to the prescription of Lee, this contribution is numerically $\Delta\kappa_\mu \cong -1.0 \times 10^{-8} \cong -0.8\left(\frac{\alpha}{\pi}\right)^3$.

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I. INTRODUCTION

In view of the increasing accuracy of current and projected measurements[1] of the muon's magnetic moment κ_μ, it is important to have reliable estimates of the weak interaction contribution.[2] In fact, if the intermediate vector boson exists, one obtains a contribution to the anomalous magnetic moment that is first order in the Fermi weak coupling constant G_F and independent of the W mass. This contribution is of order $(G_F M_N^2)(M_\mu/M_N)^2$ and is numerically the same size as the strong interaction contributions via vacuum polarization to κ_μ, which are of order[3] $(\alpha^2/3\pi)(M_\mu/M_\rho)^2$, and the sixth order electrodynamic corrections[4,5] of order $(\alpha/\pi)^3$. In contrast, the direct four field Fermi interaction contributes negligibly to κ_μ in order G_F^2.

Actually several theoretical calculations of the intermediate vector boson contribution to the muon's anomalous magnetic moment have already been given.[6] However, in view of their mutual disagreement (see our Appendix A) we are presenting here yet another calculation of this quantity. The special features and checks included in our calculation, which we feel make it especially credible, are discussed below in Section III.

It should be remarked that the W contribution to κ_μ is model-dependent since an arbitrary procedure must be used to cut off a logarithmically-divergent integral. The coefficient of this term is, however, unambigous. Our results for this coefficient agree with that of Shaffer[6] for the case of zero anomalous moment for the W. Our choice for regularizing the divergence is the gauge-invariant ξ-limiting formalism of Lee and Yang[7] which modifies both the propagators and electromagnetic vertex of the W.

II. CALCULATION

The intermediate boson contribution to the muon's electromagnetic vertex is obtained from the Feynman diagram of Fig. 1. According to the rules listed in Appendix B the matrix element is[8]

$$\mathcal{M}_\mu = - \frac{ieg_W^2}{(2\pi)^4} \int \frac{d^4k}{i} \; \bar{u}(p+Q) A_\mu u(p-Q) \tag{1}$$

where

$$A_\mu = \gamma_\beta (1+\gamma_5)(\not p - \not k)^{-1} \gamma_\alpha (1+\gamma_5) P^{\beta\tau}(k+Q) V_{\mu\sigma\tau} P^{\sigma\alpha}(k-Q) \tag{2}$$

$$V_{\mu\sigma\tau} = g_{\sigma\tau} 2k_\mu - g_{\sigma\mu}(k-Q)_\tau - g_{\tau\mu}(k+Q)_\sigma + 2\kappa_W \left(g_{\sigma\mu} Q_\tau - g_{\tau\mu} Q_\sigma \right)$$

$$+ \xi \left[g_{\sigma\mu}(k+Q)_\tau + g_{\tau\mu}(k-Q)_\sigma \right], \tag{3}$$

$$P^{\beta\tau}(\ell) = \frac{1}{\ell^2 - M_W^2} \left[g^{\beta\tau} - \ell^\beta \ell^\tau \frac{1-\xi}{M_W^2 - \xi \ell^2} \right], \tag{4}$$

$$Q^\mu = 2q^\mu$$

and

$$\sqrt{2}\, g_W^2 = G_F M_W^2 \cong 10^{-5} (M_W/M_N)^2 .$$

After the d^4k integration, $\mathcal{M}_\mu$ must take the form[9] $(M \equiv M_\mu)$

$$\mathcal{M}_\mu = - ie\, \bar{u}(p+Q) \Big[F_1(q^2)\gamma_\mu + F_2(q^2) \frac{i}{2M} \sigma_{\mu\nu} q^\nu$$

$$+ F_3(q^2) \gamma_5 (\gamma_\mu q^2 - \not q q_\mu) \frac{1}{4M_W^2} \Big] u(p-Q) \tag{5}$$

$$\equiv - ie\, \bar{u}(p+Q) J_\mu u(p-Q) ,$$

because of current conservation and CP invariance. $F_1(0)$ is part of the charge renormalization and $F_2(0)$ is the contribution to the anomalous moment that we seek

$$\Delta \kappa_\mu = F_2(0) = \Delta\left(\frac{g_\mu - 2}{2}\right) . \tag{6}$$

The form factors F_1 and F_2 can be obtained by the identities

$$\mathrm{Tr}\left\{\Lambda_j^\mu (\not p + Q + M)\ J_\mu (\not p - \not Q + M)\right\} = F_j(q^2) , \quad j = 1,2 \tag{7}$$

where

$$\begin{aligned} \Lambda_1^\mu &= \frac{1}{16\left(M^2 - Q^2\right)^2}\left[-(M^2 - Q^2)\gamma^\mu + 3M p^\mu\right] \\ \Lambda_2^\mu &= \frac{1}{16Q^2\left(M^2 - Q^2\right)^2}\left[M^2(M^2 - Q^2)\ \gamma^\mu - M(M^2 + 2Q^2)\, p^\mu\right]. \end{aligned} \tag{8}$$

Thus

$$F_j(q^2) = \frac{g_W^2}{(2\pi)^4}\int \frac{d^4k}{i}\ \mathrm{Tr}\left\{\Lambda_j^\mu (\not p + \not Q + M)\ A_\mu (\not p - \not Q + M)\right\} \tag{9}$$

We compute all traces by computer.[10]

Upon integration, we obtain

$$\Delta\kappa_\mu = \frac{g_W^2}{8\pi^2}\left(\frac{M_\mu}{M_W}\right)^2 \left\{2(1 - \kappa_W)\ln \xi + \frac{7}{3}\right\} . \tag{10}$$

In writing Eq. (10) terms of higher order in M_μ^2/M_W^2 and ξ have been neglected. The fact that $\Delta\kappa_\mu$ remains logarithmically divergent when $\kappa_W \neq 0$, instead of the expected quadratic divergence, is due to an accidental cancellation.

An attractive choice for the value of the cutoff ξ has been given by Lee.[11] He proposes that when all electrodynamic radiative corrections to Fig. 1 are included $\Delta\kappa_\mu$ will be finite for $\xi \to 0$. With this assumption one obtains for the case $\kappa_W = 0$,

$$\Delta\kappa_\mu = \frac{g_W^2}{8\pi^2}\left(\frac{M_\mu}{M_W}\right)^2\left[2\,\ell n\,\alpha + \frac{7}{3} + G(0)\right] \qquad (11)$$

$$\cong -1.0 \times 10^{-8} \cong -0.8\left(\frac{\alpha}{\pi}\right)^3 \qquad (12)$$

where G(0) is an unknown constant which can be obtained only by summing the leading divergence of all higher order electrodynamic graphs. Presumably G(0) is of order 1 as has been assumed in writing Eq. (12).

Alternatively one might choose a value for ξ based on intuitive physical arguments. One identifies $\xi^{-1} = (\Lambda/M_W)^2$ where Λ is the regulator mass in the usual cutoff approach. Choosing $\Lambda \cong 300$ BeV, which is the energy at which the four field interaction violates unitarity, and is therefore presumably an upper limit for the cutoff, and taking $M_W \cong 2$ BeV, the present lower limit to the W mass,[12] one finds a result that is a factor of 2 larger than Eq. (12).

III. CONCLUSION

As stated in the introduction, our result disagrees with all previous calculations except that in the case $\kappa_W = 0$ our $\ell n\xi$ coefficient confirms the result of Shaffer.[6] Appendix A lists these previous results.

As a check of our expression, Eq. (2), and our use of the computer trace program we have set $M_\mu = 0$ and projected $F_1(q^2)$ from Eq. (9) and find the leading $\ell n\,\xi$ term in the neutrino's electrodynamic form factor in agreement with the result of Bernstein and Lee,[13] and Meyer and Schiff.[14] In addition we note that for the case $\kappa_W = 1$ the logarithmically divergent term cancels from Eq. (10). This same behavior was obtained in the $\mu^\pm \to e^\pm + \gamma$ calculations which were carried out in the (now disproven) one neutrino theory. Finally, the trace routine[10] employed by us has been extensively checked in many other calculations.[16]

There are theoretical reasons to believe that the anomalous moment κ_W of the W must in fact be zero. The assumption of minimal electrodynamic coupling and the Lagrangian given in Ref. 7 leads to the most convergent set of Feynman rules (listed in Appendix B for reference). If we accept this version of the minimal interaction principle for the W, then any anomalous moment would arise only from dynamics. The dynamics would at the same time generate form factors for the W's anomalous moment interactions. A further argument that $\kappa_W = 0$ is given by Bernstein and Lee.[13] They show that the assumption of $\kappa_W \neq 0$, together with the assumption that the sum of all radiative corrections is finite for $\xi \to 0$, would imply a neutrino charge radius that is independent of α. The inclusion of a possible quadrupole moment for the W, which we have everywhere neglected, leads to an even more unacceptable behavior. However, we understand so little about the W boson (the origin of its large mass, for example), that we cannot afford to be too prejudiced about its behavior.

Finally we wish to remark that even if nature has chosen some way other than the W boson model to give structure to the four field Fermi interaction, one should expect weak interaction corrections to κ_μ that are of first order in G_F and quite possibly of the same order of magnitude as the result in Eq. (12).

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APPENDIX A

We list below previous calculations[6] of $\Delta\kappa_\mu$ expressed in the notation of this paper. Here $\Delta\kappa_\mu$ is the part of the anomalous magnetic moment of the muon which arises from its first order weak interactions with the W boson. The units of $\Delta\kappa_\mu$ are muon magnetons, $e\hbar/2M_\mu c$. For convenience we have defined

$$A = \left(\frac{g_W^2}{8\pi^2}\right)\left(\frac{M_\mu}{M_W}\right)^2 = \frac{1}{8\pi^2}\left(\frac{G_F M_N^2}{\sqrt{2}}\right)\left(\frac{M_\mu}{M_N}\right)^2 \cong 1.0 \times 10^{-9} \quad .$$

<u>For zero anomalous moment of the W, $\kappa_W = 0$:</u>

Byers and Zachariasen:[17]	$\Delta\kappa_\mu = -A\left\{\frac{1}{8}\ln\xi\right\}$
Amado and Holloway:[17]	$\Delta\kappa_\mu = -A\left\{\frac{1}{2}\ln\xi\right\}$
Segré:	$\Delta\kappa_\mu = -A\left\{3\ln\xi + 1\right\}$
Meyer and Schiff:	$\Delta\kappa_\mu = A\left\{\ln\xi + 1\right\}$
Pietschman:	$\Delta\kappa_\mu = A\left\{4\ln\xi + \frac{10}{3}\right\}$
Schaffer:	$\Delta\kappa_\mu = A\left\{2\ln\xi + \frac{10}{3}\right\}$
Brodsky and Sullivan: (this paper)	$\Delta\kappa_\mu = A\left\{2\ln\xi + \frac{7}{3}\right\}$

<u>For $\kappa_W \neq 0$:</u>

Segré:	$\Delta\kappa_\mu = -A\left\{(3 + 2\kappa_W)\ln\xi + 1 + 4\kappa_W\right\}$
Brodsky and Sullivan (this paper)	$\Delta\kappa_\mu = A\left\{2(1-\kappa_W)\ln\xi + \frac{7}{3}\right\}.$

APPENDIX B

For reference, we give the Feynman rules for the vector boson, using the ξ-limiting regularization, in the metric

$$g^{\mu\nu} = (1, -1, -1, -1) \quad .$$

The rules for the propagators and vertices illustrated in Fig. 2 are:

1. Vector boson propagator:

$$-i\left\{g^{\mu\nu} - m^{-2}k^{\mu}k^{\nu})(k^2 - m^2 + i\epsilon)^{-1} + (m^{-2}k^{\mu}k^{\nu})(k^2 - \xi^{-1}m^2 + i\epsilon)^{-1}\right\}$$

2. Photon propagator:

$$-i\, g^{\mu\nu}\, (q^2 + i\epsilon)^{-1}$$

3. Fermion propagator (the positive charged fermion is taken to be the particle):

$$i\,(\not p + m)\,(p^2 - m^2 + i\epsilon)^{-1}$$

4. Antineutrino propagator:

$$i\not p\,(p^2 + i\epsilon)^{-1}$$

5. 1-photon W^+ vertex:

$$ie\left\{g^{\alpha\beta}(k_1 + k_2)^{\mu} - g^{\alpha\mu}(k_1 + \kappa_W k_1 - \kappa_W k_2 - \xi k_2)^{\beta} - g^{\beta\mu}(k_2 + \kappa_W k_2 - \kappa_W k_1 - \xi k_1)^{\alpha}\right\}$$

6. 2-photon W^+ vertex:

$$-ie^2\left\{2g^{\mu\nu}g^{\alpha\beta} - (1-\xi)\,g^{\alpha\mu}g^{\beta\nu} - (1-\xi)\,g^{\alpha\nu}g^{\beta\mu}\right\}$$

7. $\text{Muon}^+ \rightarrow W^+$ + antineutrino vertex:

$$-ig_W\,\gamma^{\mu}\,(1 + \gamma_5) \quad .$$

The Lagrangian and derivations are given in Ref. 7.

FOOTNOTES AND REFERENCES

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4. Actually the sixth order $(\alpha/\pi)^3$ corrections can be enhanced by factors of $\left(\ell n\,(M_\mu/M_e)\right)^2$ and $\ell n\,(M_\mu/M_e)$. These have been studied by S. D. Drell and J. S. Trefil; unpublished.

5. The present ± 9 ppm uncertainty in the fine structure constant introduces an uncertainty $\delta\kappa_\mu = \delta(\alpha/2\pi) = \pm 0.8\,(\alpha/\pi)^3$.

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8. The notation is that of J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill Book Company, Inc., New York, 1964). It should be noted that the one-photon vertex of the vector boson is given with the wrong sign in this reference. Our normalization for $\mathcal{M}_\mu$ is such that the muon vertex in lowest order is $\mathcal{M}_\mu = -ie\,\bar{u}(p+Q)\,\gamma_\mu\,u(p-Q)$.

9. See, for example, H. Pietschmann, Ref. 6. The two other diagrams in which the photon couples to the muon line make a contribution only to $F_1(0)$ and may be ignored here.

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16. R.G. Parsons, private communication.

17. These authors computed only the leading logarithmic divergence.

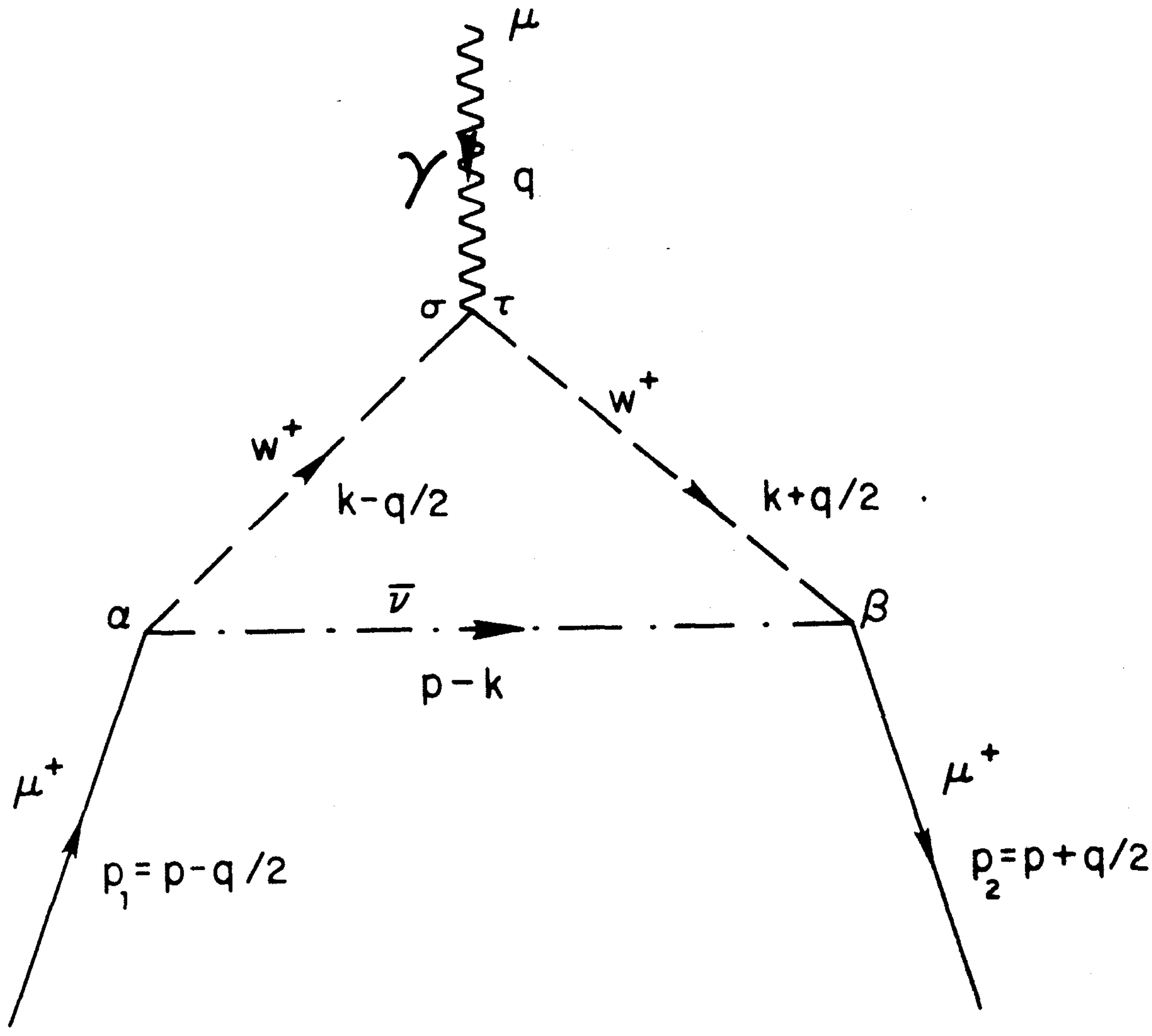

Fig. 1

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The Feynman diagram which gives the W boson contribution to the muon anomalous moment.

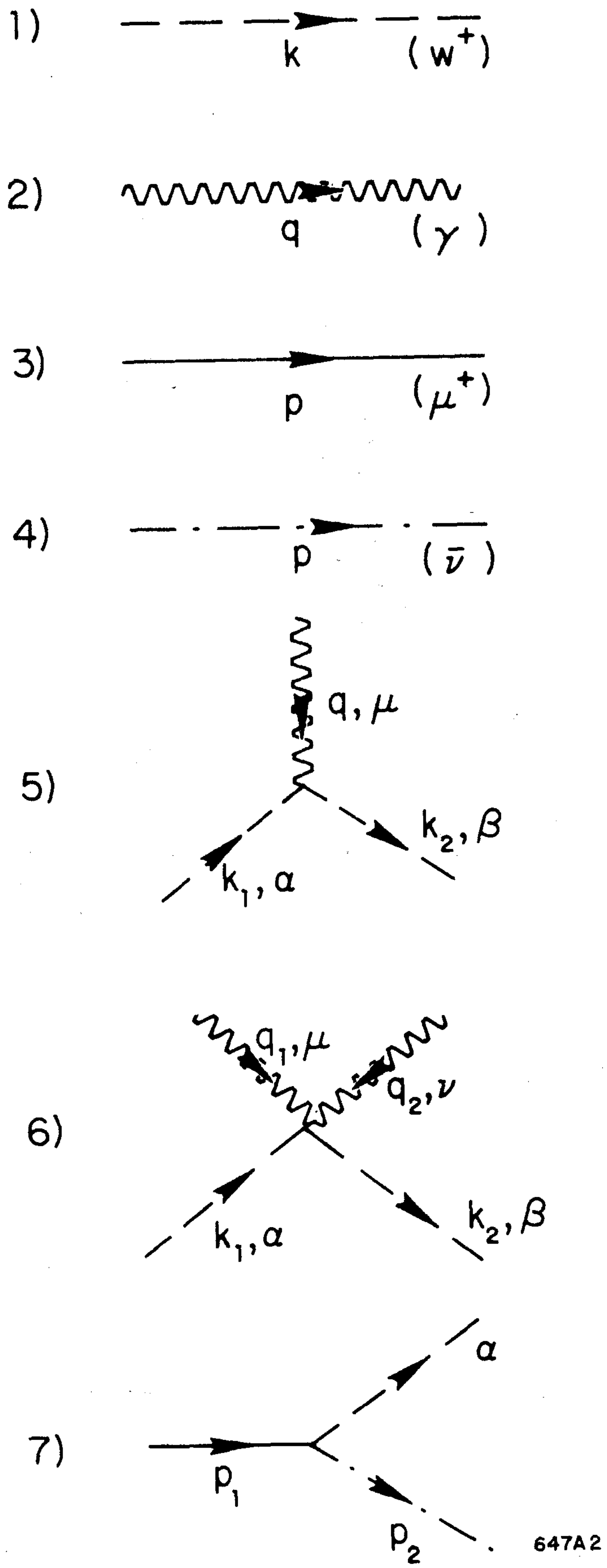

Fig. 2

Propagators and vertices for the W boson, the photon, the muon and the antineutrino.