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THE ABSOLUTE VALUE OF $A_{xx}(90^o)$ FOR PROTON-PROTON SCATTERING AT 11.4 MeV

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ABSTRACT

Because of the deterioration of the target under proton bombardment, Catillon, Chapellier, and Garreta normalize their measurements of $A_{xx}(90^o)$ and $A_{yy}(90^o)$ at 11.4, 19.15, 23.45, and 26.5 MeV to the value of $A_{xx}(90^o)$ at 11.4 MeV. In order to provide an absolute normalization for these experiments we note that at 90^o the value of A_{xx} is related to the ratio A_{yy}/A_{xx} by the equation $1 + A_{xx} = (1-A_{yy}/A_{xx})/\left[(|c|^2/|e|^2) + 1 - A_{yy}/A_{xx}\right]$ where c and e are two of the four triplet p-p helicity amplitudes defined by Raynal. This result is independent of the singlet amplitude, and further, the unknown ratio $|c|/|e|$ is shown to be well approximated by $(1+r)/(1-r)$, where r is $5/6$ of the ratio of $L\cdot S$ to tensor contributions of the ^3P phase shifts. Since we know from p-p phase shift analyses at higher energy that the $L\cdot S$ interaction is of short range compared to the known one-pion-exchange tensor interaction, we expect this ratio to be small. We believe conservative limits at this energy are set by existing p-p models fitted to the higher energy scattering, and in this way find r to lie between 0.087 and 0.162. The corresponding uncertainty in A_{xx} is $\pm$ 0.0019 which is negligible compared to the experimental uncertainty of $\pm$ 0.013 in the measurement of A_{yy}/A_{xx}. We conclude that $A_{xx}(90^o) = -0.984 \pm 0.009$ at 11.4 MeV.

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The development of a polarized beam and target at Saclay has allowed Catillon, Chapellier, and Garreta[1] to measure the two proton-proton scattering parameters $A_{xx}(90^o)$ and $A_{yy}(90^o)$ defined by Raynal[2] to high statistical accuracy at 11.4, 19.15, 23.45, and 26.5 MeV. Unfortunately it is difficult to determine the absolute polarization of the target to comparable precision, and due to radiation damage, this polarization deteriorates continuously and significantly during the course of a single run. They have therefore adpoted the expedient of dropping the beam energy back to 11.4 MeV every five seconds, and quoting all results as ratios to the value of $A_{xx}(90^o)$ at that energy. This means that for phase shift analyses at a single energy, only the ratio A_{yy}/A_{xx} is directly available, and even in energy-dependent analyses, one must refer measurements at one energy to a normalization at another, which requires non-trivial logic modifications in existing computer codes.[3] On both counts, therefore, a reliable absolute value of $A_{xx}(90^o)$ at 11.4 MeV is highly desirable.

It has already been noted by Catillon[4] that to a first approximation one has $1 + A_{xx} \approx 1 - A_{yy}/A_{xx}$ if the $L \cdot S$ scattering is small compared to the tensor scattering in the ^3P states. In this paper we replace this approximation by an exact result, and make use of theoretical arguments to limit the uncertainty in evaluating this result. As Raynal[2] shows [Eq. (14a) of R], the triplet helicity amplitudes b and d vanish at 90° c.m., and hence at this angle [Eq. (20) of R] we have

$$4\left(\frac{d\sigma}{d\Omega}\right) = |a|^2 + |c|^2 + |e|^2 \tag{1a}$$

$$4\left(\frac{d\sigma}{d\Omega}\right)A_{xx} = -|a|^2 - |c|^2 + |e|^2 \tag{1b}$$

$$4\left(\frac{d\sigma}{d\Omega}\right)A_{yy} = -|a|^2 + |c|^2 + |e|^2 \tag{1c}$$

$$4\left(\frac{d\sigma}{d\Omega}\right)A_{zz} = -|a|^2 + |c|^2 - |e|^2 \tag{1d}$$

By subtracting (1b) from (1c), rearranging the right hand side to correspond to (1a) + (1d), and dividing by $4(d\sigma/d\Omega)$, we obtain immediately

$$A_{yy} - A_{xx} = 1 + A_{zz} \tag{2}$$

Similarly, if we take the ratio of (1a) + (1b) to (1a) + (1d), we find [using Eq. (2)]

$$1 + A_{xx} = \frac{|e|^2}{|c|^2}\,(1 + A_{zz}) = \frac{|e|^2}{|c|^2}(A_{yy} - A_{xx}) \tag{3}$$

If we now factor A_{xx} out of the right hand side of (3) to obtain the experimentally measured ratio A_{yy}/A_{xx} and solve for $1 + A_{xx}$, we find

$$1 + A_{xx} = \frac{1 - A_{yy}/A_{xx}}{|c|^2/|e|^2 + 1 - A_{yy}/A_{xx}} \tag{4}$$

We emphasize that this is an <u>exact</u> equation at 90^o c.m., and note that it is independent of the singlet amplitude $|a|^2$.

In order to evaluate c and e we first note that any central interaction gives zero contribution at 90^o c.m. in the triplet-odd states so that coulomb and vacuum polarization effects play no role, and that at this low energy states with $J>2$ can be reliably estimated from one-pion-exchange to make negligible contribution. Under these restrictions we find [from Eq. (12) of R] that in terms of the nuclear-bar phase parameters defined by Stapp[5]

$$
\begin{aligned}
e^{-2i\sigma_1} k\, c(90^\circ) &= \frac{3}{2} i \left(1 - e^{2i\delta_{1,1}}\right) - \frac{3}{2} i \left(1 - e^{2i\delta_{1,2}} \cos 2\epsilon_2\right) \\
&\quad + \sqrt{6}\sin 2\epsilon_2\, e^{i(\delta_{1,2}+\delta_{3,2}+\phi_{32})} - i\left(e^{2i\phi_{32}} - e^{2i(\delta_{3,2}+\phi_{32})} \cos 2\epsilon_2\right) \\
e^{-2i\sigma_1} k\, e(90^\circ) &= i\left(1 - e^{2i\delta_{1,0}}\right) - i\left(1 - e^{2i\delta_{1,2}} \cos 2\epsilon_2\right) \\
&\quad - \sqrt{6}\sin 2\epsilon_2\, e^{i(\delta_{1,2}+\delta_{3,2}+\phi_{32})} - \frac{3}{2} i\left(e^{2i\phi_{32}} - e^{2i(\delta_{32}+\phi_{3,2})} \cos 2\epsilon_2\right)
\end{aligned}
\tag{5}
$$

with

$$
\phi_{32} = \sigma_3 - \sigma_1 = \tan^{-1} n/2 + \tan^{-1} n/3\,, \quad n = e^2/\hbar v_{LAB}
$$

Although we will evaluate the exact expression below, it is instructive to consider first the approximation given by keeping only terms linear in the $^3P_{0,1,2}$ phase shifts, which is

$$
|c|/|e| = 3(\delta_{1,1} - \delta_{1,2})/2(\delta_{1,0} - \delta_{1,2}) + 0(\epsilon_2, \delta_{3,2}, \delta^3_{1,J}) \tag{6}
$$

It has been noted by Gammel and Thaler[6] that, to the same order, the ^3P phase shifts can be expressed in terms of the contributions from the central, tensor, and spin-orbit interactions by the relations

$$
\begin{aligned}
\delta_{1,0} &= \Delta_c + 4\Delta_T - 2\Delta_{LS} \\
\delta_{1,1} &= \Delta_c - 2\Delta_T - \Delta_{LS} \\
\delta_{1,2} &= \Delta_c + \frac{2}{5}\Delta_T + \Delta_{LS}
\end{aligned}
\tag{7}
$$

or conversely

$$
\begin{aligned}
\Delta_c &= (\delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2})/9 \\
\Delta_T &= 5(2\delta_{1,0} - 3\delta_{1,1} + \delta_{1,2})/72 \\
\Delta_{LS} &= (-2\delta_{1,0} - 3\delta_{1,1} + 5\delta_{1,2})/12
\end{aligned}
\tag{8}
$$

Hence, we find immediately that in this approximation the ratio $|c|/|e|$ is independent of the central force contribution as expected, and in fact depends only on the ratio of the $L \cdot S$ to the tensor contribution; explicitly

$$
|c|/|e| \approx (1+r)/(1-r) \quad \text{with} \quad r = 5\Delta_{LS}/6\,\Delta_T \tag{9}
$$

At first sight, we are at a low enough energy to expect that centrifugal shielding will prevent all but the long-range one-pion-exchange (OPE) interaction from being effective, in which case $r=0$. Since we know experimentally[1] that $1-A_{yy}/A_{xx} = 0.024 \pm 0.013$, this would lead us to conclude that $(1 + A_{xx})_{OPE} = 0.0234 \pm 0.013$. However, Breit and Hull[7] have shown that centrifugal shielding is incomplete even at low energy for the P waves, so we must examine this assumption in more detail. We do know from unique p-p phase shift analyses at higher energy that the $^3P_{0,1,2}$ phases exhibit the +−+ OPE tensor signature below 210 MeV, and the −−+ $L \cdot S$ signature above that energy, while the $^3F_{2,3,4}$ phases retain the +−+ OPE signature over the entire elastic scattering region, both facts showing clearly and consistently that the $L \cdot S$ interaction is of considerably shorter range than one-pion-exchange. The usual physical interpretation[8] of this fact ascribes the $L \cdot S$ interaction to the exchange of the vector mesons

(ρ, ω, ϕ), and the model of Scotti and Wong[9] incorporates this interpretation directly. However, it is not clear how quantitative one can make the connection between the $L \cdot S$ interaction range and the masses of the vector mesons, so we also consider the phenomenological $L \cdot S$ hard-core potentials given by the Yale group[10] and by Hamada and Johnston.[11] A still different model is provided by Feshbach, Lomon, and Tubis,[12] who achieve the $L \cdot S$ effect by an energy-independent boundary condition at finite radius inside the static potential computed from fourth-order perturbation theory. Because of the different fitting procedures and physical assumptions whic have gone into each of these models, we believe that these models, considered as extrapolation formulae, provide a conservative estimate of the uncertainty of a knowledge of r at 11.4 MeV; energy-dependent phase-shift fits lie within this range.[3] Phase shifts for the various models, linearly interpolated from values at neighboring energies,[13] are given in Table I, in comparison with OPE.[14]

Our final result is summarized in Table II, where we compare the exact result with the approximation given in Eq. (9). Practically all the difference between the exact value of $|c|^2/|e|^2$ and this approximation comes from including ϵ_2, and only a negligible amount from the imaginary parts of the amplitudes, ${}^3F_2$, or higher powers of $\delta_{1,J}$; the difference is at worst 6%. However, the contribution from r changes the OPE value of 1 to something between 1.4 and 1.8, so should be included. Taking the central value of the spread between the models as representative, and the spread between the models as an estimate of the theoretical uncertainty, we conclude that at 90° c.m. $A_{xx}(11.4\,\text{MeV}) = -0.984 \pm 0.009 \pm 0.0019$; the first error quoted is simply the experimental error in A_{yy}/A_{xx},[1] (divided by $|c|^2/|e|^2$) while the second is the theoretical uncertainty estimated above.

TABLE I

Phase parameter values at 11.4 MeV assumed in this analysis.

Model	$\delta_{1,0}$	$\delta_{1,1}$	$\delta_{1,2}$	ϵ_2	$\delta_{3,2}$	Δ_c	Δ_T	Δ_{LS}
Yale	4.8449°	-2.7260°	0.8332°	-.2822°	0.0176°	0.0926°	1.2986°	0.2211°
Scotti-Wong	5.0361	-2.4183	1.0514	-.3031	0.0213	0.3376	1.2762	0.2033
Hamada-Johnston	4.0987	-2.2806	0.6432	-.2332	0.0147	0.0525	1.0809	0.1550
Feshbach-Lomon-Tubis	4.9072	-2.4788	0.7375	-.2658	0.0167	0.1287	1.2492	0.1091
One-Pion-Exchange	5.2727	-3.2966	0.1311	-.2726	0.0199	-0.4401	1.4282	0.0000

TABLE II

Value of $1 + A_{xx}$ predicted using Eq. (9) (approximate) and Eq. (4) (exact).

	Approximate			Exact	
Model	$(6/5)r = \Delta_{LS}/\Delta_T$	$(1+r)^2/(1-r)^2$	$1+A_{xx}$	$\|c\|^2/\|e\|^2$	$1+A_{xx}$
Yale	0.1624	1.7709	0.0134	1.6717	0.0142
Scotti-Wong	0.1593	1.7060	0.0138	1.6156	0.0146
Hamada-Johnston	0.1393	1.6108	0.0147	1.5407	0.0153
Feshbach-Lomon-Tubis	0.0874	1.3389	0.0176	1.3118	0.0180
One-Pion-Exchange	0.0	1.0	0.0234	1.0178	0.0230

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13. For Ref. 9, values at 7.357 and 14.454 MeV were supplied us by the authors; for Refs. 10, 11, and 12, we have used values computed by P. Signell at 9.69 and 14.16 MeV, which we gratefully acknowledge. For accurate work linear interpolation is clearly not good enough, but since we are concerned here primarily with the spread in values <u>between</u> models, we believe it suffices for the current work.

14. Tabulated by B. M. Johnston and M. J. Moravcsik, UCRL-5955 (unpublished). Again, higher accuracy would be obtained by multiplying by the coulomb barrier penetration factor, but we ignore this.