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REPRESENTATION DEPENDENCE OF SPONTANEOUS SYMMETRY BREAKING*

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We demonstrate in this note that the occurrence of spontaneous symmetry breaking[1][2] depends on the representation chosen for the physical states. Thus, it may happen that spontaneous symmetry breaking occurs in one representation but does not show up in another, although both representations are physically equivalent.

A transformation $U(\alpha)$ depending on a continuous parameter α is a spontaneously broken symmetry if it leaves the action integral invariant (up to boundary terms) but changes the ground state.[3] It is assumed that the $U(\alpha)$ constitute a one-parameter group. We now adopt the symmetry breaking condition[3]

$$\frac{d}{d\alpha} < 0 \left| U^{\dagger} (\alpha) \; A \; U (\alpha) \right| 0 > \neq 0 \tag{1}$$

A is an operator and $|0>$ is the ground state (A may be a product of field operators properly smeared. For more details see [3]). It can be shown that whenever the left hand side of (1) vanishes identically for any A, the transformation

$$A \to A (\alpha) \equiv U^{\dagger} (\alpha) \; A \; U (\alpha) \tag{2}$$

may be achieved by a symmetry operation, namely by a unitary transformation in the separable Hilbert space of physical states which conserves the

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ground state[3] (For a proof in axiomatic field theory see [4]). In cases where (1) is utilized, the transformation (2) may not be unitarily implementable in a separable space. We then regard it as a unitary transformation in a non-separable Hilbert space.[3]

The infinitesimal generator Q of the transformation (2) is identified through[3][5]

$$i \frac{d}{d\alpha} A\ (\alpha) = [Q,\ A\ (\alpha)] \tag{3}$$

From this and (1) we get

$$< 0 \left| [Q,\ A\ (\alpha)] \right| 0 > \neq 0 \tag{4}$$

which makes contact with the frequently used symmetry breaking condition.[1][2][6]

Consider now Haag's treatment[7] of the BCS model[8] for a superconductor. Let $\left| \alpha > \equiv U\ (\alpha) \right| 0 >$ be the translationally invariant states of well defined phase[7], namely

$$< \alpha \left| \psi\ (z)\ \psi\ (0) \right| \alpha > = \varphi_0\ (z)\ e^{2i\alpha} \tag{5}$$

where ψ (z) is a Fermi field (field operators in this example are taken at time $z_0 = 0$ and we suppress the spin indices) and $\varphi_0(z)$ is a non-trivial function. Hence

$$< \alpha \left| [N,\ \psi\ (z)\ \psi\ (0)] \right| \alpha > = -\ 2\ \varphi_0\ (z)\ e^{2i\alpha} \tag{6}$$

N is the generator of phase transformations,

$$N = \int d^3\ x\ (\psi^\dagger\ (x)\ \psi\ (x)\ -\ n) \tag{7}$$

$n = \langle 0 | \psi^{\dagger}(x)\, \psi(x) | 0 \rangle$ being the particle density in the ground state. Eq. (6) is the symmetry breaking condition for the superconductor case, and may be used in proofs on the Goldstone theorem.[1][2]

However, we may choose another representation for the physical states, that of "sharp particle number".[7] As the ground state we may take the state $|\Omega\rangle$, for which

$$
\begin{aligned}
&\langle \Omega | \psi^{\dagger}(x_1) \ldots \psi^{\dagger}(x_n)\, \psi(y_1) \ldots \psi(y_m) | \Omega \rangle \\
&= \langle \alpha | \psi^{\dagger}(x_1) \ldots \psi^{\dagger}(x_n)\, \psi(y_1) \ldots \psi(y_m) | \alpha \rangle\, \delta_{nm}
\end{aligned}
\tag{8}
$$

The right hand side is obviously independent of α. Now, in this case

$$
\frac{d}{d\alpha} \langle \Omega | U^{\dagger}(\alpha)\, A\, U(\alpha) | \Omega \rangle \equiv 0 \tag{9}
$$

for any A. It thus follows that in the representation of "sharp particle number" no symmetry breaking is realized, and the phase transformations constitute an unbroken symmetry. This is not surprising, since breakdown of phase symmetry is not expected to appear in a representation where the "particle number" is diagonal.

However, since all representations should lead to the same physical consequences, it follows that if the symmetry breaking condition (6) leads to the appearance of massless excitations[9], such excitations are present also in the case of the "sharp particle number" representation. It is thus convenient to find an analogue of condition (6) for the latter case. Such may be achieved by use of the translationally invariant state

$|\Omega_2 >$, defined by[7]

$$|\Omega_2 > = [\varphi_0^* (z)]^{-1} \lim_{V\to\infty} \frac{1}{V} \int_V \psi^\dagger(x) \psi^\dagger(x+z) d^3 x |\Omega > \qquad (10)$$

$|\Omega_2 >$ is orthogonal to $|\Omega >$, and may be equally acceptable as a ground state. The representation of "sharp particle number" is a reducible representation of the operator algebra.[7] Here

$$< \Omega | [N, \psi(x) \psi(x+z)] |\Omega_2 > = - 2 < \Omega | \psi(0) \psi(z) | \Omega_2 > \neq 0 \qquad (11)$$

Eq. (11) now replaces the usual symmetry breaking condition (6) in proofs of the Goldstone theorem.

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[9] For additional conditions needed to guarantee their appearance see ref. [3]. Let us mention here the condition $[N,H] = 0$, H the Hamiltonian. This does not necessarily follow from the invariance of H under finite phase transformations.