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K MESON NONLEPTONIC DECAYS*

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The purpose of this letter is to re-examine the predictions of a current × current Hamiltonian in the nonleptonic decays of K mesons.

It is still not clear whether the weak Hamiltonian for nonleptonic decays is of the current × current form (universality)[1] with "dynamical" octet dominance[2] or if one has to make the extra assumption[3] that it belongs to an SU_3- octet.

For s-wave hyperon decays, Sugawara[4] and Suzuki[5] showed recently that, in the current × current picture, PCAC and the $SU_3 \otimes SU_3$ algebra lead to <u>all</u> predictions of the $\Delta I = \frac{1}{2}$ rule, up to a sign.[6]

With respect to p-waves, the situation is much more ambiguous.[7] In the simplest model, namely the "strict pole approximation,"[8] it can be shown that one gets the $\Delta I = \frac{1}{2}$ predictions for Λ and Ξ decays, and that the Σ and Lee-Sugawara triangles do <u>not</u> close although the deviations are very small.[9]

Several authors[10] have shown that in the limit $m_\pi \to 0$ where all pions are off the mass shell one finds the $\Delta I = \frac{1}{2}$ rule. This result is of course easy to understand: if all pions are reduced, one must evaluate matrix elements between the vacuum and a one meson state; therefore only the octet part can contribute.

In this letter we try to improve this result by reducing only one or two pions in the $K \to 3\pi$. Our assumptions will be the following:

(1) $$H_W^{NL} = \frac{G}{2\sqrt{2}} \left[\left(J_\mu^c\right)^+ J_\mu^c + J_\mu^c \left(J_\mu^c\right)^+ \right]$$

J_μ^c is the usual Cabibbo[11] current, i.e.,

$$J_\mu^c = \cos\theta \left(j_\mu^1 + i j_\mu^2 + j_{5\mu}^1 + i j_{5\mu}^2 \right) + \sin\theta \left(j_\mu^4 + i j_\mu^5 + j_{5\mu}^4 + i j_{5\mu}^5 \right).$$

Superscripts are unitary spin indices and j_μ^i and $j_{5\mu}^i$ are respectively the vector and axial vector parts of the current.

(2) PCAC,[12] i.e., $\partial_\mu \; j^i_{5\mu} = C\phi^i$ (i = 1, 2, 3)

with ϕ^i the pion field and[13] $C = \dfrac{-i\, m_\pi^2 \; m_N \, g_A}{g_r \, K^{NN\pi}(0)}$.

(3) The space integrals of the fourth components of vector and axial currents, i.e.,

$$F^i = \int d^3 x \;\; j^i_o(\vec{x}, o) \qquad \text{and} \qquad F^i_5 = \int d^3 x \;\; j^i_{50}(\vec{x}, o)$$

generate the algebra of $SU_3 \otimes SU_3$,[14]

(4) CP invariance and Bose statistics for the mesons,

(5) The "smoothness" assumption: the ratio of two decay amplitudes approximates closely the ratio of the same amplitudes with one pion off the mass shell ($m_\pi \rightarrow 0$),[15]

(6) Universality of weak interactions: this will be stated more precisely later on.

With the help of the standard reduction technique, these assumptions lead to the following formula which holds in the limit $m_\pi \rightarrow 0$:

$$\frac{< \pi^i \pi^j + \pi^j \pi^i}{\sqrt{2}} \left| H_W \right| K > \; = \; - \frac{m_\pi^2}{C} \frac{1}{\sqrt{2}} \left[< \pi^i \left| \left[F^j_5 \, , \, H_W \right] \right| K > + < \pi^j \left| \left[F^i_5 \, , \, H_W \right] \right| K > \right] \quad (1)$$

and a similar expression for $K \rightarrow 3\pi$.

It is well known[16] that the reduction of two pions leads to a result that depends upon the order in which the reduction is carried out. The difference of the results becomes through Jocobi's identity

$$\left[F^i, \left[F^j \, , \, H_W \right] \right] - \left[F^j, \left[F^i, \, H_W \right] \right] = \left[H_W, \left[F^i, \; F^j \right] \right]$$

and thus vanishes identically if the π's are symmetrized. Furthermore, one may also argue[17] that, in the case when one pion is reduced, symmetrization minimizes the error made by going to the limit $m_\pi \to 0$.

Equation (1) and the Wigner Eckart theorem yield the following expressions for the various decay amplitudes:[18]

$$\frac{C}{m_\pi^2} A\left(K_1^o \to \pi^+\pi^-\right) = -\frac{4\sqrt{3}}{45} A_{27} + \frac{3\sqrt{2}}{10} A_{8s}$$

$$\frac{C}{m_\pi^2} A\left(K_1^o \to 2\pi^o\right) = +\frac{\sqrt{6}}{15} A_{27} + \frac{3}{10} A_{8s}$$

$$\frac{C}{m_\pi^2} A\left(K^+ \to \pi^+\pi^o\right) = -\frac{A_{27}}{3\sqrt{3}}$$

$$\frac{C^2}{m_\pi^4} A\left(K^+ \to \pi^+\pi^-\pi^+\right) = -\frac{2\sqrt{2}}{45} A_{27} + \frac{\sqrt{3}}{10} A_{8s}$$

$$\frac{C^2}{m_\pi^4} A\left(K^+ \to \pi^o\pi^o\pi^+\right) = -\frac{\sqrt{2}}{45} A_{27} + \frac{\sqrt{3}}{10} A_{8s}$$

$$\frac{C^2}{m_\pi^4} A\left(K_2^o \to \pi^+\pi^-\pi^o\right) = -\frac{A_{27}}{15} - \frac{\sqrt{6}}{20} A_{8s}$$

$$\frac{C^2}{m_\pi^4} A\left(K_2^o \to 3\pi^o\right) = -\frac{\sqrt{6}}{30} A_{27} - \frac{3}{20} A_{8s}$$

From this follows:

$$\sqrt{2}\, A\left(K_1^o \to 2\pi^o\right) - A\left(K_1^o \to \pi^+\pi^-\right) = -2A\left(K^+ \to \pi^+\pi^o\right) \qquad (2)$$

$$2\,A\left(K^+ \to \pi^o\pi^o\pi^+\right) = A\left(K^+ \to \pi^+\pi^-\pi^+\right) \tag{3}$$

$$\sqrt{2}\,A\left(K_2^o \to 3\pi^o\right) = \sqrt{3}\,A\left(K_2^o \to \pi^+\pi^-\pi^o\right) \tag{4}$$

$$\sqrt{6}\,A\left(K^+ \to \pi^+\pi^-\pi^+\right) - \frac{m_\pi^2}{C}\,A\left(K_1^o \to \pi^+\pi^-\right) = 0 \tag{5}$$

$$\sqrt{6}\,A\left(K_2^o \to \pi^+\pi^-\pi^o\right) - \frac{m_\pi^2}{C}\,A\left(K_1^o \to 2\pi^o\right) = 0 \tag{6}$$

$$A\left(K^+ \to \pi^+\pi^-\pi^+\right) + \sqrt{2}\,A\left(K_2^o \to \pi^+\pi^-\pi^o\right) = \frac{m_\pi^4}{C^2}\,\frac{\sqrt{2}}{9}\,A_{27} \tag{7}$$

Equations (2) - (4) are obtained also by reducing only one pion in the $K \to 3\pi$ amplitudes and no pion in $K \to 2\pi$. This result shows then that our assumption (5) is consistent.

Equations (5) and (6) have been predicted by several authors but either by taking all pions off the mass shell[10] or by starting from a pure octet Hamiltonian.[19] They are not too well verified experimentally (up to 15%)[10,19] but this must be expected since amplitudes with a different number of pions off the mass shell are compared.

Equations (3) and (4) coincide with the $|\Delta I| = \frac{1}{2}$ predictions and are very well confirmed by the experiments.

Without the right hand sides, Eqs. (2) and (7) would also coincide with the $\Delta I = \frac{1}{2}$ rule. It is interesting to note that precisely these predictions of the $\Delta I = \frac{1}{2}$ rule seem to be much less well confirmed experimentally.

We note that Eqs. (2) - (4) already follow[1] from the assumptions of Bose statistics for the pions and from the absence of a $\Delta I = \frac{5}{2}$ part in the Hamiltonian.

In order to test Eqs. (2) and (7) one should be able to estimate A_{27}. It is tempting to assume universality in the coupling of the weak spurion such that

$$\frac{A^{M}_{27}}{A^{M}_{8s}} = \frac{A^{B}_{27}}{A^{B}_{8s}} \qquad (8)$$

with

$$< B' \left| \left[F^{i}_{5} , H_{W} \right] \right| B > = \alpha A^{B}_{27} + \beta A^{B}_{8s} + \alpha A^{B}_{8a}$$

$$< M' \left| \left[F^{i}_{5} , H_{W} \right] \right| M > = \alpha' A^{M}_{27} + \beta' A^{M}_{8s}$$

Even if (8) does not exactly hold it is a sensible estimate.

Calculating A^{B}_{27} / A^{B}_{8s} from s-wave hyperon nonleptonic decays[5] it becomes possible through this universality assumption to express all K decay amplitudes as functions of only one reduced matrix element. One finds[20] $A^{B}_{27} / A^{B}_{8s} = \frac{1}{11}$. This leads then to the predictions for ratios of decay amplitudes summarized in Table I.[21]

TABLE I

	Experiment	$\Delta I = \frac{1}{2}$ rule	Current × Current Hamiltonian
$\frac{A\left(K^{+} \to \pi^{+}\pi^{0}\right)}{A\left(K^{0}_{1} \to \pi^{+}\pi^{-}\right)}$	0.045 ± 0.005	0	0.04
$\frac{A\left(K^{+} \to \pi^{+}\pi^{-}\pi^{+}\right)}{2\,A\left(K^{+} \to \pi^{0}\pi^{0}\pi^{+}\right)}$	1.02 ± 0.02	1	1
$\frac{\sqrt{2}\,A\left(K^{0}_{2} \to 3\pi^{0}\right)}{\sqrt{3}\,A\left(K^{0}_{2} \to \pi^{+}\pi^{-}\pi^{0}\right)}$	1.03 ± 0.04	1	1
$\frac{A\left(K^{0}_{2} \to \pi^{+}\pi^{-}\pi^{0}\right)}{\sqrt{2}\,A\left(K^{+} \to \pi^{0}\pi^{0}\pi^{+}\right)}$	0.89 ± 0.07	1	0.9

Our conclusion is that the current × current picture agrees extremely well with <u>all</u> experimental results and thus it seems to be favored over a pure octet Hamiltonian.

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17. Due to Bose statistics.

18. The antisymmetric octet amplitude vanishes due to CP invariance.

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20. With this value, one obtains $s(\Sigma_+^+) = 0.1$ in the units of Nambu.

21. Experimental numbers are taken from G. H. Trilling: Proceedings of the International Conference on Weak Interactions, Argonne National Laboratory, 1965.