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ABSTRACT

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CORRECTIVE ELEMENTS IN THE PROPOSED SLAC STORAGE RING*

P. L. Morton

The need for skew quadrupoles to correct for the linear coupling between the two transverse modes and multipole magnets to correct for the betatron oscillation frequency shift of off-momentum particles has been investigated.

The dominant linear coupling between the vertical and horizontal motion is introduced by gradients in the electric fields, used to separate the positron and electron beam. Due to the symmetry of the electric fields to be used in the SLAC storage ring, it is important that the betatron oscillation frequencies not be separated by odd integral values.

When multipole magnets are used to correct for the momentum dependences of the betatron oscillation frequencies the fields of these magnets can drive non-linear resonances. Thus care must be used locating the multipole magnets around the ring in order to avoid driving the particle beams unstable.

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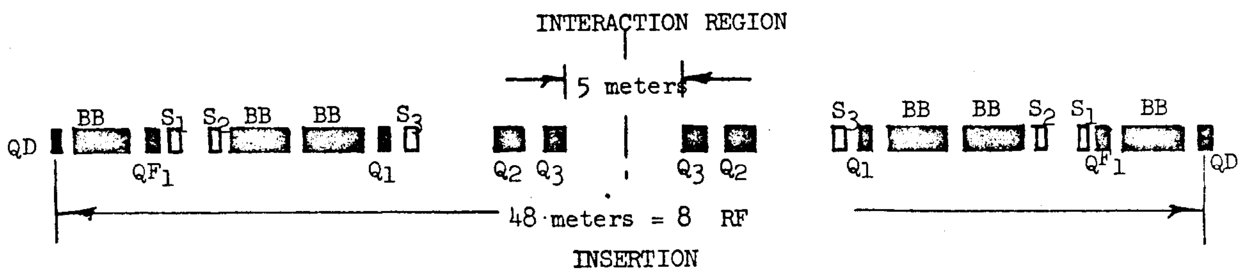
INTRODUCTION

The purpose of this paper is to determine the need for corrective elements in the SLAC storage ring. The structure of the SLAC storage ring has been described by J. R. Rees in another paper at this conference. It is made up of two superperiods--each having five normal cells and one insertion that contains the interaction region. The insertion is shown below in Figure 1 with the interaction region in the middle, BB is a bending magnet; QD, QF_1, Q_1, Q_2 and Q_3 are quadrupoles and S_1, S_2, S_3 are sextupoles.



From the interaction region to the entrance of the first bending magnet, the equilibrium orbit is independent of the particle momentum p to first order in $\Delta p/p$.

In the first section of this paper, the need for skew quadrupoles to correct for the expected linear couplings between the vertical and horizontal motion is investigated. In the second section the use of sextupole magnets to correct for the variation of betatron frequencies with momentum is discussed.

I. Linear Coupling

In order to achieve large luminosities in storage rings, it is desirable to keep the vertical size of the beam small. In electron-positron storage rings, the horizontal size of the beam is determined by quantum fluctuations, and it is important to determine the amount of coupling present that can transfer horizontal motion into vertical motion. We will let x and y represent the horizontal and vertical directions and treat the dominating linear coupling resonance in the storage ring

$$\nu_x - \nu_y \approx m \tag{1}$$

where ν_x and ν_y are the betatron oscillation frequencies, and m is an integer. In our separated function lattice the two dominating errors that can produce linear coupling are quadrupole rotation errors about the beam axis and gradient errors in the electric fields used to separate the two beams. The equations of motion including these two effects are

$$x'' + \omega_x^2 (\theta)\, x = a\, (\theta)\, y \tag{2}$$

$$y'' + \omega_y^2 (\theta)\, y = a\, (\theta)\, x \tag{3}$$

where ω_x, ω_y, and a are periodic functions of θ. For the case of small quadrupole rotation errors,

$$a = -\, 2\, \epsilon\, \frac{kR^2}{B\rho} \tag{4}$$

where k is the field gradient of the quadrupole, ϵ is the rotation angle, R is the average radius of the ring, and $B\rho$ is the magnetic rigidity of the particle. For the case of gradient errors in the electric field,

$$a = \frac{eR^2}{\beta pc}\; \frac{dE_y}{dx} \tag{5}$$

where βc, p and e are the velocity, momentum and charge of the particle, and dE_y/dx is the gradient of the electric field.

It is useful to define the strength parameter

$$M_m = \frac{1}{\pi R} \int_{2\pi} a(\theta)\, \beta_x^{1/2} (\theta)\, \beta_y^{1/2} (\theta)\, e^{im\theta}\, e^{i\varphi(\theta)}\, d\theta \tag{6}$$

where β is the usual betatron function of Courant and Snyder[1], and φ is defined as

$$\varphi(\theta) = \int_0^\theta \left[\frac{R}{\beta_x(\theta')} - \frac{R}{\beta_y(\theta')} - \nu_x + \nu_y \right] d\theta' \quad . \qquad (7)$$

If the initial y amplitude is much less than the initial x amplitude, then for the case where the coupling is entirely due to the resonance defined in equation (1), we obtain for the aspect ratio

$$A = \left(\frac{y_{max}}{\beta_y^{1/2}} \right) \left(\frac{\beta_x^{1/2}}{x_{max}} \right) = \left[\frac{|M_m|^2}{4(\nu_x-\nu_y-m)^2 + |M_m|^2} \right]^{1/2} . \qquad (8)$$

In order for the aspect ratio in the SLAC storage ring to be less than 5% for $|\nu_x-\nu_y-m| = 0.1$ it is necessary that r.m.s. rotation errors be less than 1 milliradian for the cell quadrupoles and less than 0.3 milliradian for the quadrupoles near the interaction region (Q_2 and Q_3).

The electric field used to separate the beams is antisymmetric about the interaction region so that gradient errors in the electric field can produce non-zero values for M_m only for odd m. If the betatron oscillation frequencies differ by an odd integer, the tolerances on the field gradient are too severe to be met in practice. For this reason, we have chosen to split the frequencies by 0.1 instead of 1.1. For A less than 5% this requires that $|dE_y/dx|$ be less than 0.3 kilovolts/cm^2 which corresponds to an accuracy in E_y of 3%/cm. These tolerances appear to be reasonable, so that we do not anticipate the need for skew quadrupole correcting magnets for the above mentioned effects.

II. Sextupole Corrections

The momentum spread of the injected beam in the SLAC storage ring is $\Delta p/p = \pm 1/2\%$. Without corrective elements, this momentum spread produces a spread in the betatron oscillation frequencies of $\Delta\nu_x = \mp 0.044$ and $\Delta\nu_y = \mp 0.10$. This spread in ν_x and ν_y is too large to be tolerable, and will be corrected by means of sextupole terms in the fields for all of the quadrupoles, except for the quadrupoles near the interaction region (Q_1, Q_2 and Q_3 in Fig. 1), plus the separate sextupole magnets S_1 and S_2 of Fig. 1. The reason why the separate sextupole magnets S_1 and S_2 are necessary is that there is no equilibrium orbit displacement for off-momentum particles in the quadrupoles Q_1, Q_2, and Q_3, so that an introduction of sextupole terms in the field of these quadrupoles is not effective in canceling the dependence of the betatron frequencies on momentum. The quadrupoles Q_1, Q_2 and Q_3 must be varied in order to vary the vertical beta at the interaction region, so it is important to have sextupole magnetic fields that can also be varied.

The introduction of the sextupole terms, which have a periodicity two, to cancel the dependence of betatron oscillation frequencies on momentum can drive the non-linear resonance,

$$\nu_x \approx \frac{N}{3} \tag{9}$$

where N is an even integer. For the SLAC storage ring $\nu_x \approx 5.25$ so that dominant resonance is for N = 16. We thus have included the sextupole magnets S_3 of Fig. 1 to cancel the effect of this resonance. This sextupole is in a region where there is no equilibrium orbit displacement for off-momentum particles, so that dependence of the betatron oscillation frequencies on momentum is still cancelled. If the sextupole S_3 is not used, we find that a value of $|\nu_x - N/3| = .05$ corresponds to a stability threshold for the horizontal motion of 10 cm.

BIBLIOGRAPHY

(1) E. D. Courant and H. S. Snyder, Annals of Physics 3, 1 (1958).

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