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DIVERGENCE CONDITIONS AND THE EQUAL TIME CURRENT-CURRENT COMMUTATORS*

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ABSTRACT

The equal time current-current commutation relations are deduced from equations for the divergences of the currents including electromagnetic and weak interactions, and certain assumptions for equal time commutators of electromagnetic and of weak boson fields.

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It has been shown in a previous paper[1] and, independently, by Adler and Dothan[2] that the longitudinal component of the off-mass shell pseudo-scalar meson electro-production amplitude can be determined from the equal time commutator of the vector and the axial vector currents. Furthermore, it has also been pointed out that this gauge condition can be obtained without explicit use of commutators, if we include to first order a weak interaction which gives rise to the meson decay. In this case the neutral vector current of the hadrons is no longer conserved and one finds the surprising result that the divergence of this current to first order in weak interactions is given by the equal time vector and axial vector current-current commutator. Recently Veltman[3] has introduced weak interaction contributions to the divergence of the axial current as well as to the vector current and obtained many of the results which had previously been derived from the equal time current-current commutators. The purpose of this paper is to show the fundamental connection between the equal time current-current commutators and the assumed divergence equations for the currents including first order electromagnetic and weak interactions.

We consider first the electromagnetic contributions to the divergence of the charged components of the vector current $j_\mu^{v\,\alpha}$ and the axial vector current $j_\mu^{A,\alpha}$ [2]

$$\partial^\mu j_\mu^{v,+} = -\,ie\,A^\mu j_\mu^{v,+} \tag{1}$$

$$\partial^\mu j_\mu^{A,+} = c\phi^+ - ie\,A^\mu j_\mu^{A,+} \tag{2}$$

where ϕ^+ is the charged meson field, c is a constant and A_μ is the electromagnetic potential satisfying the source equation

$$\Box A_\mu = e\left(j_\mu^{v,3} + \frac{1}{\sqrt{3}}\, j_\mu^{v,8} + j_\mu^{lept} \right) \tag{3}$$

Finally substituting in Eq. 6, Eqs. 1 and 2 for the divergence of the vector and axial vector current respectively, and using the canonical commutation relations for the electromagnetic potential

$$\left[\frac{\partial A_\mu}{\partial x_0}(x),\ A_\nu(y)\right] = -g_{\mu\nu}\ i\delta^3(x-y)$$

$$\tag{7}$$

and

$$\left[\frac{\partial A_\mu}{\partial x_0}(x),\ \varphi(y)\right] = 0$$

we arrive at the familiar current-current commutator (to zero order in e) [5]

$$\left[j_\mu^{v,3}(x) + \frac{1}{\sqrt{3}}\, j_\mu^{v,8}(x),\ j_0^+(y)\right] = j_\mu^+(x)\ \delta^3(x-y) + \frac{1}{e}\sum_{i=1}^{3}\frac{\partial}{\partial y_i}\left[\frac{\partial A_\mu}{\partial x_0}(x),\ j_i^+(y)\right] \tag{8}$$

In an entirely similar manner we obtain corresponding commutation relations between two axial vector currents if we introduce, in direct analogy with the electromagnetic potential A_μ, a weak boson field W_μ which has the weak interaction current as a source.[3]

It should ~~now~~ be clear that we can not assume the equal time commutation of $\frac{\partial A_\mu}{\partial x_0}$ with the space component of j_μ^+ vanishes, since this commutator accounts for the presence of ~~Schwinger terms~~ gradient of $\delta^3(x-y)$ terms in the current-current commutator, Eq. 8. This result can also be obtained readily if we consider, for example, the matrix elements of the divergence equations between a hadron state (a) and another hadron state (b, γ) containing a single photon. Applying the LSZ reduction formula to the photon and keeping only first order terms in e we find again Eq. 8 showing explicitly how the compensation of the gradient of $\delta^3(x-y)$ ~~Schwinger~~ terms occurs in the divergence equations.[2]

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REFERENCES

1. M. Nauenberg, Physics Letters 22, 201 (1966).

2. S. L. Adler and Y. Dothan, Caltech preprint 68-73 (to be published in Phys. Review).

3. M. Veltman, Phys. Rev. Letters 17, 553 (1966).

~~4. J. Schwinger, Phys. Rev. Letters 3, 296 (1959).~~

4. The presence of a gradient of $\delta^3(x-y)$ in the vacuum expectation value of the equal time commutator of the time ~~[illegible]~~ component with the space component of the electromagnetic current ~~was first~~ appear to have been ~~[illegible]~~ first noticed ~~[illegible]~~ by T. Goto and T. Imamura Progress of Theoretical Physics 14, 396 (1955). ~~See also J. Schwinger Phys. Letters 3, 296 (195~~ I am indebted to Professor G. Källen for calling my attention to this reference. See also J. Schwinger Phys. Review Letters 3 296 (1959)

5. We have left out of eq. 8 a term $-i\sum_{i=1}^{3} A_i(y)\left[\frac{\partial A_\mu}{\partial x_0}(x), j_i^{+}(y)\right]$ ~~on the right hand side of eq. 8~~ which is first order in e.