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POLARIZABILITY CONTRIBUTION TO THE HYDROGEN HYPERFINE STRUCTURE*

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ABSTRACT

At present there exists an apparent discrepancy of approximately 40 ± 20 ppm (parts per million) between experiment and theory for the ground state hyperfine splitting in hydrogen. The purpose of this paper is to critically examine the proton structure corrections which go into the theoretical hfs value and to try to assess and improve the accuracy to which they are known. In Section II we review the theoretical expression for the hfs and the corrections to it. The origin of the proton structure corrections is discussed along with the various physical effects: form factors, polarization, A. Bohr effect, etc. which have at various times been mentioned in regard to the hfs. In Section III we write down and solve the hfs for two non-relativistic models of proton structure. The aim of these models is to provide insight into the less transparent and much less complete relativistic calculation of the structure correction. We also discuss their relation to the recent quark model calculation of Fenster and Nambu. In Section IV we discuss the relativistic calculation of the proton structure correction, building on the recent work of Iddings. Calculation of the πN and πN^* intermediate states are presented. The problems of subtraction constants and contributions from high energy are taken up and our conclusions are presented in Section V.

I. INTRODUCTION

The hyperfine splitting in hydrogen presents an important and historic confrontation between theory and experiment. Along with positronium the hydrogen atom is the simpliest bound state system that is readily available and for which refined and detailed theoretical calculations have been and can be made. It thus affords an important test of relativistic bound state methods. The hyperfine splitting (hfs) in hydrogen is further unique in that it is sensitive to details of proton structure which are usually seen only in high energy electron scattering experiments. The hfs is therefore an important link between the usually disjoint fields of high energy and precision atomic physics.

At present there exists a discrepancy of approximately 40 ± 20 ppm (parts per million) between experiment and theory for the ground state hyperfine splitting in hydrogen.[1,2,3] There is also an apparent discrepancy with theory in the recent electron-positron pair photoproduction experiments done at the Cambridge[4] and Cornell[5] electron synchrotrons and the remeasurement of the Lamb shift by Robiscoe.[6,7] The apparent discrepancy with theory in all of these observations has refocused attention on the limits of validity of QED and places increased importance on understanding the nature of the present discrepancy in the hfs.

The purpose of this paper is to critically examine the proton structure corrections which go into the theoretical hfs value and to try to assess and improve the accuracy to which they are known. The conclusion of this paper is that hitherto uncalculated parts of the proton structure corrections may very well account for the magnitude and sign of the discrepancy but that

they do not seem to be amenable to calculation with the presently available techniques in the theory of strong interactions. The program of the paper is as follows:

Section II: We review the theoretical expression for the hfs and the corrections to it. A large class of corrections which comprise the work of many authors may be loosely described as purely radiative QED corrections. These corrections were recently reviewed and extended by Brodsky and Erickson[3] and are only briefly mentioned here. The origin of the proton structure corrections, which are the subject of this paper, is discussed along with the various physical effects: form factors, polarization, A. Bohr effect, etc. which have at various times been mentioned in regard to the hfs.

Section III: We write down and solve the hfs for two non-relativistic models of proton structure. Although these models probably bear little resemblance to reality we feel that they do contain all the physical effects which are relevant to the hfs problem and that they provide valuable insight into the less transparant and much less complete relativistic calculation of the structure correction. We also discuss their relation to the recent quark model calculation of Fenster and Nambu.[8]

Section IV: We discuss the relativistic calculation of the proton structure correction. Our work here builds on the recent work of Iddings[9] which itself builds on the work of Newcomb and Salpeter[10] and Arnowitt,[11] and of Cottingham.[12] Calculation of the πN and πN^* intermediate states are presented. We show how, in the static limit $M \to \infty$, the relativistic calculation of the structure correction goes over to the non-relativistic

Zemach[13] result.

Section V: The problems of subtraction constants and contributions from high energy are taken up. Our conclusions are presented.

II. CORRECTIONS TO THE HYPERFINE SPLITTING

The hyperfine splitting in the ground state of hydrogen has been measured to the incredible precision of 2 parts in 10^{11} by Crampton, Kleppner, and Ramsey:[2]

$$\nu = (1420.405751800 \pm 28 \times 10^{-9}) \text{ mc/sec} \tag{2.1}$$

The limit of precision to which this result can be compared with theory is set by two factors: the uncertainties in measurements of the natural constants and the uncertainties in the calculation of the corrections to the hfs calculations due to proton structure. The purely electrodynamic parts of the calculation have been carried to a higher precision than that to which these factors are known.

The total hyperfine splitting in the H ground state is given by[3]

$$\nu = \frac{16}{3} R_\infty c \left(\frac{\mu_p}{\mu_e}\right)\left(\frac{\mu_e}{\mu_o}\right)^2 \left(\frac{M}{m+M}\right)^3 \alpha^2 \left\{1 + \frac{3}{2}(Z\alpha)^2 - \alpha(Z\alpha)\left(\frac{5}{2} - \ell n 2\right) + \frac{\alpha}{\pi}(Z\alpha)^2 \left[a_o \ell n^2 Z\alpha + a_1 \ell n Z\alpha + a_2\right] - \alpha \frac{m}{M} X\right\} \tag{2.2}$$

where: $a_o = -\frac{8}{3}$; $a_1 = -\frac{37}{36} - \frac{8}{15} + \frac{16}{3}\ell n 2$; $a_2 = 18.4 \pm 5$.

In Eq. (2.2) m and M denote the electron and proton masses respectively as they will throughout this paper. The natural constants appearing in the first factor are grouped together in the way that they have been most

accurately determined.[1] All are known to better than one part in a million (ppm) except for α which was measured to an accuracy of only 9 ppm in the deuterum fine structure measurements (i.e. $2P_{3/2} - 2P_{1/2}$ splitting) by Triebwasser, Dayhoff, and Lamb.[14] In writing the correction terms in the bracket we have distinguished α and $Z\alpha$ as expansion parameters only so that binding and radiative corrections can be readily identified. The first three terms are the Breit binding correction[15] to the electron magnetic moment and the binding corrections (first and second order) to the radiative correction to the electrons electromagnetic interaction as computed by Kroll and Pollack[16] and by Karplus, Klein, and Schwinger[17] to order $Z\alpha$ and by Layzer,[18] Zwanziger,[19] and Brodsky and Erickson[3] to order $(Z\alpha)^2$. These contributions are now known to better than 1 ppm. The last term of order $\alpha \frac{m}{M}$ represents the effects of finite proton mass and structure aside from the purely kinematic reduced mass correction and must be studied if comparisons between Eqs. (2.1) and (2.2) are to be pursued to the order of 10 ppm.

In order to focus sharply on its significance we write the ratio of Eqs. (2.1) to (2.2) using Cohen and DuMond,[1]

$$\frac{\nu_{\text{theory}}}{\nu_{\text{exp.}}} = 1 - (9 \pm 20)\ 10^{-6} - \alpha \frac{m}{M} X \qquad (2.3)$$

where the uncertainty ± 20 ppm is based on a two standard deviation limit on α in Ref. 1 or a one standard deviation figure as taken from Cleland et al.[20] The question to which we address ourselves is, does the structure correction disturb the good agreement in Eq. (2.3) between experiment and

theory for a point massive proton? In studying X we must consider both the ground state average contributions as well as those arising dynamically from the polarizability of the proton. The former have been well studied and can be evaluated quite accurately in terms of the measured electromagnetic form factors of the proton. Referring to the most recent and complete study of Iddings[9] we can write

$$\alpha \frac{m}{M} X = 34 \times 10^{-6} + \frac{\Delta\nu^{(p)}}{\nu} \tag{2.4}$$

In Eq. (2.3) this gives

$$\frac{\nu_{th}}{\nu_{exp}} = 1 - (43 \pm 20) \times 10^{-6} - \frac{\Delta\nu^{(p)}}{\nu} \quad . \tag{2.5}$$

We see then that it is up to the polarizability correction $\Delta\nu^{(p)}/\nu$ to bring the theoretical number back into a clear agreement with observation.

There have been several studies[9,21,22] of $\Delta\nu^{(p)}$ all of which have come up with a negligible polarization contribution, less than 1 ppm and Eq. (2.5) stands at present as an apparent discrepancy. In analyzing this problem once more we want to shed light on the question as to whether this discrepancy should be viewed as a serious challenge to quantum electrodynamics or whether it is more properly to be viewed as a measure of the inaccuracy in our treatment of the effect of proton structure.

The specific physical idea motivating this study is the same one analyzed within the context of a massive quark model of the proton by Fenster and Nambu.[8] If the orbital electron in the hydrogen atom could completely follow the instantaneous charge and magnetization position

of the proton then the proton would appear to it as no more than a point. In this case there would be a cancellation of the finite size correction of Eq. (2.5) that introduced the discrepancy.

There is an analogous effect in deuterium, as first analyzed by A. Bohr.[23] Deuterium is a very weakly bound system and we may think of the proton and neutron as moving very slowly. The electron can thus follow the proton - i.e. by virtue of their Coulomb attraction the electron wave function at the nuclear surface can recenter on the instantaneous proton position.[24] In less graphic terms both the deuteron ground state and the electron 1S orbit are mutually polarized by mixing in higher excited state amplitudes.

The proton is less polarizable then the deuteron, its excited states lying at least m_π = 140 MeV above the ground state. However the many resonances contributing to photo and electro production cross sections are evidence that the proton is nevertheless a highly polarizable structure and there may very well be appreciable corrections to the static average ground state structure as contained in the elastic form factors.

For a proton (or deuteron) with internal dynamical degrees of freedom the Coulomb interaction with the orbital electron has off diagonal matrix elements to excited proton states. The size of the proton polarizability correction is determined by the magnitude of these off diagonal matrix elements of the Coulomb excitation as well as by the same off diagonal matrix elements of the spin dependent magnetic interaction. We therefore expect the electric as well as the magnetic multipole excitations of the proton play an important role in determining the size of $\Delta\nu^{(p)}$. Previous

studies[9,21,22] have concentrated exclusively on the magnetic dipole excitation of the 33 resonance in computing negligible contributions to $\Delta\nu^{(p)}$. We extend these considerations to a more complete study of the amplitude for electromagnetic excitation of the proton.

Sizable polarizability contributions are identified in the next section on the basis of a Schrodinger model of the proton structure. When we turn to a relativistic dispersion analysis in Section IV we follow the method of Cottingham[12] as discussed and applied to the hfs problem by Iddings.[9] This approach expresses $\Delta\nu^{(p)}$ in terms of the forward spin flip Compton scattering of virtual photons, integrated over all photon energies and space-like masses. We will find that one simply does not know enough about this virtual process to make any firm conclusions on the size of $\Delta\nu^{(p)}$. Dispersion theory is at its best when dealing with the interactions of "real" particles in a kinematic region where the absorptive amplitude is dominated by one or by at most a very few resonances. In this problem we do not have this simple circumstance and we find that many small and poorly calculable terms contribute, leading to no firm conclusion with regard to $\Delta\nu^{(p)}$. It is our belief however, derived both from the Schrodinger model considerations of Section III and the relativistic approach of dispersion theory, that one lacks the information required to infer that the discrepancy in Eq. (2.5) is at present a serious challenge to quantum electrodynamics. We know too little about proton dynamics.

III. NON-RELATIVISTIC MODEL OF PROTON STRUCTURE

We compute in this section the finite size correction to the hyperfine splitting using a non-relativistic model of proton structure. The purpose of this exercise is not to calculate a quantitatively reliable number but rather to exhibit in a concrete fashion the various physical effects which influence the hfs of a system with structure. In so far as possible the results are expressed in terms of quantities which are independent of the details of the model and which can be fitted to the observed parameters of the physical proton. In particular we want to gain some insight into the physical parameters governing the size of the polarization correction to the hyperfine splitting in hydrogen.

Our model proton is composed of a particle (called hereafter a quarkette) of charge + e and mass μ which satisfies the Schrodinger equation and is bound to a neutral, infinitely massive center of force by a non-relativistic potential $V(\underline{R})$. The infinite mass center is taken as the origin; $\underline{R}$ denotes the coordinate of the "quarkette" and $\underline{r}$ the coordinate of the electron which is bound to the physical proton to form a hydrogen atom (see Fig. 1). Depending on the choice of parity for the quarkette there are two possible cases. A positive parity quarkette is taken to have spin 1/2 and a point magnetic dipole moment. The proton is the lowest $S^{+}_{1/2}$ bound state of this quarkette. Initially we assume the quarkette to be a point particle; later we consider the possibility that the quarkette itself has finite extension. For negative parity it is most natural to consider the quarkette spinless and take the spin 1/2 as a degree

of freedom of the mass center. The proton is realized as a $P^+_{1/2}$ bound state and the potential $V(\underline{R})$ must have a spin orbit as well as a central part in order to remove the degeneracy between the $P^+_{1/2}$ ground state and a possible $P^+_{3/2}$ state. It must also have some property (perhaps a hard core) to prevent an $S^-_{1/2}$ state from being the true ground state of the proton system.

We treat the two cases of even and odd quarkette parity in Parts A and B respectively. In Part C we discuss the influence of numerical results from these models as well as their relation to a three massive quark model of the proton as analyzed recently in regard to the hfs by Fenster and Nambu.[8]

PART A

We consider in this part the positive parity quarkette model of the proton. The Hamiltonian for the full system is

$$H = H_P(\underline{R}) + H'_e(\underline{r}) + H'_c(\underline{r},\underline{R}) + H_M(\underline{r},\underline{R}) \tag{3.1}$$

where H_P is the Hamiltonian of the quarkette including the central potential $V(R)$ which binds it to the origin.

$$H_P = \frac{\underline{P}^2}{2\mu} + V(R) \tag{3.2}$$

μ is the quarkette mass and $\underline{P}$ is the momentum conjugate to $\underline{R}$. H'_e is the

free Dirac Hamilitonian for the electron which must be treated relativistically as will become clear later

$$H_e' = \underline{\alpha} \cdot \underline{p} + \beta m \ . \tag{3.3}$$

In Eq. (3.3) $\underline{\alpha}$ and β are the usual Dirac matrices; we will work in the standard representation for them; i.e.

$$\underline{\alpha} = \begin{pmatrix} 0 & \underline{\sigma}_e \\ \underline{\sigma}_e & 0 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \ .$$

H_e' contains the static electron-quarkette Coulomb interaction

$$H_e' = - \frac{e^2}{4\pi} \frac{1}{|\underline{r}-\underline{R}|} \tag{3.4}$$

and H_M which is responsible for the hfs describes the instantaneous interaction of the electron's moment with the magnetic field generated by the proton[24]

$$H_M = + \frac{e}{4\pi} \underline{\alpha} \cdot \underline{A}(\underline{r}) \tag{3.5}$$

A refinement of Eq. (3.5) to include retardation corrections to the electromagnetic interaction leads to a minor modification of our results and will be made later on. The vector potential $\underline{A}(\underline{r})$ seen by the electron is the sum of a moment and a convection current part

$$\underline{A}(\underline{r}) = - \frac{e\kappa}{2\mu} \nabla_{\underline{R}} \frac{1}{|\underline{r}-\underline{R}|} \times \underline{\sigma} + e \frac{\underline{V}}{|\underline{r}-\underline{R}|} \tag{3.6}$$

where $\underline{\sigma}$ is a Pauli matrix for the quarkette and $\frac{\kappa}{\mu} = \frac{\mu_p}{M} = \frac{2.79}{M}$ in order to

fit the total observed proton magnetic moment. As it stands Eq. (3.6) lacks a "gauge term" $-\frac{1}{2}\nabla_r \frac{\underline{V}\cdot(\underline{r}-\underline{R})}{|\underline{r}-\underline{R}|}$ which should be added to the right hand side so that the transversality condition $\nabla_{\underline{r}} \cdot \underline{A}(\underline{r}) = 0$ is satisfied. However, this "gauge term" makes no contribution to the hfs and we will drop it. Also since $\underline{V}$ and $\underline{R}$ do not commute, the convection current term of Eq. (3.6) should properly be symmetrized. Symmetrization does not alter the hfs so we shall neglect it.[24]

The Hamilitonian Eq. (3.1) is essentially a Breit Hamilitonian with one of the particles treated as very massive and then given non-relativistic internal structure. It is necessary to keep in mind that when working within the framework of the Breit theory the magnetic term H_M can appear only once in a perturbation calculation. We refer the reader to the discussion of this point to the book of Bethe and Salpeter.[25]

If one could exactly solve the Hamiltonian of Eq. (3.1), less the magnetic term H_M, exactly for the singlet and triplet ground state wave functions, $\Psi_g^{\,s}(\underline{r},\underline{R})$ and $\Psi_g^{\,t}(\underline{r},\underline{R})$ respectively, then the hyperfine splitting ν including proton structure effects to all orders would be

$$\nu = \nu_t - \nu_s = \langle \Psi_g^{\,t} | H_M | \Psi_g^{\,t} \rangle - \langle \Psi_g^{\,s} | H_M | \Psi_g^{\,s} \rangle \quad . \tag{3.7}$$

In practice it is impossible to construct these exact eigenstates in closed form because of the $\underline{r}$-$\underline{R}$ correlations and we must start an iteration solution with the electron centered at the origin $\underline{R} = 0$ of the quarkette structure. We rewrite Eq. (3.1) as

$$H = H_P(\underline{R}) + H_e(\underline{r}) + H_c(\underline{r},\underline{R}) + H_M(\underline{r},\underline{R}) \tag{3.8}$$

where

$$H_e(\underline{r}) = \underline{\alpha}\cdot\underline{p} + \beta m - \frac{e^2}{4\pi}\left(\frac{1}{r}\right) \tag{3.9}$$

and

$$H_c(\underline{r},\underline{R}) = \frac{e^2}{4\pi}\left(\frac{1}{r} - \frac{1}{|\underline{r}-\underline{R}|}\right) , \tag{3.10}$$

and treat $H_o = H_P + H_e$ exactly and the remainder $H_c + H_M$ in perturbation theory. The energies and eigenfunctions of H_e and H_P are labeled according to

$$H_e\psi_{n\pm}(\underline{r}) = E_{n\pm}\psi_{n\pm}(\underline{r}),\ n_\pm = 0,1,2\cdots \tag{3.11}$$

and

$$H_P\,\Phi_A(\underline{R}) = W_A\,\Phi_A(\underline{R}),\ A = 0,1,2\cdots \tag{3.12}$$

where $\psi_{n\pm}(\underline{r})$ are four component Dirac spinors for the positive and negative energy Coulomb solutions and the two component quarkette wave functions Φ can be further decomposed into a product of radial, orbital, and spin parts

$$\Phi(\underline{R}) = \frac{\varphi(R)}{R}\,Y(\hat{\underline{R}})\,\chi \tag{3.13}$$

where $Y(\hat{\underline{R}})$ is a spherical harmonic, χ a two component spinor and $\varphi(R)$ the

radial wave function with normalization

$$\int_0^\infty dR \left|\varphi(R)\right|^2 = 1 \ . \tag{3.14}$$

The solutions of H_o are thus simple products

$$H_o \Psi_{An\pm}(\underline{r},\underline{R}) = E_{An\pm} \Psi_{An\pm}(\underline{r},\underline{R})$$

$$\Psi_{An\pm}(\underline{r},\underline{R}) = \Phi_A(\underline{R})\ \psi_{n\pm}(\underline{r}) \tag{3.15}$$

$$E_{An\pm} = W_A + E_{n\pm} \ .$$

In particular the zeroth order approximation to the ground state wave function for hydrogen is[26]

$$\Psi_g = \Phi_o(R)\ \psi_o(\underline{r})$$

$$\Phi_o = \frac{1}{\sqrt{4\pi}} \frac{\varphi_s(R)}{R} \tag{3.16}$$

$\varphi_s(R)$ the S wave ground state radial wave function satisfying Eq. (3.14), and

$$\psi_o(\underline{r}) = \left(\alpha \frac{1}{\frac{i\sigma_e \cdot \underline{r}}{2r}}\right) \chi_e\ \psi\ (o)\ e^{-r/a_o} \tag{3.17}$$

with χ_e the 2 component electron spinor, $\psi\ (o) \equiv \sqrt{\frac{\alpha^3 m^3}{\pi}}$ and $a_o = \frac{1}{\alpha m}$ the Bohr radius; corrections of order α^2 are dropped in Eq. (3.17).

As is well known, in the product spin space of the electron and the proton the hfs reduces to the difference of the triplet and singlet spin expectation value of an operator $\sim \underline{\sigma}_e \cdot \underline{\sigma}$:

$$< \underline{\sigma}_e \cdot \underline{\sigma} >_t - < \underline{\sigma}_e \cdot \underline{\sigma} >_s = 4 < \underline{\sigma}_e \cdot \underline{\sigma} >_t \quad . \tag{3.18}$$

Hereafter, we work with the particular triplet state with both the electron and proton spins "up" along the spin axis, and write, suppressing all spin labels and terms,

$$\nu = \nu_t - \nu_s = 4\nu_t \, . \tag{3.19}$$

In first order perturbation theory the only contribution to the hfs comes from the spin part of H_M

$$\begin{aligned} \nu_1 &= 4 < \Psi_g \left| H_M \right| \Psi_g > \\ &= - 4\alpha \int d^3R \int d^3r \, \Psi_g^\dagger(\underline{r},\underline{R}) \, \frac{\mu_p}{2M} \, \underline{\alpha} \cdot \left(\underline{\nabla}_R \, \frac{1}{\underline{r}-\underline{R}} \times \underline{\sigma} \right) \Psi_g(\underline{r},\underline{R}) \, . \end{aligned} \tag{3.20}$$

The ground state expectation value of the convection current term in Eq. (3.6) has no spin dependence and does not contribute to ν in this model. Substituting Eqs. (3.16) and (3.17) we integrate over the electron coordinates with the aid of the identity

$$\int d^3r \, \psi_o^\dagger(\underline{r}) \, \frac{\underline{\alpha}}{\left|\underline{r}-\underline{R}\right|} \, \psi_o(\underline{r}) = \frac{2\pi}{3} \, \alpha \, a_o \, \psi^2(0) < \underline{\sigma}_e > \times \underline{R} \left(1 - \frac{3}{2} \frac{R}{a_o} \right) \tag{3.21}$$

This gives in Eq. (3.20)

$$\nu_1 = \nu_F \frac{1}{2} < \underline{\sigma}_e \cdot \left[\left(\underline{\nabla}_R \times \underline{\sigma} \right) \times \underline{R} \right] \left(1 - \frac{3}{2} \frac{R}{a_o} \right) >_o \tag{3.22}$$

where

$$< \cdots >_o \equiv \int d^3R \, \Phi_o^\dagger(R) \cdots \Phi_o(R) \tag{3.23}$$

denotes the proton ground state expectation value and we are instructed to evaluate the spin operators for a triplet parallel spin up state according to Eqs. (3.18) and (3.19). In Eq. (3.22)

$$\nu_F = \frac{8\pi\alpha}{3} \frac{\mu_p}{Mm} \psi^2(0) = \frac{8}{3} \alpha^4 \mu_p \left(\frac{m^2}{M} \right) \tag{3.24}$$

is the point proton (Fermi) hfs energy. Taking the spherical average in the proton ground state gives

$$\nu_1 = \nu_F \left(1 - 2 \frac{< R >}{a_o} \right) \tag{3.25}$$

where

$$< R > \equiv \int_o^\infty dR \left| \varphi_s(R) \right|^2 R \ . \tag{3.26}$$

We next consider second order perturbation theory. There are three possibilities: (1) second order in H_c, (2) second order in H_M, and (3)

mixed second order in H_c and H_M. The first involves no spin coordinates and so does not contribute to the hfs, and the second must not be computed directly, as noted earlier, when working with the phenomenological Breit Hamiltonian.

In the fully relativistic discussion in Section IV we will find that this second order magnetic dipole interaction is properly included and is only a small correction to the first order result. The mixed second order term occurs with H_c and H_M appearing in either order and gives[24]

$$\nu_2 = -\,8\,\mathrm{Re}\left\{ \sum_{\substack{A,n_+ \\ (A,n_+)\neq(0,0)}} \frac{\langle 0,0|H_M|A,n_+\rangle\langle A,n_+|H_c|0,0\rangle}{W_A - W_o + E_{n+} - E_o} - \sum_{A,n_-} \frac{\langle 0,n_-|H_M|A,0\rangle\langle A,0|H_c|0,n_-\rangle}{W_A - W_o - E_{n-} + E_o} \right\} . \tag{3.27}$$

We discuss separately the terms in Eq. (3.27) which involve only the ground state of the quarkette. Denoting this contribution by $\nu_2^{(g)}$ we have:

$$\nu_2^{(g)} = -4\,\frac{\alpha^2\mu_p}{M}\,\mathrm{Re}\left\{ \sum_{\substack{n\pm \\ n+\neq 0}} \int \frac{d^3R}{4\pi R^2}\,|\varphi_s(R)|^2 \int \frac{d^3R'}{4\pi R'^2}\,|\varphi_s(R')|^2 \int d^3r \int d^3r' \right.$$

$$\left. \left\{ \psi_o^\dagger(\underline{r}')\left[\underline{\alpha}\cdot\left[\nabla_{\underline{r}}\,\frac{1}{|\underline{r}-\underline{R}|}\times\underline{\sigma}\right]\right] \frac{\psi_{n\pm}(\underline{r})\,\psi_{n\pm}^\dagger(\underline{r}')}{E_{n\pm}-E_o} \left[\frac{1}{r'} - \frac{1}{|\underline{r}'-\underline{R}'|}\right]\psi_o(\underline{r}')\right\} \right. . \tag{3.28}$$

In arriving at the form of Eq. (3.28) we have noted that the quarkette ground state expectation value of $\underline{\alpha}.\underline{V}/\left|\underline{r}-\underline{R}\right|$ is spin independent and so does not contribute to the hfs and in view of the overall real part instruction we have freely taken the complex conjugate of the negative energy sum.

We are interested in contributions to Eq. (3.28) that are of no higher than the first power in the nuclear radius and therefore can make a series of approximations that permit us to carry out the sum over the excited electron states $n_{\pm}$. First we consider the contribution of positive energy electron bound states to the sum over states and take advantage of the spherical symmetry in $\underline{R}$ to write

$$\int d\Omega_{R'} \left(\frac{1}{r'} - \frac{1}{\left|\underline{r}'-\underline{R}'\right|}\right) = \theta(R'-r') \left(\frac{1}{r'} - \frac{1}{R'}\right) \int d\Omega_{R'} \, .$$

Only bound S state of the electron contribute in Eq. (3.28) and we find

$$\int \psi_n^{\dagger}(\underline{r}') \, \theta(R'-r) \left(\frac{1}{r'} - \frac{1}{R'}\right) \psi_o(\underline{r}') \, d^3r' \sim \frac{2\pi}{3\sqrt{\pi a_o^3}\,\sqrt{n^3}} \psi^2(0) \, R'^2 \qquad (3.29)$$

which is of order R^2. The first matrix element in Eq. (3.28) can then be evaluated in the $R \to 0$ limit and we obtain a contribution to hfs from the bound state with principal quantum number n that is approximately

$$\nu^{(g)}_{2,\text{bound}} \sim - \frac{\alpha^2 \mu_p}{M} \left|\psi(0)\right|^2 (\alpha a_o) \frac{1/n^3}{\alpha^2 m \left(1 - \frac{1}{n^2}\right)} \frac{R^2}{a_o^3} \approx \left[(R/a_o)^2 \frac{1}{n^3} \frac{1}{1-\frac{1}{n^2}}\right] \nu_F \, . \qquad (3.30)$$

The sum over all $n \neq 1$ converges and we see that the bound states contribute only in order $(R/a_o)^2$ and hence may be dropped. The low lying continuum

states of momentum $p \lesssim m$ suffer the same fate and the remaining ones can be treated in plane wave approximation up to higher order corrections in α.

$$\psi_{n\pm}(\underline{r}) \to \psi^{(\pm)}_{\underline{p}}(\underline{r}) = e^{i\underline{p}\cdot\underline{r}}\, u^{(\pm)}(\underline{p}); \qquad \left[u^{\dagger(\pm)}(\underline{p}),\, u^{(\pm)}(\underline{p})\right] = 1 \tag{3.31}$$

In this approximation we can also replace the ground state wave function by its value at $\underline{r} = 0$

$$\psi_o(r) \Rightarrow \psi\,(0)$$

and the relevant matrix elements are tabulated:

$$\int d^3r'\, \psi^{\dagger(+)}_p(\underline{r}')\left(\frac{1}{r'} - \frac{1}{|\underline{r}'-\underline{R}'|}\right) \psi_o(r') = \frac{4\pi}{p^2}\,\psi(0)\,\left[u^{(+)}(\underline{p}), u(0)\right]\left[1-e^{-i\underline{p}\cdot\underline{R}'}\right] \tag{3.32}$$

$$\int d^3r\, \psi^{\dagger}_o(r)\, \underline{\alpha}\cdot\left(\nabla_{\underline{r}}\frac{1}{|\underline{r}-\underline{R}|}\times\underline{\sigma}\right)\psi^{(+)}_p(\underline{r}) = \frac{4\pi}{p^2}\,\psi(0)\,\left[\underline{\sigma}\times\nabla_{\underline{R}}e^{i\underline{p}\cdot\underline{R}}\right]\cdot\left[u(0), \underline{\alpha}\, u^{(+)}(\underline{p})\right]. \tag{3.33}$$

The negative energy matrix elements differ only in the replacement

$$u^{(+)}(\underline{p}) \to u^{(-)}(\underline{p})\,.$$

Using

$$\sum_{n^{\pm}} = \int \frac{p^2 dp d\Omega_p}{(2\pi)^3}$$

$$\sum_{\text{spins}} u^{(+)}(\underline{p}) \left| u^{(+)}(\underline{p}) \right|^{\dagger} = \frac{\underline{\alpha}\cdot\underline{p}+\beta m+E_p}{2E_p} ; \qquad E_p = +\sqrt{p^2+m_e^2}$$

$$\sum_{\text{spins}} u^{(-)}(\underline{p}) \left| u^{(-)}(\underline{p}) \right|^{\dagger} = \frac{-\underline{\alpha}\cdot\underline{p}-\beta m+E_p}{2E_p}$$

$$E_o \approx m,$$

and retaining only spin flip terms

$$\sum_{\text{spins}} \left| u(0)\ \underline{\alpha}\ u^{\pm}(\underline{p}) \right| \left| u^{\pm}(\underline{p}),\ u(0) \right| \rightarrow \pm \frac{i\underline{p}\times\underline{\sigma}_e}{2E_p}$$

we find

$$\nu_2^{(g)} = \frac{4\alpha^2\mu_p}{M} \text{Re} \int \frac{d^3R}{4\pi R^2} \left|\varphi_s(R)\right|^2 \int \frac{d^3R'}{4\pi R'^2} \left|\varphi_s(R')\right|^2 \psi^2(0) \cdot$$

(3.34)

$$\cdot \int \frac{dp p^2 d\Omega_p}{8\pi^3} \left(\frac{4\pi}{p^2}\right)^2 \left(\underline{\sigma}\times\nabla_{\underline{R}}\right)\cdot\left(\underline{\sigma}_e\times\nabla_{\underline{R}}\right) \left(e^{i\underline{p}\cdot\underline{R}}\ e^{i\underline{p}\cdot(\underline{R}-\underline{R}')} \right) \frac{1}{2E_p} \left(\frac{1}{E_p-m} + \frac{1}{E_p+m} \right) \cdot$$

After carrying out the angular integration over $\underline{p}$ and taking the gradients, in Eq. (3.34) we have

$$\nu_2^{(g)} = - \nu_F \frac{8}{\pi a_o} < \int_o^\infty \frac{dp}{p^2} \left\{ j_o(pR) - j_o(p|\underline{R}-\underline{R}'|) \right\} >_{oo} \tag{3.35}$$

where j_o is the spherical Bessel function of zeroth order and the notation

$$<\cdots>_{AO} \equiv \int d^3R \; d^3R' \; \Phi_o^\dagger(\underline{R}) \; \Phi_A^\dagger(\underline{R}') \left\{ \cdots \right\} \Phi_A(\underline{R}) \; \Phi_o(\underline{R}') \tag{3.36}$$

has been introduced. It is convenient to split $\nu_2^{(g)}$ into two finite parts

$$\nu_2^{(g)} = \bar{\nu}_2^{(g)} + \tilde{\nu}_2^{(g)} \tag{3.37}$$

where

$$\bar{\nu}_2^{(g)} = + \nu_F \frac{8}{\pi a_o} < \int_o^\infty \frac{dp}{p^2} \left\{ 1 - j_o(pR) \right\} >_{oo} \tag{3.38}$$

and

$$\tilde{\nu}_2^{(g)} = - \nu_F \frac{8}{\pi a_o} < \int_o^\infty \frac{dp}{p^2} \left\{ 1 - j_o(p|\underline{R}-\underline{R}'|) \right\} >_{oo} \tag{3.39}$$

Using the identity

$$\int_o^\infty \frac{dx}{x^2} \left| 1 - j_o(x) \right| = \frac{\pi}{4} \tag{3.40}$$

we obtain

$$\bar{\nu}_2^{(g)} = + \nu_F \frac{2 < R >}{a_o} \tag{3.41}$$

which cancels the structure term of Eq. (3.25) and

$$\tilde{\nu}_2^{(g)} = - \nu_F \frac{2 <|\underline{R}-\underline{R}'|>}{a_o} \tag{3.42}$$

where

$$<|\underline{R}-\underline{R}'|> \equiv \int d^3R \frac{|\varphi_s(R)|^2}{4\pi R^2} \int d^3R' \frac{|\varphi_s(R')|^2}{4\pi R'^2} |\underline{R}-\underline{R}'| \quad . \tag{3.43}$$

Adding Eqs. (3.41) and (3.42) to Eq. (3.25) gives the structure correction to hfs due both to the distribution of magnetization in the proton ground state and the alteration of the electron wave function from a point Coulomb solution as a result of the proton structure. The total hfs contribution in this approximation is

$$\nu = \nu_1 + \bar{\nu}_2^{(g)} + \tilde{\nu}_2^{(g)} = \nu_F \left\{ 1 - \frac{2}{a_o} <|\underline{R}-\underline{R}'|> \right\} \tag{3.44}$$

Eq. (3.44) does not yet include any contribution arising from polarization since all excited quarkette states have been omitted in the second order perturbation treatment. For a rigid "baseball" model of the proton - i.e. a proton with the excited state spectrum far removed from the ground state to which it is only negligibly coupled by the perturbation - Eq. (3.44) is the correct result for the structure effect on hfs.

A general model independent derivation of this non-relativistic result was first given in 1956 by Zemach.[13] Specific dependence on a model such as introduced in this section is required in order to formulate the polarizability calculation which is our primary concern and to which we turn now. This involves the terms with $A \neq 0$ in Eq. (3.27) corresponding to quarkette excitation. Denoting this contribution by $\nu_2^{(e)}$ we have

$$\nu_2^{(e)} = \frac{4\alpha^2\mu_p}{M} \sum_{A\neq 0} \int d^3R\, d^3r$$

$$\times \left[\sum_{n+\neq 0} \left\{ \frac{\Psi^\dagger_g(\underline{r},\underline{R}) \left\{ \underline{\alpha}\cdot \left(\underline{\nabla}_r \frac{1}{|\underline{r}-\underline{R}|} \times \underline{\sigma} \right) \right\} \Psi_{An+}(\underline{r},\underline{R})\, \Psi^\dagger_{An+}(\underline{r}',\underline{R}') \frac{1}{|\underline{r}'-\underline{R}'|} \Psi_g(\underline{r}',\underline{R}')}{W_A - W_o + E_{n+} - E_o} \right\} \right. \tag{3.45}$$

$$\left. - \sum_{n-} \left\{ \frac{\Psi^\dagger_{on-}(\underline{r},\underline{R}) \left\{ \underline{\alpha}\cdot \left(\underline{\nabla}_r \frac{1}{|\underline{r}-\underline{R}|} \times \sigma \right) \right\} \Psi_{Ao}(\underline{r},\underline{R})\, \Psi^\dagger_{Ao}(\underline{r}',\underline{R}') \frac{1}{|\underline{r}'-\underline{R}'|} \Psi_{on-}(\underline{r}',\underline{R}')}{W_A - W_o - E_{n-} + E_o} \right\} \right].$$

Since the $1/r$ term of H_c has no matrix elements to excited quarkette states it has been dropped. Also the convection current term in Eqs. (3.5) and (3.6) does not contribute to hfs to this order and can be omitted since it contributes only a spin independent energy when inserted in Eq. (3.45). As we found in Eq. (3.28) when there was no quarkette excitation the important contributions to Eq. (3.45) come from relativistic intermediate electron energies; in this case $E_n \sim (W_A - W_o) \gg m$. For such energies

we may again replace the Dirac Coulomb states $\psi_{n\pm}$ by free particle wave functions Eq. (3.31) with a negligible error $O(\alpha)$. Furthermore, as is apparent in the form of Eq. (3.34) when the energy denominators are replaced by

$$\frac{1}{E_p \mp m}\left(\nu_2^{(g)} \to \nu_2^{(e)}\right) \frac{1}{W_A - W_o + E_p \mp m} \left(p \gg m\right) \frac{1}{W_A - W_o + p} \tag{3.46}$$

the positive and negative energy sums in Eq. (3.45) contribute equally to the order of accuracy to which we are working. Thus we need only evaluate the positive energy sum explicitly and double the result to obtain $\nu_2^{(e)}$.

Proceeding in complete analogy with the calculation of $\nu_2^{(g)}$ in Eqs. (3.28) and (3.34) we reduce Eq. (3.45) to

$$\nu_2^{(e)} = -\frac{32\alpha^2 \mu_p \psi^2(0)}{M} \sum_{A\neq 0} \int d^3R\, d^3R'\, \Phi_o^\dagger(\underline{R})\, \Phi_A^\dagger(\underline{R}')\, \Phi_o(\underline{R}')\, \Phi_A(\underline{R}) \cdot$$

$$\cdot \int_o^\infty \frac{dp}{p^3} \frac{(\underline{\sigma} \times \underline{\nabla}_R)\cdot(\underline{\sigma}_e \times \underline{\nabla}_R)}{W_A - W_o + p} \int \frac{d\Omega_p}{4\pi}\, e^{i\underline{p}\cdot(\underline{R}-\underline{R}')} \tag{3.47}$$

$$= -\nu_F \frac{8}{\pi a_o} \sum_{A\neq 0} < \int_o^\infty \frac{dp}{p} \frac{1}{(W_A - W_o + p)} \left\{1 - j_o(p|\underline{R}-\underline{R}'|)\right\} >_{A0} .$$

We may now recognize $\tilde{\nu}_2^{(g)}$ of Eq. (3.39) as the ground state term A = 0

missing from the sum in Eq. (3.47) and we can combine them

$$\tilde{\nu}_2^{(g)} + \nu_2^{(e)} = - \nu_F \frac{8}{\pi a_o} \sum_{\text{all } A} < \int_o^\infty \frac{dp}{p(W_A - W_o + p)} \left\{ 1 - j_o(p|\underline{R}-\underline{R}'|) \right\} >_{AO} \qquad (3.48)$$

Before continuing we can gain some valuable insight into the magnitude of the polarizability correction by considering Eq. (3.48). This expression makes it clear that intermediate electron momenta of

$$p \sim \frac{1}{R} \gtrsim 100 \text{ MeV} \gg m$$

are of primary importance - i.e. the hfs is sensitive to the amplitude of finding the electron near the proton as is well known. The criterion determining when the polarizability contribution will be important or not can be seen from Eq. (3.48). The important region of intermediate electron momenta extends up to $p_{max} \sim 1/R$. According to the energy denominator in Eq. (3.48) the contributions from excited states such that $\Delta W_A \lesssim p_{max} \sim 1/R$ may be of the same approximate magnitude as the ground state term whereas the contributions from highly excited states with $\Delta W_A R \gg 1$ are reduced.

The physical origin of this criterion is understood by noting that $\Delta W_A R$ is a measure of the quarkette velocity, V_q, as it moves a distance R in time $1/\Delta W_A$. Thus for

$$V_q \sim \Delta W R < 1 \qquad (3.49)$$

the electron moving near the proton surface with the velocity of light ($c = 1$) can follow the proton and re-center its wave function around the instantaneous charge position In this case the polarizability may be calculated using the recentering theory of A. Bohr[23] and Low,[24] as developed for the deuteron which satisfies Eq. (3.49) and the correction is sizable. On the other hand for

$$V_q \sim \Delta W R > 1 \tag{3.50}$$

the electron can not follow the charge which is moving too rapidly. In this case the proton approaches the hard baseball limit, the polarizability correction becomes unimportant and proton radii or form factors as measured in the elastic electron proton scattering completely characterizes the correction to the Fermi term in hfs as computed originally by Iddings and Platzman[27] and by Zemach.[13]

These remarks and observations take the following mathematical form. Introducing the integral

$$1 - j_o(p|\underline{R}-\underline{R}'|) = \int_o^{p|\underline{R}-\underline{R}'|} dz\, j_1(z) \tag{3.51}$$

into Eq. (3.48) and interchanging orders of integration one has

$$\tilde{\nu}_2^{(g)} + \nu_2^{(e)} = -\nu_F \frac{8}{\pi a_o} \sum_{\text{all } A} \left\langle \int_o^{\infty} \frac{dz\, j_1(z)}{W_A - W_o} \ell n \left(1 + \frac{(W_A - W_o)|\underline{R}-\underline{R}'|}{z} \right) \right\rangle_{AO} \tag{3.52}$$

The contribution from the proton ground state i.e. A = 0 is explicitely evaluated by observing

$$\lim_{(W_A - W_o) \to 0} \int_o^\infty dz\, j_1(z)\, \frac{1}{(W_A - W_o)}\, \ell n \left(1 + \frac{(W_A - W_o)\left|\underline{R}-\underline{R}'\right|}{z}\right) \tag{3.53}$$

$$= \left|\underline{R}-\underline{R}'\right| \int_o^\infty dz\, \frac{j_1(z)}{z} = \frac{\pi}{4} \left|\underline{R}-\underline{R}'\right|$$

which reduces to Eq. (3.44) as already evaluated. The approximate measure of the excited state contribution is given by the factor

$$\frac{1}{\chi} \ell n\, (1 + \chi) \tag{3.54}$$

where $\chi \equiv \overline{\Delta W} \cdot \overline{R}$ is the product of a mean excitation energy of the proton, $\overline{\Delta W}$ and a mean radial size $\overline{R}$. In the limit of a rigid proton $\overline{\Delta W} \to \infty$ the polarizability correction $\frac{1}{\chi} \ell n\, (1 + \chi) \to 0$.

In the opposite limit of a very polarizable structure such that for <u>all</u> A, $(W_A - W_o)\, R \to 0$ we again make the approximations of Eq. (3.53) and Eq. (3.52) becomes

$$\left(\tilde{\nu}_2^{(g)} + \nu_2^{(e)}\right)_{\Delta W \to 0} = - \nu_F \frac{2}{a_o} \int d^3R \int d^3R' \sum_{\text{all } A} \Phi_o^\dagger(R)\, \Phi_A(R) \cdot$$

$$\cdot\, \Phi_A^\dagger(R')\, \Phi_o(R') \left|\underline{R}-\underline{R}'\right| = 0 \tag{3.55}$$

when we make use of completeness of the intermediate state sum

$$\sum_{\text{all } A} \Phi_A(\underline{R})\, \Phi_A^\dagger(\underline{R}') = \delta^3(\underline{R}-\underline{R}')$$

to apply closure. What is going on in this case is that the electron and proton wave functions have been distorted - i.e. polarized - by their mutual Coulomb attractions so that they follow each others instantaneous position. There is then no finite size reduction of the hfs in contrast to the rigid proton case in which the electron sees the ground state average charge and magnetization distributions. The recentering theory of A. Bohr[23] applies in the polarizable limit in which case the Born-Oppenheimer approximation is valid for the relevant portion of the electron wave function at the proton (or nuclear) surface. The formal series generated by the recentering transformation of Low[24] is just an expansion in powers of the parameter χ in Eq. (3.54) and is appropriate for $\chi < 1$. The present treatment is less elegant in this case but is a convergent perturbation expansion in powers of the fine structure constant α and of $R/a_o \sim \alpha(mR) < \alpha$ and so can be applied to both the polarizable and the rigid "baseball" limit.[28] Model dependent numerical results can also be obtained for an intermediate case with $\chi \sim 1$ as is appropriate for the physical proton which has important resonant excitations with $\Delta W \sim 300$ MeV to several BeV and $R \sim 0.7f \sim 150\ \text{MeV}^{-1}$. We pursue the numerical discussions in more detail after looking at the negative parity quarkette model which leads to a similar form for the polarizability correction to the hfs.

For arbitrary polarizability the hfs is expressed by the sum of Eq. (3.44) and Eq. (3.52) with the ground state term A = 0 omitted from the intermediate state sum ($\Delta W_A \equiv W_A - W_o$):

$$\nu = \nu_F \left[1 - \frac{2}{a_o} < \left| \underline{R}-\underline{R}' \right| > + \frac{8}{\pi a_o} \cdot \int_o^\infty dz\ j_1(z) \sum_{A\neq 0} \left(\frac{1}{\Delta W_A} < \ell n \frac{z}{z+\Delta W_A \left| \underline{R}-\underline{R}' \right|} >_{AO} \right) \right] \qquad (3.5$$

Before leaving this spinor model we note one trivial generalization that can be made but which alters no conclusions. Instead of treating the quarkette as a point particle we may endow it with an intrinsic structure of its own. An extreme case would be to assign a large intrinsic radius to the quarkette, describing its meson cloud as extending out to the observed proton radius, but to bind the center of the quarkette tightly to the origin of the potential V(R) in Eq. (3.2). Such a model would be a return to a rigid baseball model of the proton and would not remove the apparent hfs discrepancy unless the quarkette structure cloud were itself polarizable.

Formally the effect of replacing a point by a destributed quarkette is summarized by generalizing Eqs. (3.4) and (3.6) to

$$H'_{e_o} = - \frac{e^2}{4\pi} \int d^3S \, \rho_c(\underline{S}-\underline{R}) \, \frac{1}{|\underline{r}-\underline{S}|} \tag{3.57}$$

$$\underline{A}(\underline{r}) = + \frac{ek}{2\mu} \int d^3S \, \rho_m(\underline{S}-\underline{R}) \, \underline{\nabla}_S \, \frac{1}{|\underline{r}-\underline{S}|} \times \underline{\sigma} \, . \tag{3.58}$$

where ρ_c and ρ_m are the charge and magnetic densities of the quarkette. As in the point case the structure correction in ν_1 is cancelled by a part, $\bar{\nu}_2^{(g)}$, of the second order correction. The full structure correction is obtained by replacing Eq. (3.48) by

$$\tilde{\nu}_2^{(g)} + \nu_2^{(e)} = - \nu_F \frac{8}{\pi a_o} \sum_{\text{all } A} < \int d^3S \, \rho_m(\underline{S}-\underline{R}) \int d^3S' \, \rho_c(\underline{S}'-\underline{R}') \cdot$$

$$\int_o^\infty \frac{dp}{p(W_A - W_o + p)} \left\{ 1 - j_o(p|\underline{S}-\underline{S}'|) \right\} >_{AO} \, . \tag{3.59}$$

In the limit of a quarkette of large radius but with a very polarizable wave function we may neglect $W_A - W_o$ in the denominator of Eq. (3.59) use closure and Eq. (3.40) to obtain

$$\tilde{\nu}_2^{(g)} + \nu_2^{(e)} = - \nu_F \frac{2}{a_o} \int d^3S\, d^3S'\, d^3R\, d^3R' \cdot$$

$$\Phi_o^\dagger(\underline{R})\, \rho_m(\underline{S}-\underline{R})\, \rho_c(\underline{S}'-\underline{R}')\Phi_o(\underline{R}')\, \delta^3(\underline{R}-\underline{R}')\left|\underline{S}-\underline{S}'\right| \tag{3.60}$$

$$= - \nu_F \frac{2}{a_o} < \left|\underline{S}-\underline{S}'\right| >$$

where

$$< \left|\underline{S}-\underline{S}'\right| > = \int d^3S\, d^3S'\, \rho_m(\underline{S})\, \rho_c(\underline{S}')\, \left|\underline{S}-\underline{S}'\right| \quad . \tag{3.61}$$

The total hfs is then

$$\nu = \nu_F \left(1 - \frac{2}{a_o} < \left|\underline{S}-\underline{S}'\right| > \right) \tag{3.62}$$

which is the rigid baseball model once more, only now it is the quarkette itself that is the rigid baseball.

PART B

In this part we consider the negative parity quarkette or orbital model of the proton. The full Hamilitonian is written as in Eq. (3.8) of Part A except that the quarkette is now a spin zero particle and has no intrinsic magnetic dipole moment; i.e. $\kappa = 0$ in Eq. (3.6). It is in

a P orbit about the infinite mass center which is assigned spin 1/2 and the orbital and spin angular momenta couple to form a spin 1/2 proton ground state. In this case the quarkette Hamiltonian H_p of Eq. (3.2) must contain in addition to the central potential, a spin orbit term in order to remove the degeneracy in states of the same orbital but of different total angular momenta. Therefore Eq. (3.2) is explicitly

$$H_P(R) = \frac{P^2}{2\mu} + V_c(R) + V_{SL}(R)\ \underline{\sigma} \cdot \underline{L} \tag{3.63}$$

where $V_{SL}(R)$ has the responsibility of raising the $P^+_{3/2}$ proton level to its observed excitation of $\approx$ 300 MeV above the ground state. The quarkette mass is fixed by the requirement that this model reproduce the observed proton magnetic moment. As in Part A, symmetrization and "gauge" terms may be dropped from Eq. (3.6) when dealing with the hfs. The unperturbed energies and eigenfunctions are labeled in the manner of Eqs. (3.11) and (3.15) except that in place of the simple product form of Eq. (3.13) we must couple the quarkette orbital angular momentum and the spin of the core. In particular the spin up ground state wave function is

$$\begin{aligned}\Phi_o(R) &= \frac{\varphi_p(R)}{\sqrt{4\pi}\ R}\left(\cos\theta\ \chi^{1/2} + \sin\theta\ e^{i\varphi}\ \chi^{-1/2}\right) \\ &= \frac{\varphi_p(R)}{\sqrt{4\pi}\ R^2}\ \underline{\sigma} \cdot \underline{R}\ \chi^{+1/2}\end{aligned} \tag{3.64}$$

with

$$\int_o^\infty \left|\varphi_p(R)\right|^2 dR = 1 \quad . \tag{3.65}$$

In first order perturbation theory we have

$$\nu_1 = 4 < \Psi_g \left| H_M \right| \Psi_g > = 4\alpha \int d^3R\, d^3r\, \psi_o^\dagger(\underline{r})\, \frac{\varphi_p^\dagger(R)}{\sqrt{4\pi}\, R^2}\, \underline{\sigma}\cdot\underline{R}\, \frac{\underline{\alpha}\cdot\underline{V}}{|\underline{r}-\underline{R}|}$$

$$\times\, \underline{\sigma}\cdot\underline{R}\, \frac{\varphi_p(R)}{\sqrt{4\pi}\, R^2}\, \psi_o(\underline{r}) \qquad (3.66)$$

$$= \frac{8\pi\alpha^2}{3}\, \psi^2(0)\, a_o\, \underline{\sigma}_e \cdot < \underline{R} \times \underline{V}\, (1 - 3/2\, R/a_o) >_o .$$

Recognizing that the proton magnetic moment is given explicitly by

$$\underline{\mu}_p \equiv \mu_p\, \underline{\sigma} = \frac{e}{2} < \underline{R} \times \underline{V} >_o \qquad (3.67)$$

in this model we can write Eq. (3.66) in analogy with Eq. (3.22)

$$\nu_1 = \nu_F \quad \frac{< \underline{\sigma}_e \cdot \underline{R} \times \underline{V}\, (1 - 3/2\, R/a_o) >_o}{< \underline{\sigma}_e \cdot \underline{R} \times \underline{V} >_o} \qquad (3.68)$$

$$\equiv \nu_F \left(1 - 3/2\, \frac{< R >}{a_o} \right)$$

In terms of a specific spin orbit potential Eq. (3.63) and ground state Eqs. (3.64) and (3.65) we have

$$\underline{V} \equiv i \left[H, \underline{R} \right] = \frac{-i}{\mu}\, \nabla_R + V_{SL}\, \underline{\sigma} \times \underline{R} \qquad (3.69)$$

and the moment and radius are given by

$$\left(\frac{e}{2M}\right)\mu_p = \frac{2}{3}\,\frac{e}{2\mu}\int_o^\infty dR\left|\varphi_p(R)\right|^2\left[1 - \mu\, V_{SL}\, R^2\right] \tag{3.70}$$

$$< R > = \frac{\int_o^\infty dR\left|\varphi_p(R)\right|^2 R(1 - \mu\, V_{SL}\, R^2)}{\int_o^\infty dR\left|\varphi_p(R)\right|^2 (1 - \mu\, V_{SL}\, R^2)} \tag{3.71}$$

The factor $\left[1 - \mu\, V_{SL}\, R^2\right]$ in Eq. (3.70) and Eq. (3.71) represents the effective mass correction due to the spin orbit interaction.

The second order contribution to the hfs is again divided into $\nu_2^{(g)}$ and $\nu_2^{(e)}$, the terms arising from intermediate states without and with excitation of the quarkette, respectively. The former is

$$\nu_2^{(g)} = -\,8\alpha^2\,\mathrm{Re}\sum_{\substack{n\pm \\ n+\neq 0}}\int d^3R\, d^3R'\, d^3r\, d^3r'\left\{\Phi_o^\dagger(\underline{R}')\,\psi_o^\dagger(\underline{r})\,\frac{\underline{\alpha}\cdot\underline{V}}{|\underline{r}-\underline{R}|}\,\Phi_o(\underline{R})\,\psi_{n\pm}(\underline{r})\cdot\right. \tag{3.72}$$

$$\left.\cdot\,\frac{\psi_{n\pm}^\dagger(\underline{r}')}{E_{n\pm} - E_o}\,\Phi_o^\dagger(\underline{R}')\left(\frac{1}{r'} - \frac{1}{|\underline{r}'-\underline{R}'|}\right)\Phi_o(\underline{R}')\,\psi_o(\underline{r}')\right\}\cdot$$

By an argument identical to that in Part A it can be shown that the intermediate Coulomb electron states may be replaced by plane waves - Eq. (3.31). The new matrix element that is required in place of Eq. (3.33) is

$$\int d^3r\,\psi_o^\dagger(\underline{r})\,\frac{\underline{\alpha}\cdot\underline{V}}{|\underline{r}-\underline{R}|}\,\psi_p^{(+)}(\underline{r}) = \frac{4\pi}{p^2}\,\psi(0)\left|u(0),\,\underline{\alpha}\cdot\underline{V}\,u(\underline{p})\right|\,e^{i\underline{p}\cdot\underline{R}}\cdot \tag{3.73}$$

Using Eqs. (3.32) and (3.73) we find in complete analogy with Eq. (3.34)

$$\nu_2^{(g)} = -\,64\alpha^2\,\psi^2(0)\,\mathrm{Re} < \int_0^\infty \frac{dp}{p^4}\frac{d\Omega_p}{4\pi}\,(\underline{\sigma}_e \times \underline{V})\cdot\underline{\nabla}_R\left[e^{i\underline{p}\cdot\underline{R}} - e^{i\underline{p}\cdot(\underline{R}-\underline{R}')}\right]>_{00} \cdot \qquad (3.74)$$

We further decompose $\nu_2^{(g)}$ into two parts each of which is finite:

$$\nu_2^{(g)} = \bar{\nu}_2^{(g)} + \tilde{\nu}_2^{(g)} \qquad (3.75)$$

where

$$\bar{\nu}_2^{(g)} \equiv +\frac{64\alpha^2}{3}\,\psi^2(0) < \int_0^\infty \frac{dp}{p^2}\,(\underline{\sigma}_e \times \underline{V}\cdot\underline{R})\left\{\frac{3j_1(pR)}{pR} - 1\right\}>_{00} \qquad (3.76)$$

and

$$\tilde{\nu}_2^{(g)} \equiv -\frac{64\alpha^2}{3}\,\psi^2(0) < \int_0^\infty \frac{dp}{p^2}\,\underline{\sigma}_e \times \underline{V}\cdot(\underline{R}-\underline{R}')\left\{\frac{3j_1(p|\underline{R}-\underline{R}'|)}{p|\underline{R}-\underline{R}'|} - 1\right\} >_{00} \qquad (3.77)$$

Noting that $<\underline{\sigma}_e \times \underline{V}\cdot\underline{R}'>_{00} = 0$ we see that Eq. (3.76) and Eq. (3.77) add back to form Eq. (3.74). The first term precisely cancels the first order size correction to hfs in Eq. (3.68) as we show with the help of the identity

$$\int_0^\infty \frac{dx}{x^2}\left(1 - \frac{3j_1(x)}{x}\right) = 3\pi/16 \quad . \qquad (3.78)$$

In Eq. (3.76) this gives

$$\bar{\nu}_2^{(g)} = +\,4\pi\,\alpha^2\,\psi^2(0) < (\underline{\sigma}_e\cdot\underline{R} \times \underline{V})\,R >_o \qquad (3.79)$$

which is just the opposite of the finite size correction in Eq. (3.66). The second term may be rewritten using Eq. (3.78) as

$$\tilde{\nu}_2^{(g)} = -\, 4\pi\, \alpha^2\, \psi^2(0) < \left| \underline{\sigma}_e \cdot (\underline{R}-\underline{R}') \times \underline{V} \right| \left| \underline{R}-\underline{R}' \right| >_{00} . \tag{3.80}$$

Equation (3.80) is the analog of Eq. (3.42) of Part A, namely the full proton size correction for a hard "baseball" model of the proton, that is

$$\nu_1 + \nu_2^{(g)} = \nu_F \left[1 - 3/2\, \frac{1}{a_o} < \left\{ \underline{\sigma}_e \cdot (\underline{R}-\underline{R}') \times \underline{V} \right\} \left| \underline{R}-\underline{R}' \right| >_{00} \right] \tag{3.81}$$

The excited state contribution $\nu_2^{(e)}$ may be manipulated by now familiar techniques to the form

$$\nu_2^{(e)} = +\, 64\alpha^2\, \psi^2(0) \sum_{A\neq 0} < \int_o^\infty \frac{dp (\underline{\sigma}_e \times \underline{V}) \cdot \nabla_{\underline{R}}\, j_o(p \left| \underline{R}-\underline{R}' \right|)}{p^3 (\Delta W_A + p)} >_{AO} . \tag{3.82}$$

This again is conveniently split into two parts after carrying out the gradient operation:

$$\nu_2^{(e)} = \nu_2^{(e)} + \tilde{\nu}_2^{(e)} \tag{3.83}$$

where

$$\bar{\nu}_2^{(e)} \equiv +\, \frac{64\alpha^2}{3}\, \psi^2(0) \sum_{A\neq 0} < \int_o^\infty \frac{dp}{p(\Delta W_A + p)} \left| \underline{\sigma}_e \times \underline{V} \cdot \underline{R}' \right| >_{AO} \tag{3.84}$$

and

$$\tilde{\nu}_2^{(e)} \equiv -\frac{64\alpha^2}{3}\psi^2(0)\sum_{A\neq 0} < \int_0^\infty \frac{dp}{p(\Delta W_A + p)}\, \underline{\sigma}_e \times \underline{V}\cdot(\underline{R}-\underline{R}')\left\{\frac{3j_1(p|\underline{R}-\underline{R}'|)}{p|\underline{R}-\underline{R}'|} - 1\right\} >_{A0} . \quad (3.85)$$

The first term may be reduced by carrying out the dp integration. At $p \to 0$ there is an apparent but not a real divergince as we see by putting a lower cut off at ϵ, $0 < \epsilon \ll \Delta W_A$.

$$\int_\epsilon^\infty \frac{dp}{p(\Delta W_A + p)} = \frac{1}{\Delta W_A}\ell n\left|\frac{\Delta W_A + \epsilon}{\epsilon}\right| = \frac{1}{\Delta W_A}\left\{\ell n(\bar{R}\Delta W_A) - \ell n(\bar{R}\epsilon) + 0\left|\frac{\epsilon}{\Delta W_A}\right|\right\} \quad (3.86)$$

where $\bar{R}$ is an arbitrary length. The third term vanishes as $\epsilon \to 0$ and the second logarithm $\ell n\, \bar{R}\epsilon$ gives no hfs contribution as we show by the following argument using closure and Eq. (3.69). Define

$$Q_{BA} = \int d^3R\, \Phi_B^\dagger(\underline{R})\, Q\, \Phi_A(\underline{R}) \quad (3.87)$$

Then

$$\begin{aligned}\sum_{A\neq 0}\frac{1}{\Delta W_A} < \underline{\sigma}_e\cdot\underline{V}\times\underline{R}' >_{A0} &= \sum_{A\neq 0}\frac{1}{\Delta W_A}\underline{\sigma}_e\cdot\left\{\int d^3R\, \Phi_o^\dagger(\underline{R})\, \underline{V}\, \Phi_A(\underline{R})\right\} \\ &\quad\times\left\{\int d^3R'\, \Phi_A^\dagger(\underline{R}')\, \underline{R}'\, \Phi_o(\underline{R}')\right\} \\ &= \sum_{A\neq 0}\frac{1}{\Delta W_A}\underline{\sigma}_e\cdot\underline{V}_{OA}\times\underline{R}_{AO} = -i\sum_{A\neq 0}\underline{\sigma}_e\cdot\underline{R}_{OA}\times\underline{R}_{AO} \\ &= i\underline{\sigma}_e\cdot\underline{R}_{OO}\times\underline{R}_{OO} = 0.\end{aligned} \quad (3.88)$$

Taking now the limit $\epsilon \to 0$ we find

$$\bar{\nu}_2^{(e)} = \frac{64\alpha^2}{3}\psi^2(0)\sum_{A\neq 0}\frac{\ell n(\Delta W_A \bar{R})}{\Delta W_A} < \underline{\sigma}_e\cdot(\underline{V}\times\underline{R}') >_{AO}$$

$$= \nu_F \frac{-8}{\pi a_o}\frac{M}{\mu_p}\sum_{A\neq 0}\ell n(\Delta W_A \bar{R})\; i\underline{\sigma}_e\cdot(\underline{R}_{OA}\times\underline{R}_{AO}) \qquad (3.89)$$

Finally returning to $\tilde{\nu}_2^{(e)}$ we see that the A = 0 term which is missing from the sum in Eq. (3.85) is precisely the second order ground state term $\tilde{\nu}_2^{(g)}$ in Eq. (3.77); combining yields

$$\tilde{\nu}_2^{(g)}+\tilde{\nu}_2^{(e)} = -\nu_F\frac{8}{\pi a_o}\left|\frac{M}{\mu_p}\right|\sum_{\text{all } A} < \int_o^\infty \frac{dp}{p(\Delta W_A+p)}\,\underline{\sigma}_e\times\underline{V}\cdot\left|\underline{R}-\underline{R}'\right|\left(\frac{3j_1(p\left|\underline{R}-\underline{R}'\right|)}{p\left|\underline{R}-\underline{R}'\right|}-1\right)>_{AO} \qquad (3.90)$$

In the very polarizable limit ΔW_A can be neglected in the denominator of Eq. (3.90) - i.e. the important excited states in the sum are all of sufficiently low energy that $\Delta W_A\, R \ll 1$. We can then use closure to obtain

$$\tilde{\nu}_2^{(g)} + \tilde{\nu}_2^{(e)} = 0 \quad .$$

The full proton structure correction to the hfs is then

$$\nu = \nu_1 + \bar{\nu}_2^{(g)} + \bar{\nu}_2^{(e)} + \tilde{\nu}_2^{(g)} + \tilde{\nu}_2^{(e)} = \nu_F + \bar{\nu}_2^{(e)}$$

$$= \nu_F\left(1 - \frac{8}{\pi a_o}\left|\frac{M}{\mu_p}\right|\sum_{A\neq 0}\ell n\,(\Delta W_A\bar{R})\left\{i\,\underline{\sigma}_e\cdot\,(\underline{R}_{OA}\times\underline{R}_{AO})\right\}\right)\cdot \qquad (3.91)$$

This result is analogous to the recentering theory result of Low[24] for the orbital part of the deuteron's magnetic moment. Even in this very polarizable limit there is a reduction of the point proton result due to the logarithm term in Eq. (3.91). Physically this corresponds to the fact that when the electron wave function is able to closely follow the instantaneous positon of the quarkette it sees no current and hence no magnetic field.

To handle the region of intermediate polarization we introduce

$$\frac{3j_1(p|\underline{R}-\underline{R}'|)}{p|\underline{R}-\underline{R}'|} - 1 = -\int_0^{p|\underline{R}-\underline{R}'|} \frac{dz}{z}\ j_2(z) \tag{3.92}$$

and obtain after interchanging orders of integration

$$\tilde{\nu}_2^{(g)} + \tilde{\nu}_2^{(e)} = \nu_F \frac{8}{\pi a_o}\left(\frac{M}{\mu_p}\right) \sum_{\text{all A}} < (\underline{\sigma}_e \times \underline{V})\cdot(\underline{R}-\underline{R}') \int_0^\infty \frac{dz}{z}\ j_2(z)\ \frac{1}{\Delta W_A}\ \ell n \left(1+\frac{\Delta W_A|\underline{R}-\underline{R}'|}{z}\right) >_{A0} \tag{3.93}$$

The complete expression for the proton hfs is given by the sum of Eqs. (3.91) and (3.93) and can be written using Eqs. (3.81) and (3.92)

$$\nu = \nu_F \left[1 - \frac{3}{2}\frac{1}{a_o} < \underline{\sigma}_e\cdot(\underline{R}-\underline{R}') \times \underline{V}\ |\underline{R}-\underline{R}'| >_{00} + \frac{8}{\pi a_o}\left(\frac{M}{\mu_p}\right)\int_0^\infty \frac{dz}{z}\ j_2(z) \sum_{A\neq 0}\left(\frac{1}{\Delta W_A} < \underline{\sigma}_e\cdot(\underline{R}-\underline{R}') \times \underline{V}\ \ell n \left\{\frac{\Delta W_A\ \overline{R}}{z+\Delta W_A|\underline{R}-\underline{R}'|}\right\} >_{A0}\right)\right]. \tag{3.94}$$

This is analogous to Eq. (3.56) for the even parity quarkette model.

PART C

To conclude this section we present some numerical results based on the models developed in Parts A and B. What has been accomplished so far that is of relevance for the polarizability contribution may be summarized as follows.

In both the spinor and orbital models of the proton magnetic structure, we measure the correction to the rigid "baseball" limit by a dimensionless parameter ΔWR i.e. the product of a mean electric excitation energy of the proton multiplied by its radius. The correction is appreciable for $\Delta WR \lesssim 1$ and negligible for $\Delta WR \to \infty$.

Since the meson production threshold gives a lower limit to the excitation spectrum of 140 MeV and the observed proton radius is $R \sim .7f \sim \frac{1}{300\ \text{MeV}}$ neither simple limit of a rigid baseball proton or of a very polarizable one with the electron recentered on the instantaneous charge applies. We may expect then that the actual numerical value of $\Delta v^{(p)}$ in Eq. (2.5) is dependent on details of the proton dynamics. In the orbital model the form of Eq. (3.94) indicates no dominant role being played by the M1 excitations to a $P^{+}_{3/2}$ resonance which has been a popular state in relativistic dispersion studies.

The simplest approximation for estimating the polarizability contributions from $A \neq 0$ states in Eqs. (3.52) or (3.94) would be to replace the logarithm by some mean value over the excitation spectrum as is done with the Bethe logarithm in evaluating the Lamb shift. Unfortunately in both cases orthogonality and closure may be used very simply to show

that in this approximation the contributions vanish so that this approach is inadequate. This shows again that the results depend on details of the excited state spectrum and on no easily estimated simple average parameter alone.

For purposes of an estimate we return to Eq. (3.56) of Part A and compute the contribution to hfs from an excited P state orbital triplet with the radial distribution given by the eigensolutions for a spherical harmonic ocillator well. The wave functions are taken as

$$\Phi_o(R) = \left(\frac{2\lambda}{\pi}\right)^{3/4} e^{-\lambda R^2}$$

$$\underline{\Phi}_1(R) = \left(\frac{2\lambda}{\pi}\right)^{3/4} 2\sqrt{\lambda}\,\underline{R}\, e^{-\lambda R^2} \qquad (3.95)$$

where the mean square radius is given in terms of λ by

$$< R^2 > \equiv \frac{3}{4\lambda}$$

The ground state contribution in Eq. (3.56) is [see Eq. (3.42)]

$$\tilde{\nu}_2^{(g)} = -\nu_F \frac{2}{a_o}\left(\frac{2\lambda}{\pi}\right)^3 \int d^3R\, d^3R'\, e^{-2\lambda(R^2+R'^2)} \left|\underline{R}-\underline{R}'\right|$$

$$= -\nu_F \frac{2}{a_o}\frac{4}{\sqrt{3\pi}} < R^2 >^{1/2}. \qquad (3.96)$$

The polarizability contribution in Eq. (3.56) due to the P state triplet can be expressed in a form which permits the angular and one of the radial

integrals over the wave functions to be carried out with the aid of the following identity when $A \neq 0$

$$I_A = < \int_o^\infty dz\, j_1(z) \frac{1}{W_A - W_o} \ell n \left(1 + \frac{|W_A - W_o| |\underline{R}-\underline{R}'|}{z} \right) >_{AO}$$

$$= < \int_1^\infty \frac{du}{u} \int_o^\infty \frac{dt e^{-t}}{W_A - W_o} \int_o^\infty dz\, j_1(z)\, e^{-ztu/|W_A - W_o||\underline{R}-\underline{R}'|} >_{AO} \qquad (3.97)$$

$$= \int_o^\infty d\tau \left[\frac{1}{2} \ell n \frac{1+\tau^2}{\tau^2} - 1 - \tau \tan^{-1} \frac{1}{\tau} \right] < |\underline{R}-\underline{R}'|\, e^{-|\underline{R}-\underline{R}'|(W_A - W_o)\tau} >_{AO} .$$

Inserting into Eq. (3.56) and reducing further we have for the contribution from state $A \neq 0$

$$\nu_2^A = + \frac{64 < R^2 >^{1/2}}{\pi \sqrt{3\pi}\, a_o} \int_o^\infty d\tau \left[\frac{1}{2} \ell n \frac{1+\tau^2}{\tau^2} - 1 - \tau \tan^{-1} \frac{1}{\tau} \right]$$

$$\times \int_o^\infty \rho^3\, d\rho \left(\frac{3}{2} - \rho^2 \right) e^{-\rho^2 - \rho\tau \left[|W_A - W_o| / \lambda^{1/2} \right]} \qquad (3.98)$$

In the limit $W_A - W_o \to 0$

$$\lim_{(W_A - W_o) \to 0} \nu_2^A = + \nu_F \frac{2}{a_o} \frac{2}{\sqrt{3\pi}} < R^2 >^{1/2} \qquad (3.99)$$

which cancels one-half of the finite size reduction in Eq. (3.96). In the limit $\Delta W_A < R^2 >^{1/2} \to 0$ for all A, the sum $\sum_{\text{all } A} \nu_2^A$ precisely cancels the finite size reduction, Eq. (3.96), as was shown earlier in Eq. (3.55).

For large values of $(W_A - W_o) < R^2 >^{1/2}$ the polarizability contribution decreases roughly as $1/\left[(W_A - W_o) < R^2 >^{1/2} \right]$. Fixing $< R^2 >^{1/2} \approx .8f$ we have numerically integrated Eq. (3.98) for a wide range of excitation energies

and the ratio $\nu_2^A/\nu_2^{(g)}$ is plotted in Fig. 2 as a function of $(W_A - W_o)$. For excitations up to 1 BeV the correction is still significant. At an excitation energy of $W_A - W_o = 400$ MeV, for example, this one level contributes 1/3 as much to the polarizability correction as it does in the completely polarizable limit $(W_A - W_o)\, R \to 0$ and cancels 1/6 of the finite size correction to the hfs.[30] Taking into account the fact that I_A in Eq. (3.97) is positive definite for all states $A \neq 0$ it becomes clear that the contributions of several excited states in this model may cancel an appreciable fraction of the finite size reduction in Eq. (3.96) of the hfs.

No one state A need dominate. Indeed if the density of highly excited states increases rapidly they will play a dominant role. For example if we apply Eq. (3.97) to a model of the proton in which the spinor quarkette is localized radially at a distance $\bar{R}$ from the force center we have

$$\left\langle \left|\underline{R}-\underline{R}'\right| e^{-\left|\underline{R}-\underline{R}'\right|(W_A - W_o)\tau} \right\rangle_{AO} \to \frac{\bar{R}}{2} \int_{-1}^{1} d\mu \sqrt{2-2\mu}\, e^{-\bar{R}(W_A - W_o)\tau \sqrt{2-2\mu}} \tag{3.100}$$

$$= \frac{\bar{R}}{2} \int_0^4 dz\, z^2\, e^{-z\tau\, \bar{R}(W_A - W_o)} \propto \frac{1}{(W_A - W_o)^3} \quad \text{for } W_A \to \infty$$

Thus if the density of levels grows more rapidly than $(W_A - W_o)$ the effect of the highly excited levels will be important.

Evidently the details of the dynamics of the proton cannot be avoided completely in a quantitative study of the hfs. A similar conclusion follows from the massive three quark model of Fenster and Nambu.[8] They have invoked the recentering idea of Bohr[23] to partially cancel the ground

state average form factor effect as expressed in Eq. (3.44) or Eq. (3.81). Their theory is based on a strict analogy for the proton, constructed of massive slow moving quarks, with a nucleus formed of nucleons. However, there is a fundamental difficulty in taking this analogy literally as proposed by Fenster and Nambu. This arises from the fact that the scale of excitation energies ΔW for the proton is much higher than that for a nucleus. In the deuteron or other weakly bound nuclear systems (with anomalous thresholds) the relevant parameter is $\Delta WR < 1$ and the re-centering idea of Bohr is applicable. This is not true for a proton for which the minimum ΔW is the pion production threshold, ≈ 140 MeV, so that $\Delta WR \gtrsim 1$. The Fenster and Nambu prediction is a partial cancellation of the finite size reduction in the hfs reduction based on the recentering assumption which requires $\Delta WR < 1$. Whether the proton radius R is attributed to the bound state wave function of a quarkette or to the intrinsic radius of the quarkette itself the simple recentering assumption cannot be applied to the proton as we saw below Eq. (3.55) and in Eq. (3.62).

The numbers we obtained above may have no quantitative significance whatsoever. However they do carry a warning that simplified approximate inputs in a relativistic dispersion analysis must also be viewed with great caution. At the same time these numbers also provide a strong motivation to perform a dispersion theoretic calculation of the polarizability beyond the ones that have been given and which so far have concentrated on the 33 resonance alone. We turn to the relativistic theory now.

IV. RELATIVISTIC CALCULATION

In this section we take up the relativistic calculation of the proton structure correction to the hfs. The general formulation has already been given by Iddings[9] along with the calculation of the proton ground state terms - the analogues of ν_1 and $\nu_2^{(g)}$ of the previous section. As in Section III we focus our attention on the polarizability corrections. Because we work with different invariant amplitudes and make contact with the low energy theorem in a different manner than Ref. 9 we briefly resketch the development of Iddings.

The contribution to hfs arising from proton recoil was first calculated by Newcomb and Salpeter[10] and by Arnowitt.[11] Their analysis required a study of the bound state equation for the electron-proton system. If the proton is treated as a point Dirac particle of finite mass but with zero anomalous moment the hfs contribution is

$$-\alpha\frac{m}{M}\mathcal{X} = -\frac{\alpha}{1+\kappa}\frac{m}{M}\left\{\frac{3}{\pi}\ell n\ M/m\right\} = -\ 10\ \text{ppm}\ .$$

When the proton is also assigned a point Pauli moment of $\kappa = 1.79$ nuclear magnetons the additional contribution to hfs is

$$-\alpha\frac{m}{M}\mathcal{X} = -\frac{\alpha}{1+\kappa}\frac{m}{M}\frac{1}{\pi}\left\{3(1-\kappa^2/4)\ \ell n\ M/m + \kappa^2\frac{3}{4}(-1/6 + 3\ \ell n\ \Lambda/M)\right\} \approx -\ 4\ \text{ppm}\ ,$$

A logarithmic cut off $\Lambda \sim 2M$ has been introduced as needed into the square of the Pauli moment interaction and shows a sensitivity of the numerical

results to virtual photons of high mass being exchanged between the atomic electron and the proton. This divergence is removed in a natural way when the structure of the proton is taken into account as first studied by Iddings and Platzman[27] and by Zemach.[13] In particular they show that the effect of structure analogous to the contributions of $(\nu_1 - \nu_F)$ and of $\nu_2^{(g)}$ in the Schrodinger models of the preceding section, can be summarized in terms of the electromagnetic form factors of the proton in the interaction current with the electron. In the calculation of these structure corrections to the hfs one can ignore all complications of the bound state problem and treat the electron-proton interaction in second order Born approximation. We saw this to be true in the models of the preceding section where the intermediate electron was described by free particle wave functions and energy eigenvalues. The bound state appears only in the non-relativistic factor $\psi^2(0)$ for the density of the atomic electrons at the origin in Eqs. (3.34) and (3.74).

This formulation in the relativistic calculation was presented by Iddings and Platzman[27] who showed that the complete proton structure contribution to the hfs in hydrogen can be obtained from the two photon exchange amplitude as illustrated in Fig. 3. The "blob" in Fig. 3 is the forward spin-flip Compton scattering amplitude, hereafter denoted by $C_{\mu\nu}$, for virtual photons from protons at rest. Iddings[9] exploited this result, following the method developed by Cottingham[12] for the neutron-proton mass difference, and rotated the contour of the dk_o integration in Fig. 3 from the real to the imaginary k_o axis so that the virtual photon masses that are involved are always space-like ($k^2 \leq 0$). In this regime of photon

masses the Compton amplitude $C_{\mu\nu}$ may be rigorously proved to be analytic in the entire photon energy plane except for the right and left hand cuts along the real axis required by unitarity and crossing. Anomalous thresholds are absent. The pole terms in the virtual Compton amplitude contain electromagnetic form factors $F_1(k^2)$ and $F_2(k^2)$ where $k^2 < 0$ is the $(\text{mass})^2$ of the virtual photon. The structure correction corresponding to $\nu_1 - \nu_F$ and $\nu_2^{(g)}$ in the preceding section comes from the difference between the pole terms calculated with observed values of F_1 and F_2 and with $F_1 = F_2 = 1$ as for a point proton. This difference has been studied by Iddings in detail. The non-pole terms in $C_{\mu\nu}$ contribute to the polarizability correction on which we focus our primary attention.

In place of the invariant amplitudes used by Iddings we decompose $C_{\mu\nu}$ according to

$$C_{\mu\nu}(k^2,\omega) = T_{\mu\nu}^{(1)} H_1(k^2,\omega) + T_{\mu\nu}^{(2)} H_2(k^2,\omega) \tag{4.1}$$

where

$$T_{\mu\nu}^{(1)} = \frac{1}{M^3}\left\{\left[\gamma_\mu,\gamma_\nu\right] p\cdot k - \left[\gamma_\mu,\not{k}\right] p - \left[\gamma_\nu,\not{k}\right] p_\mu\right\} \tag{4.2}$$

$$T_{\mu\nu}^{(2)} = \frac{p\cdot k}{M^3}\left\{\left[\gamma_\mu,\gamma_\nu\right] k^2 - \left[\gamma_\mu,\not{k}\right] k_\nu - \left[\gamma_\nu,\not{k}\right] k_\mu\right\} \tag{4.3}$$

ν is the polarization index for the incident photon line

μ is the polarization index for the outgoing photon line

p is the proton four momentum ($p = (M,0,0,0)$)

k is the photon four momentum

$\omega = \frac{p\cdot k}{M}$ and M the proton mass.

This choice of amplitudes is particularity convenient;[31] in the limit of physical Compton scattering ($k^2 = 0$) the amplitude H_2 is completely decoupled and the remaining amplitude H_1 satisfies a sum rule as we shall see below.

With the help of Eq. (2.2) of Ref. 9 the two photon exchange contribution to the hfs may be written (after rotation of the k_o contour of the Feynman loop integration)

$$\frac{\nu_{2\gamma}}{\nu_F} = \frac{2}{a_o}\frac{1}{M(1+\kappa)\pi^3}\int\frac{d^4K}{K^6}\left\{(2K^2 + K_o^2)\,H_1(-K^2,iK_o) - 3K^2K_o^2\,H_2(-K^2,iK_o)\right\} \tag{4.4}$$

where ν_F the point proton splitting Eq. (3.24), $\kappa = 1.79$ is the proton anomalous moment, and a_o the Bohr radius $a_o = 1/\alpha m$. In Eq. (4.4) the integration is over a four-dimensional Euclidean space K with $K_o = ik_o$, $K^2 = -k^2$, and

$$\int d^4K = 4\pi\int_o^\infty K^3dK\int_o^\pi d\psi\sin^2\psi \tag{4.5}$$

$$K_o = K\cos\psi$$

as in Eq. (2.14) of Ref. 9.

Writing disperson relations in ω for H_1 and H_2 and explicitely displaying the contribution of the intermediate one proton pole, one has

$$H_1(k^2,\omega) = - \frac{k^2 M^2 F_1(k^2)\left[F_1(k^2)+ \kappa F_2(k^2)\right]}{(k^2)^2 - 4M^2\omega^2} + \frac{1}{\pi} \int_{C(k^2)}^{\infty} \frac{d\omega'^2}{\omega'^2-\omega^2} \operatorname{Im} H_1(k^2,\omega') \qquad (4.6)$$

$$H_2(k^2,\omega) = - \frac{\kappa F_2(k^2)\left[F_1(k^2) + \kappa F_2(k^2)\right]}{(k^2)^2 - 4M^2\omega^2} + \frac{1}{\pi} \int_{C(k^2)}^{\infty} \frac{d\omega'^2}{\omega'^2-\omega^2} \operatorname{Im} H_2(k^2,\omega') \qquad (4.7)$$

where $F_1(k^2)$, $F_2(k^2)$ are the proton Dirac and Pauli electromagnetic form factors respectively and the cut starts at the pion-nucleon threshold,

$$C(k^2) = \frac{1}{4M^2}(2M\mu + \mu^2 - k^2)^2 \qquad (4.8)$$

Throughout this section μ will denote the pion mass. Iddings[9] has further shown through unitarity that Im H_1 and Im H_2 in the space-like region $k^2 < 0$ may be determined experimentally from the cross section of inelastic scattering of polarized electrons from polarized protons. It will be a long time, however, before such data exist.

In writing Eqs. (4.6) and (4.7) we have made the assumption that there need be no subtractions in ω for all $k^2 < 0$. We defer further remarks about this point until the discussion of Section V. We must subtract from Eqs. (4.4) to (4.7) the point proton result which is obtained by setting $F_1 = F_2 = 1$, Im $H_2 = 0$ and,[32] by the low energy theorem of Low[33] and Gell-Mann and Goldberger,[34]

$$-\frac{\kappa^2}{4} = \frac{1}{\pi} \int_{C(0)}^{\infty} \frac{d\omega'^2}{\omega'^2} \operatorname{Im} H_1(0,\omega') \quad . \qquad (4.9)$$

We write this difference as

$$\frac{\nu_s}{\nu_F} = S_1 + S_2 + S_3 \tag{4.10}$$

where

$$S_1 = \frac{1}{a_o M(1+\kappa)\pi^3}\int \frac{d^4K}{K^6}\left\{ + \frac{2K^2(2K^2+K_o^2)\left[F_1(F_1+\kappa F_2)-(1+\kappa)\right]+6K^2K_o^2\kappa\left[F_2(F_1+\kappa F_2)-(1+\kappa)\right]}{\frac{K^4}{M^2}+4K_o^2}\right.$$
$$\left. - \frac{(2K^2+K_o^2)\kappa^2\left[F_2^2-1\right]}{2}\right\} \tag{4.11}$$

$$S_2 = \frac{1}{a_o M(1+\kappa)\pi}\int_o^\infty \frac{dK}{K}\left\{\frac{9}{4}\kappa^2 F_2^2(-K^2)\right.$$
$$\left. - \frac{4}{\pi}\int_{C(-K^2)}^\infty \frac{d\omega'^2}{\omega'^2}\ \mathrm{Im}\ H_1(-K^2,\omega')\left(3\theta-2\theta^2-2(2-\theta)\sqrt{\theta(\theta+1)}\right)\right\}. \tag{4.12}$$

$$S_3 = \frac{-12}{a_o M(1+\kappa)\pi^2}\int_o^\infty \frac{dK}{K}\int_{C(-K^2)}^\infty d\omega'^2\ \mathrm{Im}\ H_2(-K^2,\omega')\left\{1+2\theta-2\sqrt{\theta(\theta+1)}\right\} \tag{4.13}$$

and

$$\theta \equiv \omega'^2/K^2.$$

Note that a non-pole term proportional to κ^2 has been added in S_1 and then subtracted out in S_2. (As a point of reference it might be remarked that S_1 happens to coincide with the Born approximation taken with form factors at the verticies.) The term S_1 has already been evaluated by Iddings[9] and when added to the recoil corrections S_R one has

$$S_R + S_1 = - (34.5 \pm 2) \text{ ppm} \tag{4.14}$$

Separately S_R and S_1 have logarithmic divergences in the terms proportional to κ^2 but when they are added these divergences cancel provided $F_2(k^2) \to 0$ as $(-k^2) \to \infty$, so that the integral $\int (dK/K)\, F_2^2(-K^2)$ converges for $K^2 \to \infty$. This condition is automatically satisfied however if we insist upon an unsubtracted spectral form for the photon propagator so that quantum electrodynamics is a one parameter theory and quantities such as the vacuum polarization contribution to the Lamb shift can be computed as tests of the validity of the theory.[35] The detailed numerical calculations of Iddings[9] show that once the form factors F_1 and F_2 are fitted to their observed r.m s. radii (and provided $F_2 \to 0$ $(-k^2) \to \infty$) there is suprisingly little freedom ($< \pm$ 2 ppm) in the value of S_1.

Most of the contribution to S_1 comes from the first factor in the pole term which is linear in the total moment. The structure correction from this factor contributes - 26.6 ppm using measured form factors whereas the pole term proportional to $\kappa(1+\kappa)$ subtracts another - 6.2 ppm. Finally the non pole term is responsible for only + 1.3 ppm.

For a proton without excitations we return to the rigid baseball proton limit of the models in Section III. Furthermore we can reduce the result for ν_s to the form found by Zemach[13] for structure corrections in the non-relativistic limit by taking the $M \to \infty$ limit in which case only Eq. (4.11) survives. The last or non pole term in Eq. (4.11) when added to the recoil terms S_R converges for $K \to \infty$ as remarked above. In the $M \to \infty$ limit it vanishes as a result of the $\frac{1}{M}$ factor out in front of the integral. We may do the K_o integral as a contour integral in the complex K_o plane. Writing

$$K_\mu = (K_o, \underline{k})$$
$$k = |\underline{k}|$$
$$d^4K = d^3k\, dK_o$$

we see that poles occur at the roots of $K_o^2 + K^2 = \pm 2iM K_o$ or

$$K_o = \pm \frac{ik^2}{2M}\left(1 + 0\left(\frac{1}{M^2}\right)\right) \tag{4.15}$$

and

$$K_o = \pm 2iM\left(1 + 0\left(\frac{1}{M^2}\right)\right) \tag{4.16}$$

In the limit $M \to \infty$ it is only the residue at

$$K_o = \frac{ik^2}{2M}$$

that survives as we close the contour in the upper half plane and Eq. (4.11) reduces to

$$S_1 \underset{M\to\infty}{\to} \frac{8}{\pi a_o} \int_o^\infty \frac{dk}{k^2} \left[F_1(-k^2)\, G_M(-k^2) - 1 \right] \tag{4.17}$$

where

$$G_M(-k^2) \equiv \frac{F_1(-k^2) + \kappa\, F_2(-k^2)}{1 + \kappa} \tag{4.18}$$

is the magnetic form factor of the proton. Introducing the spherical charge and magnetic densities $\rho_c(R)$ and $\rho_M(R)$ respectively

$$\begin{aligned} F_1(-k^2) &= \int d^3R e^{i\underline{k}\cdot\underline{R}} \rho_c(R) \\ G_M(-k^2) &= \int d^3R' e^{-i\underline{k}\cdot\underline{R}'} \rho_M(R') \end{aligned} \tag{4.19}$$

and using the identity (see Eq. (3.40))

$$\int d^3k \left(\frac{1 - e^{i\underline{k}\cdot(\underline{R}-\underline{R}')}}{k^4} \right) = \pi^2 \left| \underline{R}-\underline{R}' \right| \tag{4.20}$$

we obtain

$$S_1 = -\frac{2}{a_o} \left\langle \left| \underline{R}-\underline{R}' \right| \right\rangle + O\left(\frac{1}{M}\right) \tag{4.21}$$

where

$$\langle|\underline{R}-\underline{R}'|\rangle = \int d^3R d^3R' \rho_c(R)\, \rho_M(R')\,|\underline{R}-\underline{R}'| \quad . \tag{4.22}$$

This is just the form found in Eq. (3.44).

So far the only attempts at calculations beyond S_1 have been estimates[9,21,22] of the contribution of the N*(1238) intermediate state to the absorptive amplitude in the dispersion integrals. In view of the dominant role of the 33 resonance in low energy photopion processes and its prominant contribution to Compton scattering, which it enhances by roughly one order of magnitude above the pole term at photon energy $\approx$ 300 MeV, it might be expected to contribute significantly to the polarizability correction to hfs. However it was found to play a totally negligible role contributing $\lesssim$ 1 ppm. This result may be understood in terms of the discussions that have been given of the smallness of the two photon exchange contributions to electron proton scattering at high energies.[36] The arguments given there for the smallness of a narrow resonance's contribution to the dispersive amplitude may be rephrased in language appropriate to the Cottingham formulation. After rotating the k_o contour to the imaginary axis as in Eq. (4.4) the hfs contribution is expressed as an integral over the Compton scattering of a virtual spacelike photon whose energy extends over a range that never approaches the 33 resonance region. No one energy region is dominant because the energy factors in Eqs. (4.12) and (4.13) are slowly varying and it is the total area under the photopion production curve that contributes to H_1 and H_2, or equivalently to S_2 and S_3. This area exceeds that under any one resonance as shown in Fig. 4.

Another point emphasized by the models discussed in Section III is that proton states of odd parity, in contrast with even parity ones excited by M1 transitions, play a prominant role in the polarizability contribution. We therefore turn to the continuum and the odd parity resonant excited states of the proton for further corrections to the hfs.

We first discuss the term S_2 about which we may hope to get some guide from physical Compton scattering. With the help of the sum rule, Eq. (4.9), we rewrite S_2 in the convenient ratio form

$$S_2 = \frac{9\kappa^2}{a_o M(1+\kappa)\pi^2} \int_0^\infty \frac{dK}{K} F_2^2(-K^2) \left\{ 1 - \frac{\int_{C(-K^2)}^\infty \frac{d\omega'^2}{\omega'^2} \operatorname{Im} H_1(-K^2,\omega')\, \beta\left(\frac{\omega'^2}{K^2}\right)}{F_2^2(-K^2) \int_{C(0)}^\infty \frac{d\omega'^2}{\omega'^2} \operatorname{Im} H_1(0,\omega')} \right\} \tag{4.23}$$

where

$$\beta(\theta) = \frac{4}{9} \left\{ 2(2-\theta)\sqrt{\theta(\theta+1)} - 3\theta + 2\theta^2 \right\} \tag{4.24}$$

Note that β satisfies

$$0 \leq \beta(\theta) \leq 1 \text{ for } 0 \leq \theta \tag{4.25a}$$

$$\beta(\theta) \to 1 - \frac{5}{18}\,\frac{1}{\theta} + \cdots \quad \text{as } \theta \to \infty \tag{4.25b}$$

$$\beta(\theta) \to \frac{4}{9}\left| 4\sqrt{\theta} - \theta + \cdots \right| \text{as } \theta \to 0 \,. \tag{4.25c}$$

Using Eq. (4.25b) it is immediately clear that in the limit $K \to 0$ the two terms of Eq. (4.23) cancel and therefore the integrand is well behaved at $K = 0$.

To proceed further we must resort to theoretical models and assumptions. If one assumes that the dependence of Im H_1 on the photon mass is a simple scale factor, namely

$$\text{Im } H_1(-K^2,\omega) = g(K^2) \text{ Im } H_1(0,\omega) \tag{4.26}$$

then it may be seen from Eq. (4.25a) and Eq. (4.8) according to which $C(-K^2) \geq C(0)$ that $S_1 > 0$ provided $g(K^2) \leq \left|F_2(-K^2)\right|^2$, and provided Im $H_1(0,\omega)$ is always of the same sign for $\mu < \omega < \infty$. As shown in Refs. 37 and 32 Im $H_1(0,\omega)$ is proportional to $\left|\sigma_P(\omega) - \sigma_A(\omega)\right|$ where $\sigma_P(\omega)$ $\left[\sigma_A(\omega)\right]$ is the total cross section for the absorption of a circularly polarized photon of laboratory energy ω by a proton with its spin parallel (anti-parallel) to the photon spin. Using the same fit to $(\sigma_P - \sigma_A)$ as in Fig. 4 we have numerically integrated Eq. (4.23) for the cases:

(a) $g(K^2) = \left|F_2(-K^2)\right|^2$

(b) $g(K^2) = 1$, $0 \leq K^2 = 1.5$ BeV^2; $g(K^2) = 0$ $K^2 > 1.5$ BeV^2

(c) $g(K^2) = 1$, all K^2.

We approximate $F_2(-K^2)$ by the form

$$F_2(-K^2) = 1/\left|1+(K/0.96M)^2\right|^2 .$$

The results are shown in Table I. Recall that this fit uses low energy S wave parameters, puts in the N*(1238) with predominately magnetic dipole excitation as well as the second resonance[38] and takes $(\sigma_P - \sigma_A)$ equal to zero beyond 0.9 BeV.

As a crude test of the effect of assuming that the sum rule Eq. (4.9) is only apparently saturated by 0.9 BeV - that the integrand in fact is large and oscillating beyond 0.9 BeV - we show also the effect of adding a tail to the above fit

$$\frac{(\sigma_P - \sigma_A)}{2} = -50\ \mu b \quad 0.9\ \text{BeV} < \omega < 1.8\ \text{BeV}$$

which improves the fit to the sum rule. The resulting values for S_2 including this tail are shown also in Table I. The sign and magnitude of S_2 are dependent on the details of the cut off as well as on the behavior of $(\sigma_P - \sigma_A)$ which may take positive as well as negative values at high energies. However in all cases the contributions to S_2 amount to no more than 1 - 2 ppm and thus have little impact on the hfs problem.

We next turn to S_3. Because physical Compton scattering tells nothing about H_2 and there is no sum rule to help normalize the calculation, the magnitude and sign expected for S_3 is even more uncertain than for S_2. Although there is some data on electropion production, the polarized inelastic electron-proton data that is actually required does not at this time exist. We are interested in finding if any terms in Im H_2 are candidates for contributing sizably to S_3 in Eq. (4.13) or whether a sensitivity of the resulting value for S_3 to unknown features of the electropion amplitude can be identified.

In the beginning, at least, we are making only a rough search for polarization contributions, so we may use the static model and thereby simplify the kinematics and spinology. This approximation will certainly suffice for order of magnitude calculations. The small nucleon isoscalar anomalous moment will also be neglected; thus $\kappa \equiv \kappa_p \tilde{=} - \kappa_n$.

Our method for separating Im H_2 out of the electropion production amplitude is the following. We contract Eq. (4.4) with polarization vectors $\underline{e}$ and $\underline{e}'$ for the incoming and outgoing (virtual) photons respectively and, in the gauge $e_o = e'_o = 0$, this gives

$$e'_j C^{ji} e_i = \underline{e}' \cdot \underline{C} \cdot \underline{e} = \frac{2i}{M^3} \left[\left\{ \underline{k} \cdot \underline{e}' \, \underline{\sigma} \cdot (\underline{k} \times \underline{e}) - \underline{k} \cdot \underline{e} \, \underline{\sigma} \cdot (\underline{k} \times \underline{e}') \quad (H_1 - M\omega H_2) \right\} \right. \\ \left. - M\omega \, \underline{\sigma} \cdot (\underline{e}' \times \underline{e}) \, (H_1 - K^2 H_2) \right] \tag{4.27}$$

where $-K^2 = k_\mu k^\mu < 0$.

Consider first of all the nucleon - one pion intermediate state with momenta as defined in Fig. 5. The photoproduction amplitude $\underline{A}.\underline{e}$ that we work with is normalized according to

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{photoproduction}} = \frac{\alpha_f^2 q}{\mu^2 k} \sum_{\substack{\text{spin and} \\ \text{charge states}}} \left| \underline{A} \cdot \underline{e} \right|^2 \tag{4.28}$$

where

$$q = \left| \underline{q} \right|, \; k = \left| \underline{k} \right| \text{ and } f^2 = \frac{\mu^2}{4M^2} \left(\frac{g^2_{\pi NN}}{4\pi} \right) = .08$$

is the pseudo vector pion-nucleon coupling constant. Unitarity in the static limit gives

$$\text{Im}\ (\underline{e}'\cdot\underline{C}\cdot\underline{e}) = -\ f^2\left(\frac{q}{\mu^2}\right)\int\frac{d\Omega_p}{4\pi}\sum_{\substack{\text{spin and}\\ \text{charge states}}}(\underline{A}\cdot\underline{e}')^\dagger\ (\underline{A}\cdot\underline{e}) \tag{4.29}$$

If we take for A the S wave threshold (electric-dipole and longitudinal dipole) production of pions we have

$$\begin{aligned}(\underline{A}\cdot\underline{e})_{\pi^+N} &= \sqrt{2}\ \ X^\dagger\left(\underline{\sigma}\cdot\underline{e} - \frac{\underline{\sigma}\cdot\underline{k}\ \underline{k}\cdot\underline{e}}{K^2+2\omega^2}\right)X\\ (\underline{A}\cdot\underline{e})_{\pi^0P} &= 0\end{aligned} \tag{4.30}$$

where $X^\dagger$, X are nucleon two component spinors. Using Eqs. (4.29) and (4.30) we find

$$\text{Im}\ H_2^{(S)}(-K^2,\omega) = -\ g(K^2)\ \frac{f^2M^2q}{\mu^2\omega}\ \frac{1}{(K^2+2\omega^2)} \tag{4.31}$$

As a second model we take for A the full relativistic Born approximation

$$\begin{aligned}(\underline{A}\cdot\underline{e})_{\pi^+N} &= \sqrt{2}\ \ X^\dagger\left(\sigma\cdot e + \frac{\underline{\sigma}\cdot(\underline{k}-\underline{q})(2\underline{q}-\underline{k})\cdot\underline{e}}{K^2+2\omega^2-2\underline{q}\cdot\underline{k}} + \frac{i(1+2\kappa)}{2M\omega}\ \underline{q}\cdot(\underline{k}\times\underline{e})\right)X\\ (\underline{A}\cdot\underline{e})_{\pi^0P} &= -\ \frac{(1+2\kappa)}{2M\omega}\ X^\dagger\left(\omega^2\underline{\sigma}\cdot\underline{e} + \underline{\sigma}\cdot\left(\underline{q}\times(\underline{k}\times\underline{e})\right)\right)X\end{aligned} \tag{4.32}$$

We find from Eqs. (4.27) and (4.29)

$$\begin{aligned}\text{Im}\ H_2^{(B)}(-K^2,\omega) = g(K^2)\ \frac{f^2M^2}{\mu^2\omega k}\Bigg[&-\frac{1}{2}\ Q_o + \frac{3}{2}\ \frac{q}{k}\ Q_1 - \left(\frac{q}{k}\right)^2\ Q_2\\ &+\frac{(1+2\kappa)}{2M\omega}\left\{\frac{5}{6}(Q_o-Q_2) - \frac{3}{5}\left(\frac{q}{k}\right)(Q_1-Q_3)\right\}\Bigg]\end{aligned} \tag{4.33}$$

where $Q_\ell = Q_\ell\left(\frac{K^2+2\omega^2}{2kq}\right)$ are the Legendre functions of the second kind. The $\pi^o P$ amplitude of Eq. (4.32) makes only a negligible 1/M correction to the smallest terms of Eq. (4.33).

If we take for A the N*(1238) amplitude with resonant magnetic dipole excitation only

$$(\underline{A}\cdot\underline{e})_{\pi^+ N} = \frac{\sqrt{2}\, i\mu^2(1+2\kappa)e^{i\delta_{33}}}{12M\, q^3\, f^2} \sin\delta_{33}\, X^\dagger \left\{2\underline{q}\cdot(\underline{k}\times\underline{e}) - i\underline{\sigma}\cdot\left[\underline{q}\times(\underline{k}\times\underline{e})\right]\right\} X$$

$$(\underline{A}\cdot\underline{e})_{\pi^o P} = \sqrt{2}\ (\underline{A}\cdot\underline{e})_{\pi^+ N} \qquad (4.34)$$

we find

$$\mathrm{Im}\, H_2^{(R)}(-K^2,\omega) = g(K^2)\, \frac{\mu^2(1+2\kappa)^2 \sin^2\delta_{33}}{48\omega\, q^3\, f^2} \qquad (4.35)$$

with δ_{33} the pion nucleon phase shift in the 3-3 channel.

Finally taking for A the Fubini, Nambu, Wataghin[39] approximation to the full static model photoproduction amplitude

$$(\underline{A}\cdot\underline{e})_{\pi^+ N} = \sqrt{2}\, X^\dagger \left\{\sigma\cdot e + \frac{\underline{\sigma}\cdot(\underline{k}-\underline{q})(2\underline{q}-\underline{k})\cdot\underline{e}}{K^2+2\omega^2-2\underline{k}\cdot\underline{q}}\right.$$

$$\left. + \frac{i\mu^2(1+2\kappa)\, e^{i\delta_{33}} \sin\delta_{33}}{12M\, q^3\, f^2}\left[2\underline{q}\cdot(\underline{k}\times\underline{e}) - i\underline{\sigma}\cdot\left[\underline{q}\times(\underline{k}\times\underline{e})\right]\right]\right\} X \qquad (4.36)$$

$$(\underline{A}\cdot\underline{e})_{\pi^o P} = \frac{i\mu^2(1+2\kappa)\, e^{i\delta_{33}} \sin\delta_{33}}{6M^2\, q^3\, f^2}\, X^\dagger \left[2\underline{q}\cdot(\underline{k}\times\underline{e}) - i\underline{\sigma}\cdot\left[\underline{q}\times(\underline{k}\times\underline{e})\right]\right] X$$

we find

$$\text{Im } H_2^{FNW}(-K^2,\omega) = g(K^2)\left\{\frac{f^2M^2}{\mu^2\omega k}\left[-\frac{1}{2}Q_o+\frac{3}{2}\left(\frac{q}{k}\right)Q_1-\left(\frac{q}{k}\right)^2 Q_2\right]\right.$$
$$\left.+\frac{\mu^2(1+2\kappa)^2\sin^2\delta_{33}}{48\omega\, q^3 f^2}\right\}. \qquad (4.37)$$

In Eqs. (4.31), (4.33), (4.35) and (4.37) an ad hoc form factor $g(K^2)$ has been added. Table II gives the results for S_3 for each smplitude taken with three choices of $g(K^2)$:

(a) $g(K^2) = \left|F_2(-K^2)\right|^2$

(b) $g(K^2) = 1$, $0 \leq K^2 \leq 1.5$ BeV^2; $g(K^2) = 0$ $K^2 > 1.5$ BeV^2

(c) $g(K^2) = 1$, all K^2.

We also make a rough estimate of the N*(1238) - π intermediate state.[38] The momenta are labeled in Fig. 6. The amplitude is

$$(\underline{B}\cdot\underline{e})_{\pi^+N^{*o}} = X_j^\dagger\left(e_j+\frac{(k-q)_j\,(2\underline{q}-\underline{k})\cdot\underline{e}}{K^2+2\omega^2-2\underline{k}\cdot\underline{q}}\right)X$$

$$(\underline{B}\cdot\underline{e})_{\pi^-N^{*++}} = (\underline{B}\cdot\underline{e})_{\pi^+N^{*o}} \qquad (4.38)$$

$$(\underline{B}\cdot\underline{e})_{\pi^oN^{*+}} = 0.$$

where X_j is the two componant polarization spinor for the N* satisfying

$$\sum_{\text{Spins}} X_j X_i^\dagger = \left(\delta_{ji} - \frac{1}{3} \sigma_j \sigma_i \right) \quad . \tag{4.39}$$

We find

$$\text{Im } H_2(-K^2,\omega) = g(K^2) \frac{1}{3} \frac{f_*^2 M^2}{\mu^2 \omega k} \left[+ \frac{1}{2} Q_o - \frac{3}{2} \left(\frac{q}{k}\right) Q_1 + \left(\frac{q}{k}\right)^2 Q_2 \right] \tag{4.40}$$

where f_* is the NπN* coupling constant $f_*^2 = 0.39$. Numerical results are shown at the bottom of Table II.

V. CONCLUSIONS

There can be but one conclusion from the varied numbers in these tables: We don't know enough about the amplitude of virtual photon absorption by a proton to calculate the polarizability correction to hfs with any confidence using dispersion theory. Recall from Eq. (4.12) and Eq. (4.13) that the input for the structure contribution ν_S/ν_F is virtual photon absorption by a proton integrated over all energies and space-like masses of photons. The weighting function in the integrals is only slowly convergent at high energies and masses. This is the analogue of our earlier observation in the Schrodinger models that high momentum components of the intermediate electron state corresponding to the electron at the proton surface are important and that numerous excited proton states play an important role in hfs.

With the aid of the sum rule Eq. (4.9) based on the low energy theorem and the no subtraction assumption in the dispersion relation for H_1 the S_2 contribution can be tied down to a small value as shown in Table I. Barring a very large and unanticipated enhancement in Im $H_1(-K^2,\omega)$ for large $-K^2 < 0$ we cannot hope to find any significant contribution to the hfs from this term.

We turn next to S_3 and the entries of Table II. Photoexcitation of the N*(1238) is the best known and calculable of the amplitudes but plays only a minor role, as noted earlier, contributing < 1 ppm.

The electric dipole production of S-wave charged pions has a threshold value fixed by the Kroll-Ruderman theorem for real photons (and in the limit of a massless π^+ being produced). Depending on the form factor appropriate

to this amplitude it may contribute up to 2-3 ppm in the direction of removing the apparent discrepancy. However much of its contribution is apparently cancelled when we also include in the pion photoproduction amplitude the meson current contribution in Born approximation and magnetic dipole photoproduction to the 33 resonance state only as in Eq. (4.37). Nevertheless very different and cut-off dependent results are found from the model of Eq. (4.33) which includes a Pauli coupling to the isovector moment of the proton in Born approximation. This is because the magnetic dipole amplitude continues to grow with increasing photon energy in this model and the integration over Im $H_2(-K^2,\omega)$ acquires very sizable contributions for large ω. Furthermore the numerical results obtained in this way are sensitive both in sign and magnitude to modifications of the Born approximation amplitudes by poorly known phase shifts in the non - 33 channels. The last row in Table II shows that an N*(1238) plus pion state in the photo absorption amplitude is also not a promising candidate for a hfs contribution.

While completing this write up of our calculations we have received a letter from Dr. Francoise Guérin[40] at Orsay describing a dispersion analysis of the polarizability contribution to hfs along parallel lines to our discussion of this section. She reports a contribution of $\lesssim 1$ ppm from the electric dipole photopion production amplitude as well as a negligible contribution from higher resonances.

We find ourselves then with no clear candidate to add even 10-20 ppm to the hfs. At the same time there is enough sensitivity in the poorly calculable amplitudes so that we can't be sure that such a contribution

isn't present. In this our dilemma is similar to that encountered by attempts[41] to use the Cottingham approach to calculate the neutron-proton mass difference. For this problem it is the isovector part of the non spin flip forward virtual Compton amplitude that must be determined and approximations similar to those we've made above also fail to predict a neutron that is more massive than the proton. As is abundantly clear from the recent rash of analyses[42] triggered by the work of Dashen and Frautschi[43] the detailed behavior of the high energy and photon mass region is very important.

We might try to achieve further progress by looking into the possibility of a subtracted dispersion relation for H_2 in Eq. (4.13). At high energies the K^2 dependence of Im $H_2(-K^2,\omega)$ can only be guessed and as a liberal guess we may ignore form factors altogether setting

$$\text{Im } H_2(-K^2,\omega') = \text{Im } H_2(0,\omega') \quad \text{for } \omega' > 2M. \tag{5.1}$$

The orders of integration in Eq. (4.13) can then be interchanged and we obtain

$$(S_3) \xrightarrow[\text{high energy}]{} -\frac{6}{a_o(1+\kappa)\pi^2} \int_{2M}^{\infty} d\omega' \text{ Im } H_2(0,\omega') . \tag{5.2}$$

Whether or not this integral converges is unknown although it exists for all of the models in Table II. A numerical evaluation based on Eq. (4.31) gives < 2 ppm to the hfs from Eq. (5.2).

With the assumption that the dispersion integral in Eq. (4.7) doesn't exist we make a subtraction writing in place of Eq. (4.7)

$$H_2(k^2,\omega) = - \frac{\kappa F_2(F_1+\kappa F_2)}{(k^2)^2 - 4M^2\omega^2} + G(k^2) \tag{5.3}$$

where we drop the dispersion integral itself as having contributed negligibly to the hfs. Our idea here is to determine G so that in Eq. (4.4) it adds 20-40 ppm to hfs; this fixes the integral

$$- \frac{6}{a_o M(1+\kappa)\pi^3} \int \frac{d^4K}{K^4} K_o^2 \, G(-K^2) = (20\text{-}40) \times 10^{-6} \tag{5.4}$$

or

$$\int_o^\infty dK^2 \, G(-K^2) = - (30\text{-}60) \quad .$$

However an inescapable implication of this assumption is that there is a sizable difference between electron-proton and positron-proton scattering. Since we introduce the subtraction constant in order to increase the attraction between the electron and proton and thereby to increase the hfs it is not surprising that the effect of G is to increase the cross section for electron-proton scattering above that for positron-proton scattering.[44] This appears to be in conflict with observation[45] and so we abandon the idea of a simple subtraction term. This is not a rigid conclusion for two reasons. First of all one can tailor the K^2 dependence of $G(-K^2)$ so that the hfs is accounted for by the integral Eq. (5.4) while at the

same time the observed cross sections at large K^2 are not altered. Secondly the scattering is observed at finite angles and so requires, for a complete analysis of the two photon exchange contribution to the electron-positron difference, knowledge of virtual Compton scattering in non forward directions as well as for time like photon masses. Thus many more amplitudes and parameters are introduced beyond those appearing in the forward spin flip Compton amplitude for virtual space like photons. For these reasons it does not appear profitable to explore this approach beyond the discussion of Ref. [44]. The introduction of the subtraction constant $G(k^2)$ in Eq. (5.3) leads us directly back to the results computed there if we keep only the leading terms for small momentum transfer in the electron (positron) - proton scattering amplitude.

Further counsel against searching for significant contributions to the hfs from high energy parts of the two photon amplitude come from the current commutator sum rules of Bjorken.[31]

The situation as we view it presently is as follows: A relativistic dispersion approach fails to provide any real insight or quantitative contributions towards a resolution of the hfs problem.

The non relativistic Schrodinger models show that sizable albeit model dependent polarizability contributions may very well be present and it is indeed puzzling as to where they have been lost in the dispersion approach. These models also suggest that one should not expect to find the predominant polarizability contributions to the hfs in the magnetic dipole excitation of the N*(1238) resonance. The electric excitations are also important.

Both in the hfs as well as the Lamb shift studies[6,46] the apparent discrepancies between theory and experiment are comparable to the size of the proton recoil current and structure contributions. The possibility cannot be ruled out with certainty that a better understanding and formulation of the contributions due to proton dynamics won't resolve these discrepancies. We cannot claim to understand or to have computed their contributions to the hfs to an accuracy of better than ~ 10 ppm.

A new determination of the fine structure constant from the fine structure[14] of H or D together with the Ruderman[47] analysis of the chemical shift of the muon moment measurements may provide a purely experimental resolution of the hfs problem itself but there would then still remain the discrepancy in the comparison of theory with experiment in the Lamb shift.[48]

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FOOTNOTES AND REFERENCES

1. E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).
2. S. B. Crampton, D. Kleppner and N. F. Ramsey, Phys. Rev. Letters 11, 338 (1963).
3. S. J. Brodsky and G. W. Erickson, Ann. Phys. (N.Y.) in press (1966).
4. R. Blumenthal, D. Ehn, W. Faissler, P. Joseph, L. Lanzerotti, F. Pipkin, and D. Stairs, Phys. Rev. 144, 1199 (1966).
5. R. Talman, B.A.P.S., Ser. II, Vol. 11, No. 3, p. 380 (1966).
6. R. T. Robiscoe, Phys. Rev. 138, A22 (1965).

   R. T. Robiscoe, Phys. Rev. Letters (July 11 Issue) (1966).

   Measurements of the Lamb shift, i.e. the $^2S_{1/2} - {}^2P_{1/2}$ splitting, in deuterium have also been reported (R. Robiscoe and B. Cosens, B.A.P.S., Ser. II, Vol. 11, No. 1, p. 62 (1960)).
7. A. J. Layzer, J. Math Phys. 2, 292, 308 (1961). G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 271, 447 (1965).
8. S. Fenster and Y. Nambu, Supp. Prog. Theo. Phys. (Kyoto) (Yukawa Festschrift) p. 250 (1965).
9. C. K. Iddings, Phys. Rev. 138, B446 (1965).
10. W. Newcomb and E. Salpeter, Phys. Rev. 97, 1146 (1955).
11. R. Arnowitt, Phys. Rev. 92, 1002 (1953).
12. W. Cottingham, Ann. Phys. (N.Y.) 25, 424 (1963).
13. A. C. Zemach, Phys. Rev. 104, 1771 (1956).
14. S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., Phys. Rev. 89, 98 (1953).

    Their value of the fine structure constant is $\alpha^{-1} = 137.0388$ and

the error ± 9 ppm is a two standard deviation figure. This figure is arrived at by adding the frequencies for the $^2P_{3/2} - {}^2S_{1/2}$ and the $^2S_{1/2} - {}^2P_{1/2}$ transitions in deuterium and comparing with the theoretical formula for the fine structure splitting $^2P_{3/2} - {}^2P_{1/2}$. If the additional + 0.3 MH found by Robiscoe (Ref. 6) in H is finally confirmed in D and if the $^2P_{3/2} - {}^2S_{1/2}$ transition does not decrease by a compensating amount there will be an increase in the measured value of α of + 13 ppm. The theoretical hfs in Eq. (2.2) is increased by 26 ppm which would bring it into agreement with Eq. (2.1) to within 20 ppm. This would remove the hfs anomaly but a serious disagreement would remain in the Lamb shift itself between theory and experiment.

15. G. Breit, Phys. Rev. 35, 1447 (1930).

16. N. Kroll and F. Pollack, Phys. Rev. 84, 594 (1951); 86, 876 (1952).

17. R. Karplus, A. Klein and J. Schwinger, Phys. 84, 597 (1951).
R. Karplus and A. Klein, Phys. Rev. 85, 972 (1952).

18. A. J. Layzer, Nuovo Cimento 33, 1538 (1964).

19. D. E. Zwanziger, Nuovo Cimento 34, 77 (1964).

20. W. E. Cleland, J. M. Bailey, M. Eckhause, V. W. Hughes, R. M. Mobley, R. Prepost and J. E. Rothberg, Phys. Rev. Letters 13, 202 (1964).

21. C. Iddings and P. Platzman, Phys. Rev. 115, 919 (1959).

22. A. Verganelakis and D. Zwanziger, Nuovo Cimento 39, 613 (1965).

23. A. Bohr, Phys. Rev. 73, 1109 (1948).

24. F. Low, Phys. Rev. 77, 361 (1950).

25. H. A. Bethe and E. E. Salpeter, "Quantum Mechanics of One- and Two-Electron Atoms", Academic Press (N.Y.) 1957.

26. J. D. Bjorken and S. D. Drell, Relativistic Quantum Mechanics (McGraw-Hill Co., N.Y.) 1964.

27. C. Iddings and P. Platzman, Phys. Rev. 113, 192 (1959). See also R. Faustov, Nuclear Physics 75, 669 (1966).

28. We have recalculated the structure corrections to the deuterium hfs reproducing the Low results in the polarizable limit $\overline{\Delta W R} \to 0$ as is appropriate in this case.

29. There is a simple modification of Eq. (3.56) and the development leading to it if we include retardation corrections in the interaction Hamiltonian as remarked below Eq. (3.5). This refinement was suggested by D. R. Yennie. Instead using a static magnetic potential in Eq. (3.5) we can compute the interaction between the electron and proton by coupling them directly to the radiation field. The transverse photon being exchanged between electron and proton has the same vertices as given by the numerator factors in Eq. (3.5) but the interaction now has an additional energy denominator from the photon propagator itself. In the light of our discussions below Eqs. (3.45) and (3.46) we can compute the interaction very simply since the electron can be treated relativistically in Born approximation. According to Eq. (3.5) the photon exchanges mumentum $\vec{p}$ between the electron and proton with an amplitude $\frac{1}{|\underline{p}|^2}$, the fourier transform of $\frac{1}{|\underline{r}-\underline{R}|}$ and in Eq. (3.45) the combination

$$\left(\frac{1}{p^2}\right) \frac{1}{p + W_A - W_0} \left(\frac{1}{p^2}\right)$$

represents the two static interactions with the intermediate energy

denominator sandwiched in between. If we now allow the transverse photon to propagate between the electron and proton there are six time orderings of its vertices - i.e. the electron or proton can emit or absorb the photon before or after the instantaneous coulomb interaction described by Eq. (3.4). The combination of energy denominators gives in place of the above

$$\frac{1}{2p}\frac{1}{p^2}\left[\frac{1}{2p(p + W_A - W_0)} + \frac{1}{(p + W_A - W_0)2p} + \frac{1}{(p + W_A - W_0)^2}\right] =$$

$$\frac{1}{p^2}\frac{1}{p + W_A - W_0}\frac{1}{p^2}\left\{\frac{p + \frac{W_A - W_0}{2}}{p + W_A - W_0}\right\}$$

The additional factor

$$\frac{2p + W_A - W_0}{2p + 2(W_A - W_0)}$$

is unity in the polarizable proton limit and falls to a minimum of $\frac{1}{2}$ in the rigid baseball limit. In Eq. (3.56) it changes the logarithm as follows:

$$\ell n \frac{z + \Delta W_A \left|\underline{R}-\underline{R}'\right|}{z} \Rightarrow \frac{1}{2}\left\{\ell n \frac{z + \Delta W_A \left|\underline{R}-\underline{R}'\right|}{} + \frac{\Delta W_A \left|\underline{R}-\underline{R}'\right|}{z + \Delta W_A \left|\underline{R}-\underline{R}'\right|}\right\}.$$

30. If the retardation effects of Footnote (29) are included the numerical values plotted in Fig. 2 are reduced by approximately 10% for $W_A - W_0 = 400$ MeV and by close to 20% for $W_A - W_0 \sim 1$ BeV.

31. J. D. Bjorken, Phys. Rev. in press (1966).

32. S. D. Drell and A. C. Hearn, Phys. Rev. Letters 16, 908 (1966).

33. F. Low, Phys. Rev. 96, 1428 (1954).

34. M. Gell-Mann and M. L. Goldberger, Phys. Rev. 96, 1433 (1954).

35. S. D. Drell, A. C. Finn, and A. C. Hearn, Phys. Rev. 136, B1439 (1964).

36. c.f. S. D. Drell and F. Zachariasen, "Electromagnetic Structure of Nucleons", Oxford Univ. Press (1961) Chapter 2.

37. M. Gell-Mann, M. L. Goldberger, and W. Thirring, Phys. Rev. 95, 1621 (1954).

38. M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 (1963).
Ph. Salin, Nuovo Cimento 28, 1294 (1963).

39. S. Fubini, Y. Nambu and V. Wataghin, Phys. Rev. 111, 329 (1958).

40. F. Guérin (private communication)

41. A. Finn (unpublished)

42. H. Pagels, Phys. Rev. 144, 1261 (1966).
H. Fried and T. Truong, Phys. Rev. Letters 16, 559 (1966).
G. Shaw and D. Wong, Phys. Rev. - to be published

43. R. Dashen and S. Frautschi, Phys. Rev. 135, B1190 (1964).
R. Dashen, Phys. Rev. 135, B1196 (1964).

44. S. D. Drell and J. D. Sullivan, Physics Letters 19, 516 (1966).

45. A. Browman, F. Liu, and C. Schaerf, Phys. Rev. 139, B1079 (1965).
Further experiments on this ratio are in progress at C.E.A. (L. Hand) and Cornell (J. deWire).

46. E. E. Salpeter, Phys. Rev. 87, 328 (1952); 89, 92 (1953).
T. Fulton and P. C. Martin, Phys. Rev. 95, 811 (1954).

47. M. A. Ruderman, Phys. Rev. in press (1966).

48. The latest published theoretical number on the Lamb shift in H is 1057.64 ± .21 MH [G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 271, 447 (1965)]. This number is further reduced by 0.14 MH to 1057.5 MH by the completion of the fourth order radiative correction by M. Soto (to be published). The experimantal result of Ref. 6 exceeds this ~ 0.5 ± 0.1 MH and that of Ref. 14 by ~ 0.3 ± 0.1 MH. Since the Lamb shift itself is proprotional to $\alpha^3 Ry$ an increase in α of ~ (70-200) ppm would be required to remove this discrepancy, short of theoretical formula modifications or addenda, and this would disrupt the hfs comparison by (an intolerable) ~ (150-400) ppm.

Table I

Model Calculations of S_2 in Parts per Million

	(a) $g(K^2)=\left[1+(K/0.96M)^2\right]^{-4}$	(b) $g(K^2)=\theta(1.5\text{BeV}^2-K^2)$	(c) $g(K^2)=1$
Sum Rule Fit to $(\frac{\sigma_P-\sigma_A}{2})$ Gives $\kappa = 2.1$	0.5	-1.5	-1.5
Sum Rule Fit with tail to $(\frac{\sigma_P-\sigma_A}{2})$ Gives $\kappa = 1.8$	1	-1	-1

Table II

Model Calculations of S_3 in Parts per Million

Nucleon-Pion	(a) $g(K^2)=\left[1+(K/0.96M)^2\right]^{-4}$	(b) $g(K^2)=\theta(1.5\text{BeV}^2-K^2)$	(c) $g(K^2)=1$
S Wave Threshold Electric Dipole Amplitude	+0.9	+2.1	+2.5
Born Amplitude	-0.6	-0.6	-27.0
N*(1238) Magnetic Dipole Amplitude	-0.3	-0.6	-0.6
Born Amplitude with Final State Enhancement in 33 Channel	-0.1	+0.2	+0.3
N*(1238) - Pion	-.01	-0.2	-0.2

FIGURE CAPTIONS

Fig. 1 - Model of proton structure with "quarkette" of charge +e and mass μ with coordinate $\underline{R}$ moving about a fixed center of mass at the origin. The electron in the hydrogen atom is at $\underline{r}$.

Fig. 2 - Ratio of polarizability to ground state contribution to the hfs for the one quarkette excited P state in the Gaussian model (Eq. 3.98).

Fig. 3 - Two photon exchange amplitude for the proton structure contribution to the hfs in hydrogen. $C_{\mu\nu}$ is the forward spin-flip Compton amplitude for virtual photons of momentum k_μ.

Fig. 4 - Photopion production cross section in H according to the model of Gourdin-Salin (Ref. 37); taken from (Ref. 31), σ_P (σ_A) is the cross section for absorbing a circularly polarized photon with its spin parallel (antiparallel) to the proton spin.

Fig. 5 - Kinematics for amplitude of single pion photoproduction by a virtual photon.

Fig. 6 - Kinematics for amplitude of virtual photoproduction of the N*(1238) plus a pion.

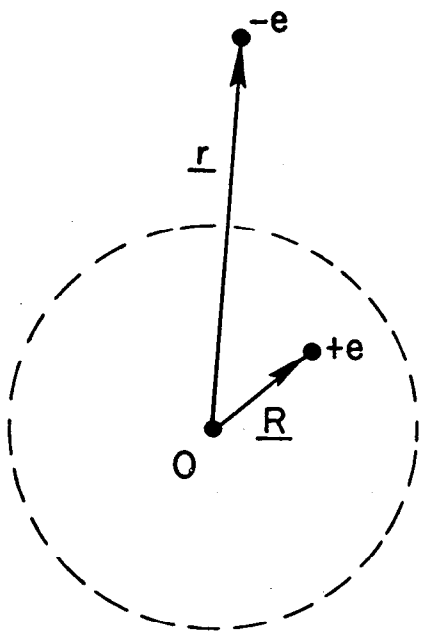

Fig. 1

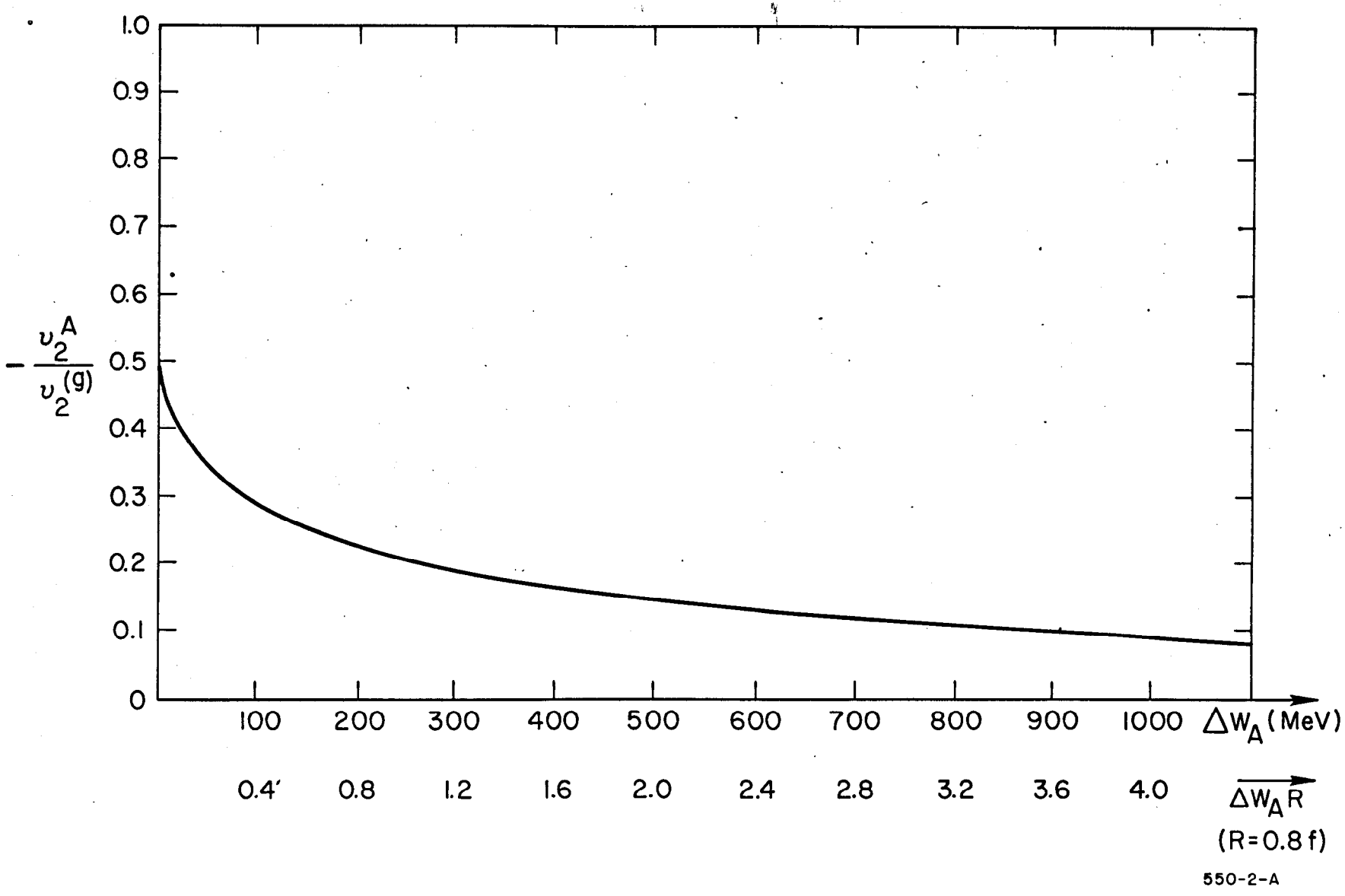
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$-\frac{v_2^A}{v_2^{(g)}}$
100
200
300
400
500
600
700
800
900
1000
ΔW_A (MeV)
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
$\overline{\Delta W_A R}$
(R=0.8 f)
550-2-A

Fig. 2

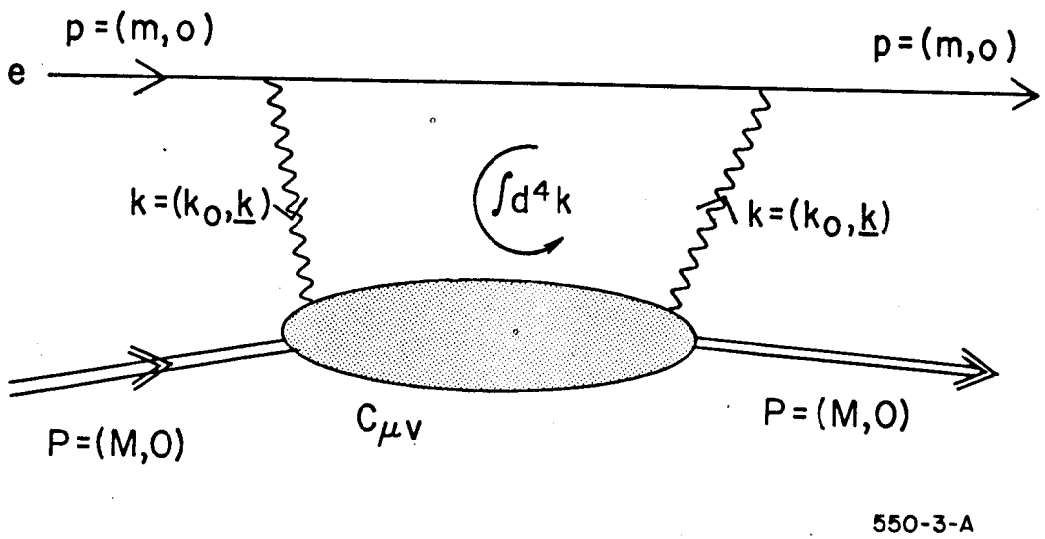

Fig. 3

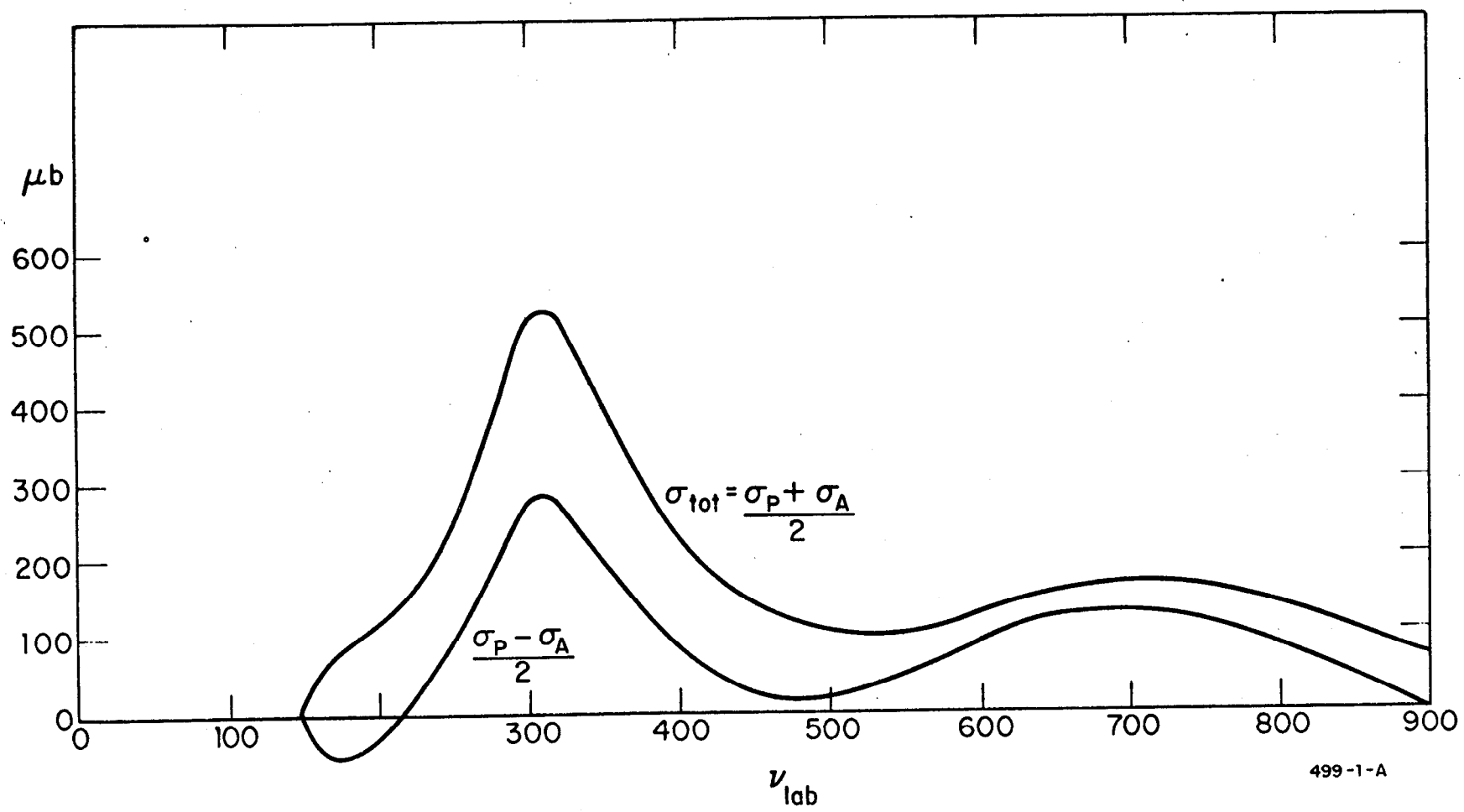
μb
600
500
400
300
200
100
0
0
100
300
400
500
600
700
800
900
$\sigma_{tot} = \frac{\sigma_P + \sigma_A}{2}$
$\frac{\sigma_P - \sigma_A}{2}$
ν_{lab}
499-1-A

Fig. 4

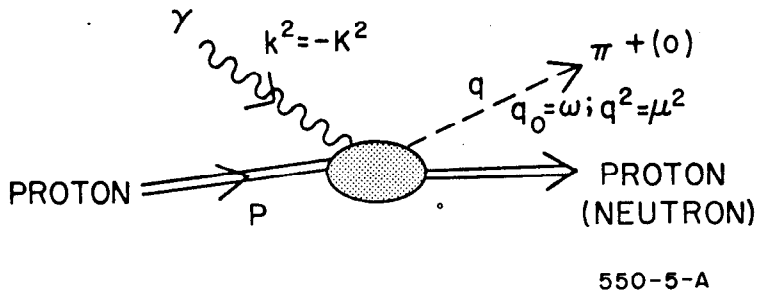

Fig. 5

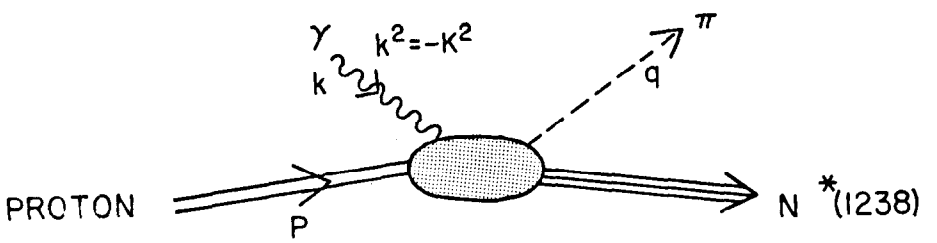

Fig. 6