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INELASTIC MODEL OF THE n-p MASS DIFFERENCE*

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Recently, two new derivations of the n-p mass difference have been given[1,2] which attempt to include inelastic Compton effects by exhibiting a so-called feedback term with the effect of reversing the overall sign of earlier estimates. The purpose of this note is to give a simple, field theoretic example of how the feedback mechanism may be expected to operate. We consider the difference, $\Delta\Sigma(\omega)$, of the nucleon self-energies obtained by iterating the difference, $\Delta\Sigma^{(0)}(\omega)$, of the FS-type[3] electromagnetic self-energies between an increasing number of virtual pions. This difference of sums over the subset of proper self-energy rainbow (or ladder) graphs, illustrated in Fig. 1, is supposed to approximate the n-p mass difference, and provide the mechanism for the reaction of inelastic, one photon plus many pion states back on the strong interaction determination of the nucleon masses. These graphs are chosen only because they can be generated by the iteration of a linear integral equation, which we do not attempt to solve.[4] Rather, we observe that, if a solution exists for which $\mathrm{Im}\Delta\Sigma$ is not pathological near threshold and vanishes rapidly for large ω, then the feedback mechanism can be sufficient to reverse the sign of the initial $\Delta\Sigma^{(0)}$ estimate. When evaluated with the aid of reasonable charge and moment form factors, the latter leads to the incorrect result for $\Delta m = m_p - m_n$, of amount $\Delta m^{(0)} = -Z\Delta\Sigma^{(0)}(m) \sim +\frac{1}{2}$ MeV, where Z denotes the nucleon wave function renormalization-constant.

We represent by $\delta\Sigma = x_p \delta\Sigma_p + x_n \delta\Sigma_n$ the sum of such self-energy graphs for the nucleon, obtained by the iteration of the simpler, electromagnetic $\delta\Sigma^{(0)} = x_p \delta\Sigma_p^{(0)} + x_n \delta\Sigma_n^{(0)}$. Here, $x_{p,n}$ denote isotopic projection operators for proton and neutron, respectively, while $\delta\Sigma_{p,n}$ and $\delta\Sigma_{p,n}^{(0)}$ are the corresponding self-energy functions. $\delta\Sigma^{(0)}$ is illustrated by the first term on the right side of Fig. 1; it is of order e^2 and is understood to be given in terms of realistic charge and moment distributions of the electromagnetic vertex. The model may thus be defined by the equation[5]

$$\delta\Sigma(-i\gamma\cdot p) = \delta\Sigma^{(o)}(-i\gamma\cdot p) + \frac{ig^2}{(2\pi)^4}\sum_{i=1}^{3}\int \frac{d^4k}{k^2+\mu^2}\,\gamma_5\tau_i\,\frac{1}{m+i\gamma\cdot(p+k)}\,\delta\Sigma\left(-i\gamma\cdot(p+k)\right)\frac{1}{m+i\gamma\cdot(p+k)}\,\gamma_5\tau_i \tag{1}$$

where we neglect the variation of pion and nucleon masses in Eq. (1) because $\delta\Sigma$ is already of order e^2. Both e^2 and g^2 are taken as renormalized, with $g^2/4\pi = 15$. Inserting isotopic representations, one easily finds, for $\Delta\Sigma = \delta\Sigma_p - \delta\Sigma_n$,

$$\Delta\Sigma(-i\gamma\cdot p) = \Delta\Sigma^{(o)}(-i\gamma\cdot p) - \frac{ig^2}{(2\pi)^4}\int \frac{d^4k}{k^2+\mu^2}\,\gamma_5\,\frac{1}{m+i\gamma\cdot(p+k)}\,\Delta\Sigma(-i\gamma\cdot(p+k))\,\frac{1}{m+i\gamma\cdot(p+k)}\,\gamma_5\,, \tag{2}$$

where $\Delta\Sigma^{(o)} = \delta\Sigma_p^{(o)} - \delta\Sigma_n^{(o)}$.

It is useful to employ for $\Delta\Sigma(\omega)$ the form of the very general representations[6] valid for $\Sigma(\omega)$,

$$\Delta\Sigma(\omega) = \int_{m+\mu}^{\infty} dn\left[\frac{\Delta\rho_+(n)}{n-\omega} + \frac{\Delta\rho_-(n)}{n+\omega}\right] \tag{3}$$

where $\Delta\rho_{\pm}(n) = \pm(n\mp m)^2\,\Delta\tau_{\pm}(n)$. Insertion of Eq. (3) into Eq. (2), followed by the computation of an elementary Feynman integral, produces

$$\Delta\Sigma(\omega) = \Delta\Sigma^{(o)}(\omega) + \frac{g^2}{8\pi^2}\int_{m+\mu}^{\infty} dn\left[\frac{\Delta\rho_+(n)}{n-m}\,f_+(n,\omega,\mu) + \frac{\Delta\rho_-(n)}{n+m}\,f_-(n,\omega,\mu)\right], \tag{4}$$

where

$$f_{\pm}(n,\omega,\mu) = \int_0^1 dx\left\{\frac{xm\,[m-(1-x)\omega]}{\mu^2(1-x)+xm^2-x(1-x)\omega^2} - \frac{1}{2}\,\frac{[n\mp(1-x)\omega]}{(n\mp m)}\,\ln\left[\frac{\mu^2(1-x)+xn^2-x(1-x)\omega^2}{\mu^2(1-x)+xm^2-x(1-x)\omega^2}\right]\right\} \tag{5}$$

which relations provide a linear equation for $\Delta\rho_{\pm}$, in terms of the absorptive part of $\Delta\Sigma^{(o)}(\omega)$ and the cuts of $f_{\pm}$. If a solution exists, we are interested in $\Delta\Sigma(m)$,

$$\int_m^\infty dn \left[(n-m)\Delta\tau_+(n) - (n+m)\Delta\tau_-(n)\right] = \Delta\Sigma^{(o)}(m) + \frac{g^2}{8\pi^2}\int_m^\infty dn \Big[(n-m)\Delta\tau_+(n)f_+(n,m,o) - (n+m)\Delta\tau_-(n)f_-(n,m,o)\Big], \tag{6}$$

where the $\Delta\rho_{\pm}$ have been replaced by the $\Delta\tau_{\pm}$ and, for simplicity, all dependence on μ has been dropped; this latter approximation does not lead to an infrared logarithm in $\Delta\Sigma(m)$, but introduces errors proportional only to $(\mu/m)^2$. Writing

$$f_{\pm}(n,m,o) = 1 - \ell_{\pm}(n) , \tag{7}$$

where

$$\ell_{\pm}(n) = \frac{1}{2}\int_o^1 dx \left[\frac{n\mp(1-x)m}{n\mp m}\right] \ln\left(1 + \frac{n^2-m^2}{xm^2}\right), \tag{8}$$

and defining the average value, ℓ, of Eq. (8),

$$\ell \equiv \frac{\int_m^\infty dn \left[(n-m)\Delta\tau_+(n)\ell_+(n) - (n+m)\Delta\tau_-(n)\ell_-(n)\right]}{\int_m^\infty dn \left[(n-m)\Delta\tau_+(n) - (n+m)\Delta\tau_-(n)\right]} \tag{9}$$

we may use the definition $\Delta m = -Z\Delta\Sigma(m)$ to obtain

$$\Delta\Sigma(m) = \Delta\Sigma^{(o)}(m) + \frac{g^2}{8\pi^2}(1-\ell)\,\Delta\Sigma(m) ,$$

or

$$\Delta m = \Delta m^{(o)}\left(1 - \frac{g^2}{8\pi^2}(1-\ell)\right)^{-1}. \tag{10}$$

For negative ℓ , or for sufficiently small positive $\ell(\ell < .57)$, Eq. (10) exhibits the desired change in sign. The value of ℓ depends on the detailed solution to the model, but it is apparent that the effect depends upon reasonable threshold behavior together with strong damping, at higher energies, of $\Delta\tau_{\pm}$.[7] For example, if a solution exists with $\Delta\tau_{+} \approx \Delta\tau_{-} = \Delta\tau$, where $\Delta\tau$ increases from threshold in a smooth way, and is essentially cut-off for $n > 2m$, then we may approximate $\Delta\tau$ by

$$\Delta\tau\,(n) \approx 2\Delta\tau_{o} \left\{ \left(\frac{n}{m} - 1 \;\; \theta\left(\frac{3}{2} - \frac{n}{m}\right) \theta\left(\frac{n}{m} - 1\right) + \left(2 - \frac{n}{m}\right) \theta\left(2 - \frac{n}{m}\right) \theta\left(\frac{n}{m} - \frac{3}{2}\right) \right\}$$

where $\Delta\tau_{o}$ is some constant. This, together with the relation

$$\frac{1}{2} \int_{o}^{1} dx(1-x)\, \ell n \left(1 + \frac{n^2 - m^2}{xm^2}\right) \approx \frac{n}{m} - 1 \;\; ,$$

approximately valid in the region $2m > n > m$, leads to the value $\ell = \frac{1}{2}$, satisfying our criterion. The magnitude of Eq. (10) should not be taken seriously; the model itself has only the virtue of providing a simple example of the feedback mechanism.

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2. H. M. Fried and T. N. Truong, Phys. Rev. Letters 16, 559 (1966), and Erratum, 16, 884 (1966). The relation of this method to that of the conventional calculation is described in the Brown University preprint "Feedback Mechanism for the n-p Mass Difference" by the same authors.

3. R. P. Feynman and G. Speisman, Phys. Rev. 94, 500 (1954). References 1 and 2 contain or lead to an exhaustive list of the relevant literature.

4. For simple $\Delta\Sigma^{(o)}$, this equation has been studied by many authors; e.g., D. Falk, Phys. Rev. 115, 1069 (1959).

5. More precisely, one should begin with the unrenormalized m_o, g_o, and include self-energy corrections to the pion and nucleon propagators and to the ps vertex; extract the appropriate renormalization constants which convert g_o to g; and then approximate the renormalized propagators by their respective pole terms, and each renormalized ps vertex by $\gamma_5 \tau_i$.

6. See, for example, M. Ida, Phys. Rev. 136, B1767 (1964).

7. Similar damping is essential to the success of the feedback mechanisms of references 1 and 2. Even if a solution with these properties does not exist here, it might still be possible to find a sufficiently small ℓ; otherwise, a damping factor, corresponding, e.g., to the use of more realistic ps vertex functions, must be included.

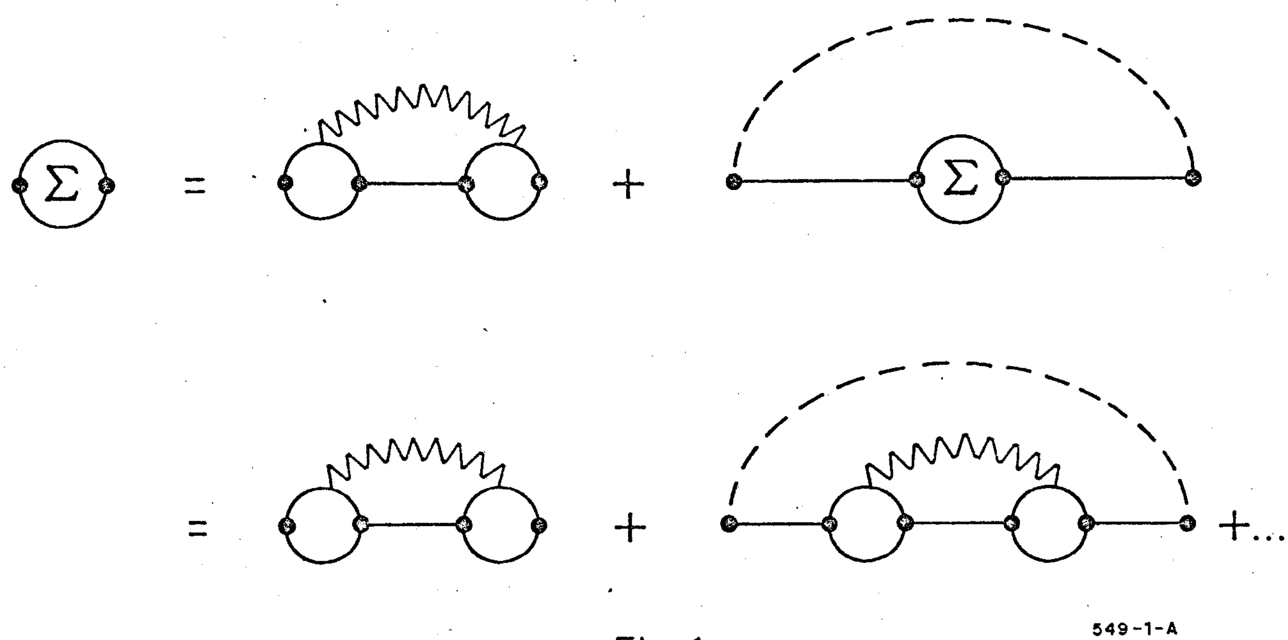

Fig. 1

Pictorial representation of an approximate integral equation for $\delta\Sigma$,
and its iterative solution.