GEOMETRICAL RECONSTRUCTION OF HIGH ENERGY INTERACTIONS IN SPARK CHAMBER MAGNET ARRANGEMENTS *

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SUMMARY

We describe a geometrical reconstruction procedure for track coordinates, taken from spark chambers in a non-uniform magnetic field. By a least-squares method we fit simultaneously all parameters (momenta, spatial angles, and the vertex coordinates) of the observed trajectories, which originate at the same vertex. A test run has been performed using simulated high energy events in a non-uniform magnetic field where the interaction vertex was assumed not to be measured.

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I. INTRODUCTION

The analysis of high energy interactions of elementary particles, using spark chambers in combination with an analyzing magnetic field, usually requires a geometrical optimization procedure in order to obtain from the measurements of track coordinates an estimate of the relevant physical quantities, such as momenta and spatial angles.[1] Complications arise when the magnetic fields are non-uniform and the interaction vertex cannot be measured. For example, in the case of the recently developed technique[2] of using streamer track chambers placed in magnetic fields, one might be interested to reconstruct trajectories passing through non-uniform field regions. Moreover, since the interaction usually takes place outside the sensitive region of the streamer track chamber, the vertex cannot be measured and has to be reconstructed.

Here we discuss a least-squares fitting procedure for a digital electronic computer, which has the following features:

   a. it fits coordinates to trajectories traversing regions of non-uniform magnetic fields;

   b. it optimizes simultaneously all measured trajectories and the co-ordinates of the interaction vertex, from where the trajectories originate.

Thus, the number of unknown parameters in our fitting problem is 3N + 3, where N is the number of observed trajectories due to one interaction; each trajectory carries three parameters (momentum and two spatial angles), and three parameters are due to the vertex.

It should be noted that due to the coupling of the vertex to the different trajectories, a strong correlation of the parameters can be expected. Therefore an essential result of the fitting program will be the covariant matrix of the parameters which has to be known for further kinematical optimization.

II. UNDERLINE METHOD

The basic idea applied is to fit by a least-squares method the coordinates of all observed trajectories originated at one vertex to the numerical solution of a differential equation. The differential equation describes the path of a charged particle through an arbitrary magnetic field; the numerically computed trajectory may be represented by a vector function F :

$$\left[x(s),\ y(s),\ z(s)\right]_J = F_J(\vec{\alpha},\ \vec{r}_o,\ s) \tag{1}$$

$J = 1, 2, \ldots, N$ refers to the different tracks, $\vec{\alpha}$ stands for the vector of

the parameters $1/P$, λ, ϕ of the different tracks

P = momentum (BeV/c)

λ = dip angle

ϕ = scattering angle projected in a horizontal plane, defined in $\vec{r}_o$.

$\vec{r}_o = (x_o,\ y_o,\ z_o)$ represents the coordinates of the unmeasured common vertex. The unmeasured vertex ties the initial conditions of the different trajectories together and was introduced as three more parameters to be fitted for. The differential equation to be solved for each trajectory has the general form:

$$\frac{d\vec{u}}{ds} = \frac{2.9979 \times 10^{-2}}{|P|}\ (\vec{u} \times \vec{B}) \tag{2}$$

$\vec{B}$ = the magnetic field (kG)

s = arc length (m)

$\vec{u} = \left(\frac{dx}{ds},\ \frac{dy}{ds},\ \frac{dz}{ds}\right).$

At the vertex one has the relations:

$$u_x = \cos \lambda \cdot \cos \phi$$

$$u_y = \cos \lambda \cdot \sin \phi$$

$$u_x = \sin \lambda\ .$$

In the non-linear least-square method[3] one finds the minimum of

$$\chi^2 = (m - m^*)^T \, G \, (m - m^*).\tag{3}$$

by an iterative procedure. $m_J = (x, y, z)_J$ is the column vector of measured quantities.

m^* stands for the coordinates of the computed track (1) and corresponds to the current choice of the parameters; G is a weight matrix for which usually the inverse of the covariance matrix containing the measurement errors is taken. For this program it is assumed that the (possibly reconstructed) spatial coordinates x, y, z, and their standard deviations are known. Let us define a vector β for the set of trajectories containing all parameters to be fitted for:

$$\beta = (\vec{\alpha}, \vec{r}_0).$$

By expanding Eq. (1) around a first estimate β_0, one obtains:

$$m^* \approx m_0^* + \frac{\partial F_J}{\partial \beta_0} \, \Delta\beta_0 + \ldots O(\Delta\beta_0^2).$$

The iterative procedure then yields corrections $(\Delta\vec{\alpha}, \Delta\vec{r}) = \Delta\beta$, which one relates to the residuals $\Delta c = (m - m^*)$ by:

$$\Delta\beta = (A^T G \, A)^{-1} \, A^T G \, \Delta c,\tag{4}$$

where   T   stands for "transposed"

$$A = \frac{\partial F_J}{\partial \beta}.$$

The covariance matrix of the fitted parameters can be obtained as:

$$E(\delta\beta \; \delta\beta^T) = (A^T G \, A)^{-1}.\tag{5}$$

where E means exspectation value, and $\delta\beta$ indicates the error in the vector of the parameters. Let us look at some details of the fitting procedure:

A. The Integration Method

Two methods of numerical integration of Eq. (2) have been tested for convergence and computer economy: a second order, Adams method (predictor-corrector method), and a third order Runge-Kutta method. Due to better convergence as a function of step-size, the somewhat more elaborate Runge-Kutta method was preferred. The step-size can be set depending on the average radius of curvature. If momentum loss can be expressed as a function of s, it introduces no particular complication to fit tracks for which the initial momentum changes either continuously or discontinuously at a known rate.

B. The Magnetic Field

The reading of the magnetic field in each integration step requires an appreciable amount of computer time and storage space. The most economical way to represent $\vec{B}$ seems to be to fit the measured field in different regions to simple functions (e.g. rational functions); thus the field components do not need to be stored, but can be computed at each intergration point quite quickly.

C. The Residuals

For evaluation of the points m* in any iteration , the following criteria were set: in each integration step the distance vector $\vec{d}$ from the point on the trajectory to the next measured point m was computed. m* is defined on the trajectory at the closest distance to m. It appears only necessary to test the sign of the cosine of the spatial angle between the distance vector $\vec{d}$ and the tangent unit vector $\vec{u}$. That is to say one tests the sign of $\vec{d} \cdot \vec{u}$. Whenever the sign of the cosine becomes negative, the integration has passed the point of the closest distance to m; one then computes the accurate m* by a further integration step using the appropriate step size.

D. The Partial Derivatives

The derivatives $\partial m_J/\partial\beta$ have also been computed numerically. This was done by varying the current choice of β by $\delta\beta$ and integrating (1) again. The criteria to compute points on the perturbed trajectory are the same as described in (C). By this method s does not enter the fitting problem as a further parameter. It appeared unnecessary to compute the partials in each iteration. In some cases one can approximate the partials $\partial\vec{r}/\partial\vec{r}_0$ by using the partials for the case of a uniform field, which can be represented at points $s = $ constant by a matrix:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

One observes though that due to the criteria used to find the points m^* , one has to replace this matrix in each point by:

$$I - \vec{u}^T\vec{u} .$$

This can be seen as follows:

Introducing a variation $\delta\vec{r}_0$ at the vertex gives in each point of the trajectory a variation $\delta\vec{r}$, for which in first approximation one has

$$\delta\vec{r}_0 = \delta\vec{r} + \vec{u}\lambda .$$

See Fig. 1. Using the fact $\vec{u}^T \cdot \delta\vec{r} = 0$ we find:

$$\lambda = \vec{u}^T \delta\vec{r}_0$$

hence

$$\delta\vec{r} = \delta\vec{r}_0 (I - \vec{u}^T\vec{u})$$

$$\frac{\partial\vec{r}}{\partial\vec{r}_0} \simeq \frac{\delta\vec{r}}{\delta\vec{r}_0} = I - \vec{u}^T\vec{u} . \tag{6}$$

Both possibilities to use the approximate partials, Eq. (6) and the numerically computed ones have been provided in our computer program.

E. The Matrix Inversion and Convergence of the Iterative Procedure

For matrix inversions and the test on convergence the SLAC library procedure SOLVE was used, which by means of proper scaling avoids numerical singularities.[4] As the main check for convergence it is tested whether the corrections $\Delta\beta$ lie within the error ellipsoid of the parameter defined by $\Delta\chi^2 = 1$, that is to say, the inequality

$$1 \leq \Delta\beta^T (A^T G A)\Delta\beta \tag{7}$$

is being examined in each iteration. This requires us to choose the inverse of the covariance matrix of the measuring errors for G (at any rate this is advisable if one wants to interpret the final χ^2 in terms of probability).

F. The First Estimate

Depending on the geometrical configuration, first estimates on the parameters have to be computed from the data input. The first estimate (starting) parameters were computed assuming a uniform magnetic field. By fitting circles to the first few measured points of the different tracks in a plane perpendicular to the main field direction, one obtains, from the intersection point, an approximation for the vertex coordinates. At this vertex approximate ϕ's and λ's can be evaluated as well. The final result of the best fit parameters has shown no dependence on the starting values.

III. TEST

Presently an ALGOL version of the described fitting procedure with the name CIRCE exists at SLAC. CIRCE was tested using simulated input data. A simulation procedure for high-energy interactions generates track coordinates with

random errors, due to an interaction model in a chosen configuration of target, streamer track chamber, and magnetic field.[5] We represented the components of the B field in different regions of the magnet by simple functions, fitted to actual field measurements. The B_z component has, in the radial direction, about 30% deviation from uniformity. The B_r component reaches 20% of $B_{z_{max}}$.

In a test run photoproduction of ρ mesons on protons was simulated:

$$\gamma + p \to \rho + p \to \pi^+ + \pi^- + p,$$
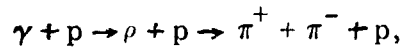
generating randomly photon momenta from a bremsstrahlung spectrum, an exponential four momentum transfer distribution and an isotropic decay of the ρ in its c.m. system. The optimization of fake trajectories serves as a test of the proper function of the fitting procedure; in particular the occurrence of biases in the fitted quantities, due to the method, can be tested. Since this method was also applied to compute the inherent resolution of the invariant mass distribution for a given experimental set-up, a ρ mass (750 MeV) with zero width was used for the simulation. The random Gaussian errors of the coordinates had assumed standard deviations:[2]

$$< \Delta x > \approx < \Delta y> = 0.5 \text{ mm} \quad < \Delta z > = 1.00 \text{ mm}.$$

The other parameters and dimensions were chosen corresponding to an actual experimental set-up*

$$B_{z_{max}} = 15 \text{ kG}.$$

Dimensions of the streamer track chamber are 200 cm $\times$ 150 cm $\times$ 60 cm. The invariant mass distribution computed from the fitted momenta and angles of the

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* Proposal for an experiment to study high-energy photoproduction on protons at SLAC, R. Mozley, I. Derado, D. Drickey, D Fries, A. Odian, F. Villa , and D. Yount.

decay pions are shown in Fig. 2. The mass distribution is centered at about 750 MeV. The "experimental" resolution is approximately

$$\Delta M^*_{(750)} = \pm \ 7 \ MeV.$$

We tested for about 100 fitted fake events the distribution of $\sqrt{2\chi^2} - \sqrt{2n-1}$ (n number of degrees of freedom), which can be used as a normal deviate with unit variance for n's which are greater than 30. The distribution was centered at sero and had approximately Gaussian shape and unit half width. The covariance matrix of the fitted parameters showed the pronounced cross-correlation of the errors of all the parameters.

CIRCE takes about 45 seconds per event on a Burroughs 5500 computer. A FORTRAN IV version of CIRCE is presently being prepared. Due to the difference of the basic cycle time, we expect less than 15 seconds running time on an IBM 7090 computer. On faster computers (like the IBM 360 model 75 or 91 or CDC 3600) the running time per event can be expected to be of the order of a few seconds per event.

ACKNOWLEDGEMENT

REFERENCES

1. See, for example, B. Zacharov "Optimization of magnet trajectories in analysis of spark chamber data," Nucl. Instr. and Methods 33 136-140 (1965).

2. F. Bulos, A. Boyarski, R. Diebold, A. Odian, B. Richter and F. Villa, "Streamer chamber development," 5th International Conference on High Energy Accelerators, Frascati, Italy, September 1965.

3. An account of the non-linear least squares fitting problem is given by R. Böck, "Application of a generalized method of least-squares for kinematical analysis of tracks in bubble chambers," CERN 60-30.

4. SLAC Library program SOLVE in ALGOL and FORTRAN IV written by C. Moore (unpublished).

5. D.E.C. Fries, "Digital simulation of high-energy interactions and trajectories in arrangements of spark chambers and magnets," to appear as SLAC Report No. 60.

FIGURE CAPTIONS

Fig. 1--Variation of the vertex coordinates of a trajectory for the approximate evaluation of the partial derivatives $\partial \dot{r}/\partial \dot{r}_o$.

Fig. 2--Invariant $\pi\pi$ mass distribution of the fitted fake events

$$\gamma + p \rightarrow \rho + p \rightarrow \pi\pi p ,$$

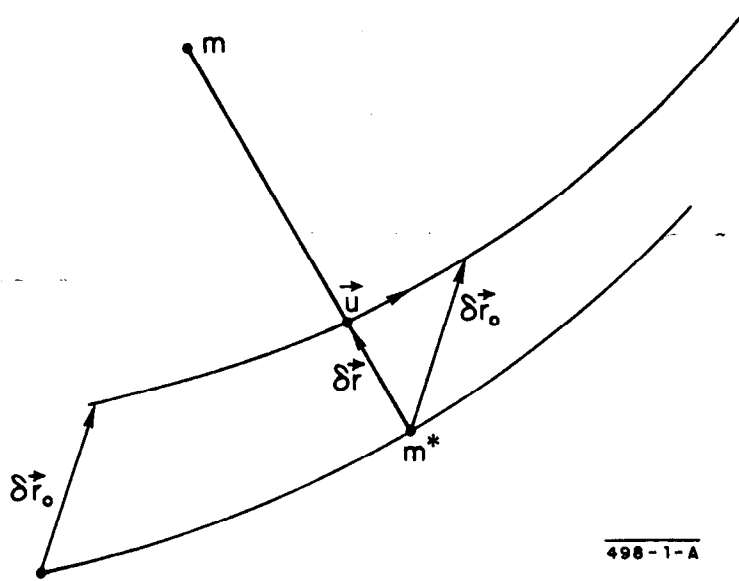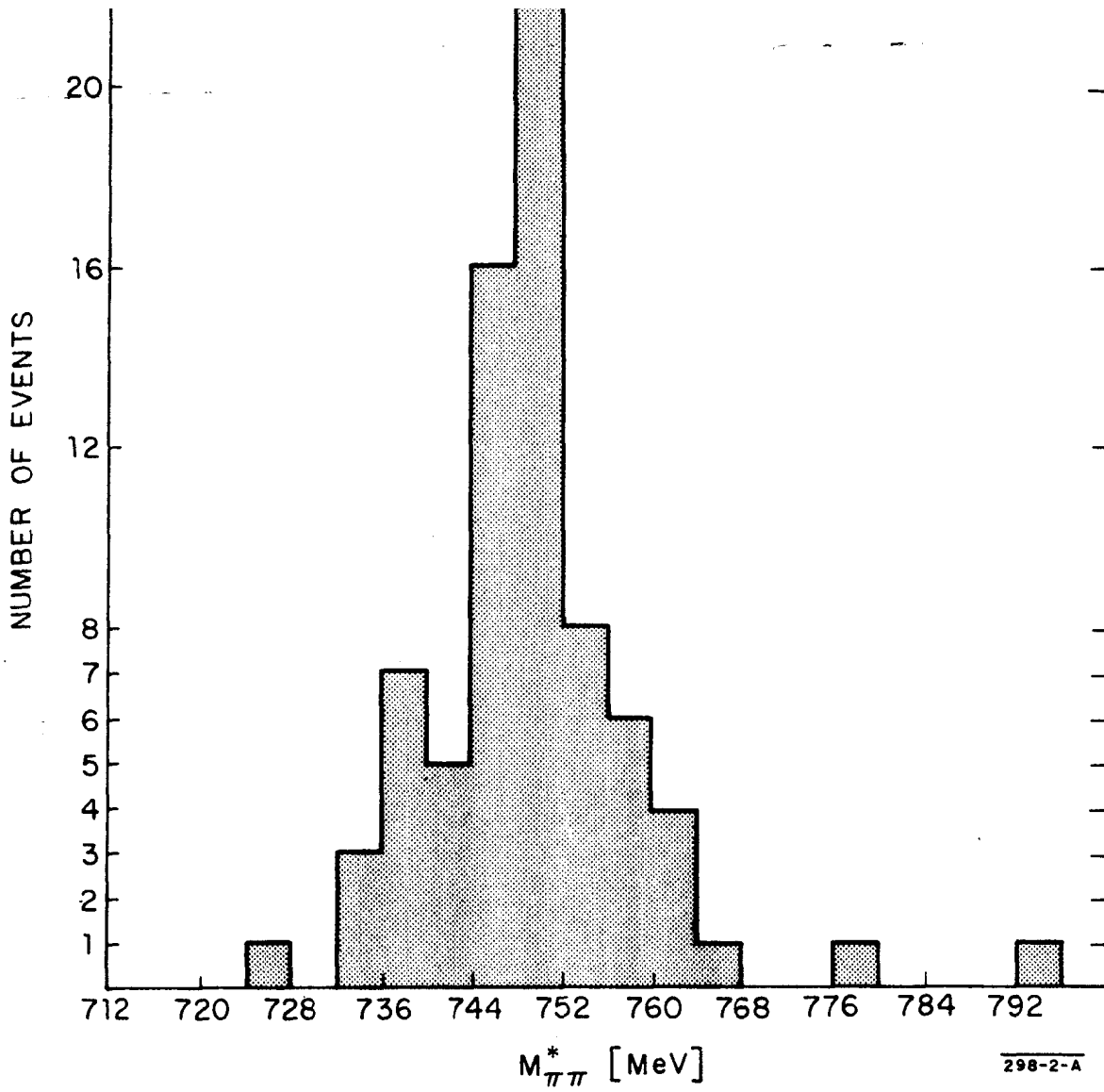where E_γ was simulated as a bremsstrahlung spectrum $3 \leq E_\gamma \leq 15$ BeV.

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FIG. 1

FIG. 2