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AN INEQUALITY FOR ELECTRON AND MUON SCATTERING FROM NUCLEONS*

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by

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Adler[1] has derived a fixed-momentum transfer sum rule for inelastic neutrino scattering from a nucleon. By isotopic rotation this may be turned into a useful inequality for inelastic electron-nucleon (or μ-nucleon) scattering:

$$\lim_{E\to\infty} \frac{d(\sigma_p + \sigma_n)}{dq^2} > \frac{2\pi\alpha^2}{q^4} = \frac{1}{2} \lim_{E\to\infty} \frac{d\sigma_p^{NS}}{dq^2} \tag{1}$$

E and E' are incident and final laboratory energies of the electron, θ the scattering angle, and $q^2 = 4EE'\sin^2\frac{\theta}{2}$. σ_p is the total (elastic + inelastic) electron-proton cross-section and σ_p^{NS} the cross-section from a point, spinless proton. More generally, writing

$$\frac{d(\sigma_p + \sigma_n)}{dq^2\,dE'} = \frac{E'}{E}\left\{\cos^2\frac{\theta}{2}\,F_1(q^2,E-E') + \sin^2\frac{\theta}{2}\,F_2(q^2,E-E')\right\} \tag{2}$$

the inequality, Eq. (1), is

$$\int_0^\infty d\nu\,F_1(q^2,\nu) > \frac{2\pi\alpha^2}{q^4} \tag{3}$$

Equation (1) has the classic structure of a sum rule except for the factor of 2 mismatch. It occurs because half of the point cross-section comes from the isoscalar current, about which isotopic spin commutation rules (the basic input to the sum rule) shed no light.

The key to arriving at Eq. (1) lies in the inequality

$$\left\langle P \middle| Q^2 \middle| P \right\rangle + \left\langle N \middle| Q^2 \middle| N \right\rangle > \frac{1}{2} \sum_{n;T=\frac{1}{2}} \left| \left\langle P \middle| T^+ \middle| n \right\rangle \right|^2 - \frac{1}{3} \sum_{n;T=\frac{3}{2}} \left| \left\langle P \middle| T^- \middle| n \right\rangle \right|^2$$

$$= \frac{1}{2} \left\langle P \middle| \left[T^+, T^- \right] \middle| P \right\rangle = \frac{1}{2} \qquad (4)$$

Using the idea expressed in Eq. (4) (on current densities instead of charges) and Adler's calculation[2] it is straightforward to obtain Eq. (3) and Eq. (1).

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REFERENCES

1. S. L. Adler, CERN preprint 65/1319/5.
2. The main assumptions in that derivation are; (a) local commutation relations for the isovector charge densities, and (b) unsubtracted dispersion relations for certain components of the odd part of the forward scattering amplitude of the isovector current from a nucleon.