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NON-COLLINEAR DIAGRAMS AND THE JOHNSON-TREIMAN RELATIONS*

by

H. Harari
Stanford Linear Accelerator Center, Stanford University, Stanford, California

and

H. J. Lipkin
Argonne National Laboratory, Argonne, Illinois

and

Weizmann Institute of Science, Rehovoth, Israel

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if we assume that the differences between the elastic cross sections are dominated by the exchange of any system which belongs to an SU(3) octet, we obtain:[5]

$$(K^+p|K^+p) - (K^-p|K^-p) = (K^0p|K^0p) - (\overline{K}^0p|\overline{K}^0p) + (\pi^+p|\pi^+p) - (\pi^-p|\pi^-p) \quad (2)$$

This is well satisfied by experiment. On the other hand, exact SU(3) predicts:[6,7]

$$(K^+p|K^+p) - (\pi^+p|\pi^+p) = (\pi^+p|K^+\Sigma^+) \quad (3)$$

$$(K^-p|K^-p) - (\pi^-p|\pi^-p) = (K^-p|\pi^-\Sigma^+) \quad (4)$$

$$(K^+p|K^+p) - (K^-p|K^-p) = (K^-p|K^+\Xi^-) + \frac{3}{2}(K^-p|\eta\Lambda) + \frac{\sqrt{3}}{2}(K^-p|\eta\Sigma^0) + \frac{\sqrt{3}}{2}(K^-p|\pi^0\Lambda) + \frac{1}{2}(K^-p|\pi^0\Sigma^0) \quad (5)$$

These relations are not satisfied by the data. The inelastic amplitudes are negligible at high energies, whereas the differences between the elastic amplitudes are, in some cases, of the order of 15-20%. It is, hence, clear that SU(3) symmetry breaking is involved here. This does not mean, however, that it affects both sides of Eqs. (3)-(5). One oversimplified explanation for the discrepancy between Eqs. (3)-(5) and the data may be based on the assumption that the symmetry-breaking contributions somehow reduce the amplitudes of the inelastic processes and do not affect the differences between the elastic amplitudes. This is the only interpretation of Eqs. (3)-(5) which is consistent with the Johnson-Treiman relations as obtained from exact $SU(6)_W$, and with the successful relation (2).

If we try to include an SU(3)-octet symmetry-breaking term in the matrix elements for the elastic processes we obtain no relations.

However, if we restrict our discussion to energies which are high enough so that all inelastic processes with one meson and one baryon in the final state are negligible, we obtain from broken SU(3), a Gell-Mann Okubo-type relation:

$$2(Kp|Kp) + 2(\overline{K}p|\overline{K}p) = (\pi p|\pi p) + 3(\eta p|\eta p) \tag{6}$$

$(Kp|Kp)$ is either the $(K^+p|K^+p)$ or the $(K^op|K^op)$ amplitude (these are equal in the absence of the charge exchange amplitudes); $(\overline{K}p|\overline{K}p)$ and $(\pi p|\pi p)$ represent, respectively, elastic scattering amplitudes of any charged state of $\overline{K}$ or π on protons. Equation (6) cannot be compared with experiment, as there is no way of measuring the η-p scattering cross section.

We now turn our attention to the non-collinear terms which break $SU(6)_W$. For the forward elastic scattering processes appearing in the Johnson-Treiman relations, the simplest non-collinear diagrams are those of two-meson exchange shown in Fig. 1. We now show that the contribution of these two-meson exchange diagrams also satisfy the Johnson-Treiman relations if the constituent three-point vertex functions are invariant under $SU(6)_W$.

The Johnson-Treiman relations have two peculiar features which distinguish them from other relations obtained from higher symmetries.

1. The momentum transfer for all amplitudes considered is rigorously zero, even when the physical masses of the particles are used;
2. The linear combinations of amplitudes which appear in the relations are chosen so that there is only a single contribution in the t-channel in $SU(6)_W$; namely a $\underline{35}_F$ (antisymmetric) in SU(6), octet in SU(3),

singlet in W-spin. This occurs because the $\underline{35}$ is the only SU(6) representation which is common to the antisymmetric product of $\underline{35} \times \underline{35}$ and the product $\underline{56} \times \overline{\underline{56}}$. These features also simplify the analysis of the two-meson exchange graph.

We consider the diagram as consisting of two four-point vertices[5] as indicated in Fig. 1c: a four-meson vertex, which we denote by M, and a two-meson two-baryon vertex, which we denote by B. Because of the zero momentum transfer, each of two four-point vertices is collinear in a suitable Lorentz frame; namely the rest frame of the baryons for the B-vertex and the rest frame of the initial and final mesons for the M-vertex.

We define $SU(6)_W$ groups for each vertex, and denote them by $SU(6)_W^B$ and $SU(6)_W^M$. These are defined to have the z-axis in the direction of the intermediate meson momenta in the rest frame of the initial baryon or initial meson respectively. Note that these two $SU(6)_W$ groups are different, and that neither is the same as the one which is used normally to obtain the Johnson-Treiman relations. The latter is the group defined in the rest frame of the baryons with the z-axis in the direction of the initial and final meson momenta. If each of the four three-point vertices is invariant separately under its own $SU(6)_W$ group, the B-vertex is seen to be invariant under $SU(6)_W^B$ and the M-vertex under $SU(6)_W^M$. However, there is no $SU(6)_W$ group under which the entire diagram is invariant.

Consider first the B-vertex and assume invariance under $SU(6)_W^B$. The exchanged mesons are each classified either in the $\underline{35}$ or singlet representations of $SU(6)_W^B$. We first examine the case where both are in $\underline{35}$'s. The two mesons couple together to some linear combination of the representations which arise in the product $\underline{35} \times \underline{35}$, namely $\underline{1}$, $\underline{35}_D$, $\underline{189}$, $\underline{405}$,

$\underline{35}_F$, $\underline{280}$ and $\underline{\overline{280}}$. The relation between these representations and those present in the coupling of the two mesons in the initial and final state is not simple because $SU(6)_W^B$ is not conserved in the M-vertex. Even if we assume conservation of $SU(6)_W^M$ in the M vertex, the classification of a given two-meson state in $SU(6)_W^M$ is different from the classification in $SU(6)_M^B$. However, we can see that only the antisymmetric two-meson states in $SU(6)_W^B$ can contribute to the Johnson-Treiman relations, namely $\underline{35}_F$, $\underline{280}$, and $\underline{\overline{280}}$. From the assumption of invariance of the M-vertex either under charge conjugation or under $SU(6)_W^M$, it follows that the symmetric state of two exchanged mesons contributes equally to the elastic scattering amplitudes $(MB|MB)$ and $(\overline{M}B|\overline{M}B)$. These contributions cancel in the differences which appear in the Johnson-Treiman relations.[9]

We note that invariance of the B-vertex under $SU(6)_W^B$ requires that the contributions of $\underline{280}$ and $\underline{\overline{280}}$ vanish as these do not appear in the product $\underline{56} \times \underline{\overline{56}}$. We are therefore left with a pure $\underline{35}$ contribution. This means a pure octet in SU(3), since the $\underline{35}$ contains only SU(3) octets and singlets, and the SU(3) singlet contribution to the Johnson-Treiman relations cancels if the M-vertex conserves SU(3).

These conclusions are unchanged if one of the exchanged mesons is a singlet in $SU(6)_W$. The two-meson system then must be a $\underline{35}$. If both mesons are singlets, the contributions to the Johnson-Treiman relations cancel.

We have now established that the coupling of the B vertex in the t-channel is pure $\underline{35}$ in $SU(6)_W^B$ and pure octet in SU(3). We now show that only the W-spin singlet contributes. In the rest system of the baryons all the four meson momenta lie in a plane, which we can call the yz-plane. Invariance of the scattering amplitude under rotations and reflections requires that it be invariant under a reflection in this plane, or the equivalent combination of a space inversion and a 180° rotation about the x-axis.[10] This transformation has been shown to be equivalent to a 180° W-spin rotation about the x-axis.[11] The W-spin eigenstates with $W_z = 0$ are eigenstates of this tranformation with the eigenvalue +1 if W is even and -1 if W is odd. Thus, even though W-spin is not conserved in the overall process, even and odd eigenvalues of W are not mixed.

If we choose our z-axis in the direction of the initial and final meson momenta, these mesons are in a state which is a mixture of W = 0 and W = 2 with no W = 1 component. The W = 1 component of the coupling of the two baryons therefore does not contribute to this amplitude. Although the W-spin in this system is different from that used in $SU(6)_W^B$, the difference in the baryon classification is only a rotation of the z-axis, keeping the baryons at rest. Since the W-spin of a baryon at rest is the same as its ordinary spin, the rotation cannot introduce a W = 1 component. Thus only the W = 0 component contributes to the amplitude in $SU(6)_W^B$.

The Johnson-Treiman relations now follow directly from the result that the only nonvanishing contribution in the t-channel is an SU(6)-35, SU(3)-octet and W-spin singlet.[12] The coupling at the B-vertex is unique, under the assumption of $SU(6)_W^B$ invariance. The coupling at the M-vertex is unique, just from SU(3), as the symmetric D-type coupling cancels out in the Johnson-Treiman relations and only the pure octet F remains.

Similar arguments can be applied to a larger class of diagrams, and <u>in particular to the exchange of a two-meson ladder</u>. Although there is no $SU(6)_W$ group considered in going from one rung of the ladder to the next, the antisymmetry of the two meson coupling must be preserved. This is all that is required.

FOOTNOTES AND REFERENCES

1. K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).
2. H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965).
3. The Johnson-Treiman relations were obtained from various versions of broken U(6,6) and collinear $SU(6)_W$ by: J. Charap and P. T. Matthews, Physics Letters 16, 95 (1965); R. F. Dashen and M. Gell-Mann, Physics Letters 17, 145 (1965); H. Harari and H. J. Lipkin, Phys. Rev., to be published. The explicit $SU(6)_W$ derivation can be obtained either by using SU(6) Clebsch-Gordan coefficients or by the method described in: H. Harari, to be published in the Proceedings of the High-Energy Physics Symposium, Boulder, Colorado, August 1965.
4. Note that differences between the total cross sections are related by the optical theorem to differences between the imaginary parts of the elastic amplitudes. These cannot be contributed by one particle exchange diagrams which are described by real amplitudes.
5. It was noted by R. F. Sawyer, Phys. Rev. Letters 14, 471 (1965), that if we assume the exchange of one octet vector-meson Regge trajectory we obtain Eq. (1), provided that the vector-nucleon coupling is pure F. It should also be remembered that the simplest one-vector exchange picture cannot contribute to differences between the imaginary parts of the amplitude.

6. H. Harari and H. J. Lipkin, Phys. Rev. Letters <u>13</u>, 208 (1964).

7. H. Harari, to be published in the Proceedings of the Trieste Seminar. 1965.

8. We disregard the time orderings of the various three-point vertices. These are irrelevant in our treatment which is valid for all time orderings. The SU(3) and SU(6) couplings are described in the t-channel as a convenient algebraic device. The t-channel couplings are related to s-channel couplings by algebraic transformations which reverse signs of SU(3) and SU(6) quantum numbers but which do not change particles into anti-particles or reverse momenta. Despite the confusing use of the term "crossing" to describe these algebraic transformations, they have nothing to do with crossing symmetry. Thus the initial and final baryon states are coupled together in $SU(6)_W$ as $56 \times \overline{56}$, but both states remain <u>baryons</u>. The W-spin of the $\overline{56}$ is related to the ordinary spin with <u>the baryon phases, not the anti-baryon phases</u>. The initial and final state mesons and the two exchanged mesons are likewise coupled, and the term "state of the two-meson system" is used loosely to decribe the couplings obtained, even though the two mesons in the "system" do not exist at the same time.

9. An equivalent alternative argument is to note that the contributions of two-meson states which are eigenstates of C with the eigenvalue +1 cancel in the Johnson-Treiman relations. These include $\pi^0\pi^0$, $\eta\eta$, $\pi^0\eta$, X^0X^0, $X^0\pi^0$, $X^0\eta$, $\rho^0\rho^0$, $\rho^0\omega$, $\rho^0\Phi$, $\Phi\Phi$, $\Phi\omega$, and $\omega\omega$. The particular linear combination of two-meson states which contributes to the Johnson-Treiman relation can be defined as

$$|X> = V_M \left|(M\bar{M} - \bar{M}M)\right>$$

where V_M is the operator describing the action of the M-vertex function and $|M\bar{M} - \bar{M}M>$ is the antisymmetric combination of the initial and final mesons which contributes to the Johnson-Treiman relations in the t-channel. From the requirement that the exchanged meson state $|X>$ have vanishing components for all the $C = +1$ states described above, it follows that the expansion of $|X>$ in the eigenstates of $SU(6)_W$ contains only the antisymmetric multiplets 35_F, 280 and $\overline{280}$.

10. This transformation has been used by T. D. Lee and C. N. Yang, Phys. Rev. 104, 254 (1956); A. Bohr, Nuclear Physics 10, 486 (1959). For detailed discussion of the application of this transformation to W-spin see ref. 11.

11. H. J. Lipkin and S. Meshkov, to be published.

12. H. Harari, ref. 3.

FIGURE CAPTION

1. Two-meson exchange diagrams.

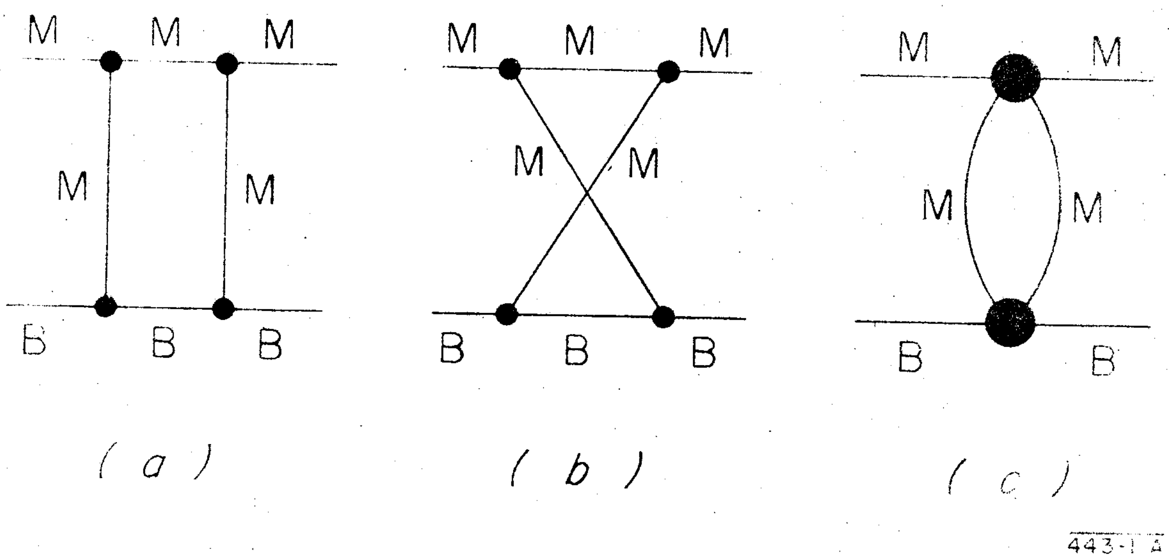

FIG. 1