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IMMITTANCES OF REDUCED DEGREE AND NONPOSITIVE REAL IMMITTANCES IN NETWORK SYNTHESIS*

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Some years ago the author encountered an interesting paradox of network design. A symmetrical parametric band-pass filter of degree eighteen was required; its attenuation response and circuit schematic were as shown in Fig. 1. The first ladder realizations resulted in two negative inductance values and it was found that neither the open-circuit impedance nor the short circuit admittance was positive-real — they both had a pair of complex conjugate poles. Yet it was found that by a judicious choice of the order of realization of the poles of loss a network with all positive elements could be obtained and it was judged that the values of the elements were correct to five places of decimals. Thus a real network with all positive elements which performed exactly as prescribed had been found from two nonpositive real real immittances!

For the resolution of this paradox it is useful to consider a simple symmetrical low-pass filter and an effect first mentioned by Darlington[1] but not much discussed since; with the usual notation, the transfer function

$$H = \frac{A + pB}{P}$$

the characteristic function,

$$K = \frac{pB'}{P}$$

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and the open and short circuit immittances

$$Z_{11} = Z_{22} = \frac{A}{p(B+B')}$$

and

$$Y_{11} = Y_{22} = \frac{A}{p(B-B')}$$

The equation which relates the polynomials A, B, B' and P is

$$A^2 - P^2 = p^2(B^2 - B'^2) \tag{1}$$

If now, Z_{11} is infinite at a frequency of infinite loss (f_∞), then

$$B + B' = 0 \qquad \text{when} \qquad P = 0$$

Therefore A must have a zero at this frequency; therefore the left-hand side of Eq. (1) must have a double zero at this frequency; therefore the right hand side and in particular, its factor $(B + B')$ must also have a double zero at this frequency.

Therefore the numerator and denominator of Z_{11} have one and two factors of the form $(p^2 + \omega_\infty^2)$ respectively and one each of these may be cancelled. Thus Z_{11} is of reduced degree and all the elements of the filter cannot be calculated from it. Nevertheless, the network is perfectly "well behaved" and in general all the elements can be calculated from Y_{11}.

Similarly, if Y_{11} is infinite at a frequency of infinite loss, $B - B'$ has a double zero at that frequency and Y_{11} is of reduced degree. In both cases a pole-zero-pole triplet is coincident with a pole of loss and may be considered as a single pole.

In practice perfect coincidence does not occur because of rounding errors in the calculations and often the two poles become complex.

As an example a series of symmetrical low-pass filters of seventh degree with m values* = 0.36143, 0.52033 and 0.81336 and equal ripple pass bands† was designed. The size of the ripple was varied so as to cause the special behaviors described above. The results are summarized in Table I. As the ripple was reduced from a large value the singularities of both Z_{11} and Y_{11} moved upward in frequency until for $t = 0.120555$** a pole-zero-pole triplet of Z_{11} almost coincided at $z = 0.36143$; then for $t = 0.047318$ the same triplet almost coincided at $z = 0.52033$; finally for $t = 0.0111$ a pole-zero-pole triplet of Y_{11} almost coincided at $z = 0.36143$. In each case this near coincidence resulted in a pair of complex conjugate poles and any realization of the "offending" immittance resulted in a negative inductance value at the pole of loss in question and meaningless element values beyond that point in the realization; nevertheless, the realization of the other immittances produced the well-defined (if not all physically realizable) networks shown in Fig. 2.

Furthermore, the element values found from the nonpositive real immittances agreed very well with these from the positive real immittances up to the realization of the pole of loss. This demonstrates practically what is obvious from a consideration of the arithmetic involved — that anomalous behavior in a very small region about one pole of loss has

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*These m values are the values of the z variable, as defined by this author[2], at the frequencies of infinite loss; that is, $z^2 = 1 - 1/\omega^2$ and $m^2 = 1 - 1/\omega_\infty^2$.

**t is the maximum value of the characteristic function in the pass band. The pass band ripple is $10 \log (1 + t^2)$ db.

†These filters are almost elliptic function filters with $\theta = 70^\circ$ but this is irrelevant to the point of the example.

almost no effect upon calculations at the other poles of loss and the realization can proceed normally and accurately.

TABLE I

$t = 0.120555$

Z_{11} — poles at $z^2 = -0.41443943$ and $0.13063198 \pm 0.000035046j$ *

zeros at $z^2 = 2.320305, -0.0416367$ and 0.13063244

Y_{11} — poles at $z^2 = -.33624535$ and $.02362285$

$t = 0.047318$

Z_{11} — poles at $z^2 = -0.11305091$ and $0.27073947 \pm 0.0013496j$ *

zeros at $z^2 = -1.3195111, 0.0434051$ and 0.27074247

Y_{11} — poles at $z^2 = -.12890869$ and $.06947354$

$t = 0.0111$

Y_{11} — poles at $z^2 = 0.13061413 \pm 0.0019937j$

zeros at $z^2 = -0.22337576, 0.1305910$ and 0.47677876

Z_{11} — poles at $z^2 = 0.0694031, 0.4210358$ and 0.5225697

It is now apparent what happened in the case of the large parametric filter; two poles and a zero of Y_{11} and Z_{11} almost coincided at $\omega_{\infty 3}$ and $\omega_{\infty 6}$ respectively as shown in Fig. 3,† consequently almost seven eighths of the element values could be found from each end by placing $\omega_{\infty 3}$ at port 1 and $\omega_{\infty 6}$ at port 2 and then realizing Z_{11} and Y_{22} .

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*Note that $0.36143^2 = 0.13063164$ and $0.52033^2 = 0.27044331$.

†This is not so much of a coincidence as it seems because the symmetry of the attenuation response and the narrow bandwidth ensure that the behavior of Z_{11} in the upper stop-band is mirrored by that of Y_{11} in the lower.

Some conclusions can be drawn from these examples about one way of improving ladder network realization methods. The main cause of inaccuracy in a realization is the proximity of a pole of an immittance and a pole of loss at which the immittance is being evaluated; this results in a near zero and inaccurate value of the denominator and therefore the values of the element removed and all subsequent elements are inaccurate. If the proximity of a pole of the (open-circuit, say) immittance and a pole of loss is due to the near coincidence of a zero-pole-zero triplet* there, the values of the numerator and denominator of the immittance will be very small and their ratio almost meaningless. However, the other (short-circuit) immittance has only a zero near that frequency and its evaluation as a small number will be accurate enough.

This suggests that the method of factorizing the chain matrix proposed by Watanabe[3] would be complete proof against this sort of behavior and would allow the specification of any order of poles of loss. Watanabe suggested that the numerator and denominator of both the open and short circuit immittances be evaluated for the realization of each pole of loss and the element value deduced from the ratio of the sum of the numerators to the sum of the denominators. An element of this value is then removed from both the open and short circuit immittances and the realization continued. Thus if the value of any numerator or denominator is small when it should not be, then the value of the other numerator or denominator predominates in the sum and the value of the element is

*The two zeros may be real or complex conjugates.

automatically determined by the "well-behaved" immittance. It would appear that this method of ladder realization is necessary in order to obviate the careful analysis and a second pass through a realization program with a different order of poles of loss that would otherwise be necessary in some special cases.

REFERENCES

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