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THE W-SPIN RELATIVISTIC VERSION OF SU(6) AND ITS EXPERIMENTAL TESTS*

by

Haim Harari
Stanford Linear Accelerator Center, Stanford University, Stanford, California

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I.

Since the introduction of the static SU(6) symmetry scheme[1] many attempts have been made to formulate a relativistic version of the theory, incorporating both the internal SU(3) symmetry and Lorentz invariance into a larger symmetry group. It is now clear that all attempts to write a complete theory which will describe all strong interaction processes in an SU(3) and Lorentz invariant way, and will imply a symmetry larger than the direct product of these two groups are doomed to failure unless some revolutionary unconventional new ideas are introduced into the game.[2] On the other hand, it had been pointed out[3,4] that certain sets of processes could be treated according to a covariant version of SU(6). In these notes we shall discuss some of the difficulties encountered in the various versions of "relativistic SU(6)" arriving at the conclusion that a non-trivial chain of subgroups is obtained[3,4] which may serve as a chain of approximate symmetries for some special sets of processes. In particular we study the group $SU(6)_W$[5,6] which has been proposed as a possible underlying symmetry for collinear processes, and we discuss some of its experimental tests, showing that in most cases a good agreement is obtained between the theoretical predictions and the experimental data.

In the original static version of SU(6) the relevant algebra is defined by the set of operators $\sigma_i\lambda_\alpha$; $i = 0,1,2,3$; $\alpha = 0,1,2,\ldots,8$; where σ_i are the Pauli matrices, λ_α are the U(3) generators and $\sigma_0 = \lambda_0 = 1$. We denote this SU(6) group by $SU(6)_S$. The low-lying baryons and mesons are represented by the well-known 35 and 56 dimensional supermultiplets of $SU(6)_S$ and it is clear that this description can be meaningful only when the particles are at rest, as only then their intrinsic spins are well-defined and might be considered as good quantum numbers. It is not surprising that discussions of decay processes or scattering reactions lead to catastrophes (e.g. the forbiddenness of the $\rho \to 2\pi$ and $N^* \to N\pi$ decays). One cannot expect to get any useful information concerning such processes by using this static approach, as in all cases, at least one of the particles which are involved in the process is highly relativistic. The first difficulty of the symmetry scheme is hence obvious: Some of the quantum numbers (namely those related to the intrinsic spin space) are well-defined only for particles at rest. The classification is static in its nature and cannot be applied to nonstatic problems.

It should be noted, however, that this difficulty is only one part of the complete picture. Another obvious reason for the failure of the theory stems from the fact that the "intrinsic" degrees of freedom are completely decoupled from the "space-time" degrees of freedom and the model implies separate conservation of the intrinsic spin $\vec{S}$ and the orbital angular momentum $\vec{L}$ instead of conserving their vectorial sum $\vec{J} = \vec{L} + \vec{S}$. While studying $SU(6)_S$ it seems that the two difficulties mentioned above are identical or at least intimately related. However, we shall see later that it is possible to overcome the first difficulty by defining a relativistic generalization of $SU(2)_S$ which will be adequate for describing a particle with an arbitrary momentum. However, the separate conservation of an "intrinsic spin space" operator again leads to predictions which are in clear contradiction with experimental data.

The difficulties can be summarized briefly by noting that $SU(6)_S$ fails to describe high energy processes because:

a) It can be applied only to static situations.

b) It leads to the conservation of quantum numbers defined by operators which are applied only to the "intrinsic spin space" (i.e. they act only on the spin indices of the wave functions).

The first step toward a relativistic generalization of the "static" approach is trivial: Instead of talking about a "spin-unitary-spin" six component quark, we discuss a relativistic free quark which satisfies the Dirac equation. This is a twelve-component object which might be regarded as our basic multiplet and could imply that some kind of a twelvefold symmetry exists. We then write the free field equation for an arbitrary particle while its spin is mathematically constructed from an appropriate number of basic spinors (or spin indices, or quarks). This can be done according to the generalized[7] Bargman-Wigner formalism.[8] While doing this it becomes clear that the various spin states can be classified according to the finite dimensional representations of SU(4) or any of its noncompact versions, including SU(2,2). (As far as only the classification is concerned, the differences among these groups are irrelevant.) If we further want to include the SU(3) quantum numbers we may classify all the states according to the representations of U(12) or U(6,6). Again, both classifications are equivalent.

The real difficulties arise when we try to suggest that one of these groups is a good symmetry of nature. It is clear that even the simple one-quark Dirac equation is not invariant under the transformations of these groups and that we cannot expect SU(4) or U(2,2) to be an exact or an approximate symmetry of the strong interactions. Similarly, a complete U(2,2) invariance cannot be reconciled with the known symmetry properties of the kinetic energy term of the total Lagrangian. We could, however, try to formulate a phenomenological field theoretic approach in which we distinguish between the kinetic energy part which is definitely not invariant under anyone of these groups and the interaction part which might be invariant under one of them.[7,9] In other words, instead of trying to find a universal symmetry which is the basic symmetry of physics we might adopt the less ambitious approach of developing a formalism which allows us to calculate simple diagrams for various processes while using the first Born approximation and inserting the proposed symmetry only into the interaction vertices,[10] thus obtaining new relations between coupling constants and form factors. In order to do this we notice that the "spin group" could be U(2,2) which contains the homogeneous Lorentz group and the best choice of a total spin-unitary spin symmetry of the interaction would apparently be U(6,6). The U(6,6) algebra is obtained by considering all possible products of the nine U(3) generators λ_α and the sixteen U(2,2) operators which can be identified with the sixteen Dirac gamma matrices. The unitarity problem which arises[11] should not worry us more than it worries us while doing any ordinary calculation in the first Born approximation. It is well known that this approximation usually leads to solutions which do not satisfy the unitarity condition, and it is usually expected that higher contributions will correct this crude approximation. On the other hand we find that the $\rho\pi\pi$ and $N^* N\pi$ vertices are no more forbidden by this approximation, thus curing one of the major practical difficulties of $SU(6)_S$. A detailed investigation of the selection rules implied by this "vertex-symmetry" shows, however, that one difficulty in principle still remains. The theory still implies the conservation of a set of quantum numbers (those of U(2,2)) which apply only to the intrinsic spin degrees of freedom. Once more, the orbital angular momentum is not coupled to the "intrinsic" U(2,2). Although p-wave decays are usually allowed by U(2,2) because of the inclusion of first-order derivatives within the U(2,2) multispinors of the low-lying states, higher ℓ-values are forbidden in most cases. An analysis of this point[12] shows that the following vertices are forbidden by U(2,2): $2^+ \to 0^- + 0^-$; $2^+ \to 1^- + 0^-$; $2^+ \to 0^- + 0^- + 0^-$; $\frac{3}{2}^- \to \frac{1}{2}^+ + 0^-$; $\frac{5}{2}^\pm \to \frac{1}{2}^+ + 0^-$; $\frac{5}{2}^\pm \to \frac{3}{2}^+ + 0^-$; $\frac{5}{2}^\pm \to \frac{1}{2}^+ + 0^- + 0^-$ etc. (2^+, 0^-, etc. are the J^P = spin-parity values of the particles involved.) These forbid the direct coupling of most of the known resonances to their decay products[12] (e.g. $f^0 \to 2\pi$, $A_2 \to \rho\pi$, most N^*'s cannot decay into $N\pi$, most $Y^* \not\to \Sigma\pi$, $\Lambda\pi$ etc.) One might argue that the introduction of vertex corrections such as triangular diagrams could allow these decays to proceed. However, in this case the predictive power of the approximate symmetry is reduced to nothing.

The inevitable conclusion of all this is, of course, that even the interaction part of our Lagrangian cannot preserve the U(2,2) (consequently the U(6,6)) symmetry. We must break it by introducing derivative couplings to all orders and we have no apriori reason to believe that the low derivatives are dominant. We find ourselves facing a completely broken symmetry in which the symmetry breaking terms might be much larger than the symmetrical contributions. However, we know the exact nature of all our symmetry breaking terms! They are linear combinations of products of momentum terms p_μ or derivative terms ∂_μ. All these terms transform under U(2,2) or U(6,6) like products of the four gamma matrices γ_μ

Our next step is, hence, obvious. We try to analyze the symmetry breaking effects of our non-symmetrical terms and see if we can find a subsymmetry which is unaffected by these terms. As we do not believe that low-order contributions, either in the kinetic energy or in the derivative couplings are dominant and we do not wish to introduce a perturbative approach for dealing with such terms, we assume symmetry breaking to all orders and look for a good subsymmetry which survives after including all these corrections. This amounts to solving the following problem: Which generators of U(6,6) commute with the four gamma matrices γ_o, γ_x, γ_y, γ_z (and consequently with all products of these matrices). Only such a set of operators can be regarded as a set of approximately conserved quantities even in the presence of all our symmetry breaking terms. Unfortunately, the answer to our question is very simple. In fact, it is too simple: The only generators of U(6,6) which commute with <u>all four</u> γ_μ matrices are the nine U(3) generators λ_α. This is, of course, an extremely poor final result for the enormous effort which has been done in the direction of combining the spin and SU(3) symmetries. However, this result is not as bad as it sounds. We must not forget that most of the interesting processes (at least from the various symmetries point of view) are restricted to a plane and the assumption of having symmetry breaking terms transforming like <u>all four</u> γ_μ matrices is too general for such processes. Moreover, there are no more than half a dozen known SU(3) predictions concerning five-point functions and almost all the work that has been done in particle symmetries was done for two, three and four point functions. The fact that we can say nothing more than what is predicted by SU(3) for five (or more) point functions is not such a terrible loss. But what happens if we consider co-planar processes? We assume that all our symmetry breaking terms transform like γ_o, γ_y, γ_z or their products but not like γ_x. The mathematical problem is, again, extremely simple: We have to find all U(6,6) generators which commute with the three matrices $\gamma_o, \gamma_y, \gamma_z$. There are eighteen such operators: λ_α and $\gamma_o \sigma_x \lambda_\alpha$, and they form a $U(3) \otimes U(3)$ algebra. The two commuting U(3)'s are defined by $(1 \pm \gamma_o \sigma_x) \lambda_\alpha$. This symmetry is larger than the ordinary SU(3) symmetry and we expect it to hold for all coplanar processes including three-body decays of resonances and scattering processes with a two-body final state. It is here that we are almost helpless. Although the symmetry is larger than that of SU(3), its additional predictive power is fairly small. We are forced to define most spin states of the physical particles as mixtures of various irreducible representations of $U(3) \otimes U(3)$ and it is difficult to derive clear predictions of the symmetry. The only conclusion of $U(3) \otimes U(3)$ which has been derived so far states that the spin component perpendicular to the scattering plane is conserved in certain sets of scattering processes.

If we now restrict our interest to a still smaller number of dimensions, we may ask: What is the residual symmetry which survives for collinear processes? In this case we look for all U(6,6) generators which commute with γ_z and γ_o but not necessarily with γ_x and γ_y. We find a full U(6) group, different from $U(6)_S$. It is defined by the matrices λ_α, $\gamma_o\sigma_x\lambda_\alpha$, $\gamma_o\sigma_y\lambda_\alpha$, and $\sigma_z\lambda_\alpha$. Following Lipkin and Meshkov,[5] we shall call this group $U(6)_W$, and its "W-spin" subgroup defined by $\gamma_o\sigma_x$, $\gamma_o\sigma_y$, σ_z - $SU(2)_W$. We can use this subsymmetry for analyzing two-body decays or forward and backward scattering amplitudes, and we can derive a tremendous amount of predictions which can be tested experimentally at present or in the near future. In the second part of these notes we will mainly concentrate on the application of this symmetry to various processes. But before that we carry our analysis one more step further and discuss the "degenerate" case of a "zero-dimensional" physical problem, namely - the problem of describing particles at rest. Here we require that our symmetry generators commute only with γ_o and we obtain a $U(6) \otimes U(6)$ group defined by the matrices $(1 \pm \gamma_o)\,\sigma_i\lambda_\alpha$. This group includes both $SU(6)_S$ and $U(6)_W$ as its subgroups and may serve as a generalization of $SU(6)_S$ for classifying particles at rest. Its application to physical problems is limited to cases like $\bar{p}p$ annihilation at rest, and even there - at most to one vertex of the assumed dynamical mechanism. The classification of the low-lying meson and baryon states is, however, clear. The mesons are in a $(\underline{6},\underline{\bar{6}})$ representation which splits into $\underline{35}+\underline{1}$ under both $SU(6)_S$ and $SU(6)_W$.

(We shall see later that these two $\underline{35}$'s are not identical.) The low-lying baryonic states are assumed to be in a $(\underline{56}, \underline{1})$ while their anti-baryons are in $(\underline{1}, \overline{\underline{56}})$. Higher states can be in the $(\underline{15}, \overline{\underline{15}})$ or the $(\underline{21}, \overline{\underline{21}})$ for mesons and in $(\underline{70}, \underline{1})$ or $(\underline{126}, \overline{\underline{6}})$ for baryons. These representations are reduced by $SU(6)_S$ or $SU(6)_W$ in the following way:

$$(\underline{15}, \overline{\underline{15}}) \rightarrow \underline{189} + \underline{35} + \underline{1}$$
$$(\underline{21}, \overline{\underline{21}}) \rightarrow \underline{405} + \underline{35} + \underline{1}$$
$$(\underline{126}, \overline{\underline{6}}) \rightarrow \underline{700} + \underline{56}$$

Once again - the $SU(6)_S$ multiplets are not necessarily identical to the $SU(6)_W$ multiplets. Note that there are no $U(6) \otimes U(6)$ invariant $B\bar{B}M$ and MMM vertices. However, we do not expect to find such vertices as there is no vertex in which all three particles are at rest.

Before we proceed to the actual detailed analysis of various processes, let us summarize our conclusions and see precisely what our assumptions are, how we managed to overcome the two basic difficulties of $SU(6)_S$ and what is the price that we had to pay for it. We notice that U(6,6) cannot serve as a good symmetry even for one free particle. It can provide us, at most, with a classification scheme for the free particles, but this classification is completely equivalent to that of the static $U(6) \otimes U(6)$. In fact, we could forget about U(6,6) and start our analysis from $U(6) \otimes U(6)$ and we would still get the same chain of subgroups[3] which are the suggested approximate symmetries for 0,1,2 and 3 dimensional processes, namely:

$$U(6) \otimes U(6) \supset U(6)W \supset U(3) \otimes U(3) \supset U(3)$$

When we apply these groups as approximate symmetries, we essentially make a very strong dynamical assumption. We assume that if a certain process is "externally" collinear (i.e., all the real particles which are involved are moving along the same line in a certain frame of reference), then all its intermediate states are also collinear. This is certainly true for one-pole diagrams; it is even true for a superposition of any number of pole diagrams in all possible channels of the process. However, we cannot expect, for example, that a two-pion exhange diagram for a scattering process will be strictly collinear with its external lines. The most that we can say is that we hope that somehow the one-pole diagrams plus other collinear contributions of more complicated nature are the dominant mechanisms for most processes. In this respect we are in a better position than that of most phenomenological calculations which usually include, at most, a few one-pole-diagrams (whereas we allow any number of them, in all channels). However, our situation is worse than the one usually encountered in most symmetry schemes, in which at least some of the predictions are valid irrespective of any dynamical assumptions. In any case, we would like to point out that the ultimate test of our assumptions is the experimental situation and this looks at present favorable.

It is interesting to see, at this point, how we have solved the basic difficulty of $SU(6)_S$ and $U(6,6)$, namely - the separate conservation of quantities which are defined only in the intrinsic spin space. If we consider $U(6)_W$, which is practically the only new useful symmetry in our chain we find that apart from the $SU(3)$ quantum numbers the only conserved additive quantum number is W_z which is identical with the ordinary helicity. It is well known that the helicity is conserved in collinear processes, as the orbital angular momentum has no component along the direction of motion. This means that our symmetry does not include any new additive quantum number. As for the other symmetry operators which are involved, namely $\gamma_0\sigma_x$ and $\gamma_0\sigma_y$, we shall see that they lead to a classification of the particles which will allow, in principle, all ℓ-values to occur in a given vertex, a feature which is not surprising in view of the inclusion of an arbitrary number of derivative couplings in the interaction. The Lorentz group is, of course, not a subgroup of any group in our chain. It is also clear that a collinear process can be transformed into a non-collinear one by a Lorentz transformation. This only means that we are not able to solve our original problem, namely - to introduce a symmetry larger than $SU(3) \otimes L$. This does not mean that our chain of residual symmetries is not fully covariant. The "W-spin laws of nature" are not changed when we go from one Lorentz frame to another. They are, however, formulated most easily in the special frame in which all particles are collinear, and there we obtain our relatively simple $U(6)_W$ group. When the same system is described in any other frame we will obtain a group which is isomorphic to $U(6)_W$ but is defined in terms of more complicated functions of the gamma matrices.

We have obtained a covariant version of SU(6) which does not contradict either the basic principles of physics or the known experimental data (to the extent that it was tested). However, we paid the price of being able to deal effectively, almost only with collinear processes.

II.

We now turn our attention to a detailed study of $U(6)_W$, the collinear symmetry group. The three W-spin operators are defined by:

$$W_z = \frac{1}{2}\sigma_z$$

$$W_x = \frac{1}{2}\gamma_o\sigma_x$$

$$W_y = \frac{1}{2}\gamma_o\sigma_y$$

It is easy to verify that these operators satisfy the commutation relations of ordinary spin operators and hence constitute an SU(2) algebra. We note that, apriori, there is a certain sign ambiguity in the definitions of W_x and W_y. We could change the signs in the definitions of W_x and W_y, define them by $W_x = -\frac{1}{2}\gamma_o\sigma_x$; $W_y = -\frac{1}{2}\gamma_o\sigma_y$ and they would still form an SU(2) algebra and commute with γ_o and γ_z, as required. In order to resolve this ambiguity we go back to $U(6) \otimes U(6)$ and diagonalize γ_o and σ_x, simultaneously. We find, of course, that W_x is also diagonal in the same scheme. The question now arises: Is it possible to define W_x as $+\frac{1}{2}\gamma_o\sigma_x$ for <u>all</u> representations of $U(6) \otimes U(6)$? This is a special case of a more general problem: Given two diagonal generators of an algebra (which, consequently, represent two additive quantum numbers), we define a third additive quantum number by taking their product. Can we use this <u>same</u> definition of our new quantum number for all the representations of the given algebra?

Let us study a trivial example. Consider the original Wigner super-multiplet theory,[13] based on identifying the four spin-isospin components of the nucleon with the components of a four-dimensional representation of SU(4). We know that as a group of rank 3, SU(4) contains three independent additive quantum numbers. In Wigner's theory, two of them are t_z and σ_z, the third components of isotopic spin and ordinary spin. How is the third additive quantum number, X, defined? It is easy to find its eigenvalues for the nucleon representation by requiring that X cannot be expressed as a linear combination of σ_z and t_z. The values of t_z, σ_z and X for the nucleon are (up to multiplicative factors):

	t_z	σ_z	X
$p\uparrow$	1	1	1
$p\downarrow$	1	-1	-1
$n\uparrow$	-1	1	-1
$n\downarrow$	-1	-1	1

We see that, for the nucleon, X can be defined as:

$$X(\text{nucleon}) = + t_z \circ \sigma_z$$

The antinucleon is in the conjugate $\underline{4}$ representation of SU(4). Recalling that all the additive quantum numbers change sign when we go to the conjugate states, we find the following set of values:

	t_z	σ_z	X
$\bar{p}\downarrow$	-1	-1	-1
$\bar{p}\uparrow$	-1	1	1
$\bar{n}\downarrow$	1	-1	1
$\bar{n}\uparrow$	1	1	-1

Consequently:

$$X(\text{antinucleon}) = - t_z \cdot \sigma_z$$

It is easy to see the reason for this sign-flip: We cannot reverse the signs of all quantities in a relation of the form A = B.C, while preserving the relation itself. The relevant conclusion for us is, of course, that in the framework of U(6) ⊗ U(6) there is no generator which can be defined as $+\frac{1}{2}\gamma_o\sigma_x$ for all the representations. If we define:

$$W_x(\text{quark}) = +\frac{1}{2}\gamma_o\sigma_x$$

$$W_y(\text{quark}) = +\frac{1}{2}\gamma_o\sigma_y$$

It implies:[14]

$$W_x(\text{antiquark}) = -\frac{1}{2}\gamma_o\sigma_x$$

$$W_y(\text{antiquark}) = -\frac{1}{2}\gamma_o\sigma_y$$

for higher representations (35, etc.) neither relation is correct, in general, and more complicated forms are usually obtained.

Now that we know the exact definition of the W-spin generators for quarks and antiquarks we can classify all particles by using the quark algebra as a mathematical device (without ever assuming that these elusive creatures exist).[15] We notice that all W-spin operators commute with the Lorentz transformation in the z-direction. Consequently, we may classify all particles at rest, knowing that the obtained classification will be valid for a particle moving in the z-direction with an arbitrary momentum.

We first discuss the quarks and the antiquarks. The eigenvalues of γ_o for both a quark and an antiquark at rest are + 1.[16] We can see this easily if we start from a positive energy ($\gamma_o = +1$) solution for the Dirac equation for a quark and then go to the conjugate state which is an $E < 0$, $\gamma_o = -1$ antiquark. We are, however, interested in a positive energy state of the antiquark, and this, of course, has $\gamma_o = +1$. We can, therefore, drop all γ_0's in the definitions of the W-spin operators for particles at rest and obtain:

For a quark <u>at rest</u>: $W_{\pm} = W_x \pm iW_y = + \frac{1}{2}\sigma_{\pm}$

For an antiquark <u>at rest</u>: $W_{\pm} = W_x \pm iW_y = - \frac{1}{2}\sigma_{\pm}$

The total W-spin of a quark or an antiquark is, hence, equal to $\frac{1}{2}$, and they belong to the $\underline{6}$ and $\underline{\bar{6}}$ representations of $SU(6)_W$, respectively. We also conclude that the W-spins of all states which are either pure multi-quark states (with no antiquarks) or pure multi-antiquark states, are equal to the ordinary spins of these states. The baryons are, consequently, in a $\underline{56}$ of $SU(6)_W$ which contains a $W = \frac{3}{2}$, $S = \frac{3}{2}$ decuplet. The antibaryons are in a $\underline{\overline{56}}$ and their W-spins are the same. The peculiar sign-flip in the definitions of $W_{\pm}$ for the antiquarks induces only one change in the classification of the <u>low-lying</u> states. This is found in the mesonic quark-antiquark system.[5] A quark and an antiquark can be combined to form two ordinary spin multiplets: $S = 0$ and $S = 1$. They also construct two W-spin multiplets: $W = 0$ and $W = 1$. In order to find the relation between these two sets of multiplets, let us consider the four different helicity states which are obtained: There are two states with $S_z = \pm 1$ which necessarily belong to the $S = 1$ triplet. Now, $W_z = S_z$ for all particles, hence the W_z-values of these two states are also ± 1 and they belong to the $W = 1$ triplet. The two other helicity states have $S_z = W_z = 0$. We identify the zero-helicity state of the $S = 1$ triplet by applying the lowering operator S_- to the $S_z = 1$ state. The obtained state is $\frac{1}{\sqrt{2}}\left\{\left|\frac{1}{2},-\frac{1}{2}\right> + \left|-\frac{1}{2},\frac{1}{2}\right>\right\}$ where the two numbers in each state vector denote the helicities of the quark and the antiquark, respectively. The orthogonal state $\frac{1}{\sqrt{2}}\left\{\left|\frac{1}{2},-\frac{1}{2}\right> - \left|-\frac{1}{2},\frac{1}{2}\right>\right\}$

is, consequently, the S = 0 singlet. The same procedure can be followed while resolving the W = 1 state from the W = 0 state. This time we apply W_- to the $S_z = W_z = 1$ state and find that the S-spin singlet is a member of the W-spin triplet and the W-spin singlet is the zero helicity component of the S-spin triplet.[5] The W-spin triplet is constructed from the following helicity states:

$$W = 1 \begin{cases} W_z = 1 & \left|\frac{1}{2}, \frac{1}{2}\right> & S = 1;\ S_z = 1 \\ W_z = 0 & \frac{1}{2}\left\{\left|\frac{1}{2}, -\frac{1}{2}\right> - \left|-\frac{1}{2}, \frac{1}{2}\right>\right\} & S = 0;\ S_z = 0 \\ W_z = -1 & \left|-\frac{1}{2}, -\frac{1}{2}\right> & S = 1;\ S_z = -1 \end{cases}$$

For example: the four helicity states of the $p'\bar{n}'$ system (which are the π^+ and ρ^+ mesons) are grouped in the following W-spin multiplets:

$$W = 1: \left\{\rho^+_1, \pi^+_0, \rho^+_{-1}\right\} \quad W = 0: \left\{\rho^+_0\right\}$$

where the subscripts denote the helicities.

The $\underline{35}$ representation of $SU(6)_W$ contains:

1. A W = 0 octet of the zero helicity vector meson octet $V_0^{(8)}$.
2. A W = 1 octet which includes:
   a. The non-zero helicity states of the vector meson octet $V_{+1}^{(8)}, V_{-1}^{(8)}$.
   b. The pseudoscalar meson octet $P_0^{(8)}$.
3. A W = 1 unitary singlet which includes:
   a. The non-zero helicity state of a unitary singlet vector meson $V_{+1}^{(1)}, V_{-1}^{(1)}$.
   b. A unitary singlet pseudoscalar meson $P_0^{(1)}$ (presumably the $X^0(959)$ meson).

This last state of the $\underline{35}$ of $SU(6)_W$ belongs to an $SU(6)_S$ singlet. On the other hand, the zero helicity, unitary singlet, vector meson $V_0^{(1)}$, which is an $S = 1$ state of a $\underline{35}$ of $SU(6)_S$ is the $W = 0$ state of an $SU(6)_W$ singlet.

This completes the classification of the low-lying mesons and baryons. The classification of higher states can be done in the same way, using the raising and lowering W-spin operators, and the higher W-spin states are usually found to be mixtures of various ordinary spin states. We will not go here into the details of all this but simply note that the calculations of processes involving these higher resonances are usually more tedious but not more complicated in principle. In particular, particles with high spins include components with small values of W, thus allowing the coupling of a high-spin state to two low-spin states. Such couplings may involve, in general, any value of the orbital angular momentum ℓ.

Now that we have classified all relevant states according to the representations of $SU(2)_W$ and $SU(6)_W$, we go on to discuss various experimental tests on the symmetry. We start by listing the predictions which are obtained by assuming $SU(2)_W$ invariance (without referring to the SU(3) part).

1. The $P_0P_0P_0$, $V_0V_0P_0$ and $T_0V_0P_0$ vertices are forbidden by $SU(2)_W$ where P_0, V_0, T_0 are the zero-helicity states of a pseudoscalar, a vector and a $J^P = 2^+$ meson, respectively. More generally: W-spin conservation forbids any vertex of the form $C_0 \rightarrow A_0 + B_0$ if $P_A \cdot P_B \cdot P_C \cdot (-1)^{S_A+S_B+S_C} = -1$, where A_0, B_0, C_0 are three zero-helicity states of mesons A, B, C with intrinsic parities P_A, P_B, P_C and spins S_A, S_B, S_C.

This prediction of W-spin is "trivially" satisfied by the experimental data as the same selection rules can be obtained from conservation of ordinary angular momentum and parity. Consequently, it cannot serve as a test of the symmetry.

2. Photoproduction of $N^*(1238)$. In general, there are two independent amplitudes in the photoproduction of a $J^P = \frac{3}{2}^+$ state. These can be defined either as helicity amplitudes (which are the same as the W-spin amplitudes) or as the usual M1 and E2 transition amplitudes, where 1 and 2 represent the <u>total</u> angular momentum of the photon. The two helicity states of the real photon belong to the same W-spin triplet, thus leading us to a unique ratio between the two helicity amplitudes. This ratio is obtained from the unique way of coupling a $W = 1$ state (photon) and a $W = \frac{1}{2}$ state (proton) to a $W = \frac{3}{2}$ object (N^*). The prediction, expressed in terms of the usual electromagnetic multipoles, is that the transition is a pure M1 transition[4] and that the E2 contribution vanishes. This is known to be consistent with the experimental data to a good accuracy.[17]

3. Electroproduction of $N^*(1238)$. The γNN vertex in the process $e^- + p \rightarrow e^- + N^{*+}$ differs from our previous case of photoproduction only in the existence of a third amplitude, contributed by the longitudinal <u>virtual</u> photon. This term is, however, forbidden by W-spin conservation as the zero-helicity component of the photon is a $W = 0$ object and cannot be coupled to a $W = \frac{1}{2}$ state to form a $W = \frac{3}{2}$ particle. Consequently, the only amplitude which contributes to the electroproduction of the N^* is, again, the M1 amplitude,[4] and the experimental data, again, indicates that the transition is predominantly a magnetic dipole one.[18]

4. Neutrino production of $N^*(1238)$. We consider separately the contributions of the vector and axial vector parts of the coupling of the

$N^*\bar{N}$ current to the leptonic V - A current. For the vector part, again, only the M1 contribution is allowed[4] by W-spin. For the axial vector part, W-spin conservation predicts that there is no M2 transition amplitude, while all other contributions are, in principle, allowed.[4]

5. The coupling of a vector meson to a baryon and a baryon resonance with $S = W = \frac{3}{2}$ (e.g. the SU(3) decuplet states) is also predicted to be of the M1 type.[4] This can be tested, of course, only in cases where the one-particle exchange picture is known to give a good description of the experimental situation. In various such cases, M1 dominance is found to be in agreement with the data.[19]

We now proceed to the predictions of the full $SU(6)_W$ group.

6. The Johnson-Treiman[20] relations. Using $SU(6)_W$ we can easily prove the Johnson-Treiman relations for the forward elastic amplitudes of $K^{\pm}p$, $\pi^{\pm}p$ and $K^{\pm}n$ scattering (the $K^{\pm}n$ amplitudes are the same as the K^0p, $\bar{K}^0p$ amplitudes by isospin invariance). The derivation is easiest if we look at the t-channel for these processes, in which, in $SU(6)_W$ language, the reaction is:

$$\underline{56} + \overline{\underline{56}} \rightarrow \underline{35} + \underline{35}$$

The allowed channels are $\underline{405}$, $\underline{35}_F$, $\underline{35}_D$, $\underline{1}$. However, the differences between the K^+K^- and K^-K^+ states, etc., are contributed only by representations which are antisymmetric in the two mesons, i.e., only by the 35_F. The two mesons have $W = 1$, $W_z = 0$ and can be coupled only to $W = 0$ (there is no $W = 2$ in the 35, and $W = 1$ is forbidden because the C-G coefficient $\binom{111}{000}$ vanishes). The transition, hence, goes via the $(\underline{8},\underline{1})$ part of the 35_F. It can go through $I = 0$ or $I = 1$ channels, but using the V-spin subgroup of SU(3) we find that it goes only through

$V = 1$. That means that for all three pairs of elastic processes the difference between the mp and the $\overline{m}p$ amplitudes is given only by a $V = 1$, $V_z = 0$, $W = 0$, SU(3)-octet term of a $\underline{35}$ of $SU(6)_W$. Using the Wigner-Eckart theorem we obtain the final conclusion:

$$A(mp) - A(\overline{m}p) = \alpha(m \left| V_z \right| m)$$

Namely, the difference between the amplitudes is proportional to the eigenvalues of the operator V_z for the appropriate meson, where α is a common term which includes the reduced matrix element for the processes and the C-G coefficients for the $p\overline{p}$ system which are the same in all cases. We know that:

$$(K^+ \left| V_z \right| K^+) = 1; \ (K^0 \left| V_z \right| K^0) = \frac{1}{2}\ ; \ (\pi^+ \left| V_z \right| \pi^+) = \frac{1}{2}$$

Hence:

$$\frac{1}{2}\,[A(K^+p) - A(K^-p)] = A(\pi^+p) - A(\pi^-p) = A(K^0p) - A(\overline{K}^0p) = A(K^+n) - A(K^-n)$$

Using the optical theorem we get:

$$\frac{1}{2}\,[\sigma_t(K^+p) - \sigma_t(K^-p)] = \sigma_t(K^+n) - \sigma_t(K^-n)$$

This is very well satisfied by the experimental data over a wide energy range.[21] The other relation, concerning the Kp and πp cross sections, does not contradict the data, but the agreement here is poorer.[21]

We should add that this success of $SU(6)_W$ is extremely mysterious as the experimental situation with respect to SU(3) for these reactions indicates that strong SU(3) symmetry breaking effects are involved.[22]

7. $\bar{p}p$ annihilation at rest into two pseudoscalar mesons. The annihilation goes via a $C = -1$, ^3S state and $SU(6)_W$ predicts:[4]

$$(\bar{p}p|\pi^+\pi^-) : (\bar{p}p|K^+K^-) : (\bar{p}p|K^0\bar{K}^0) = 1:2:1$$

The derivation of these ratios follows the same lines as that of the Johnson-Treiman relation, and the only contributing $C = -1$ channel is the 35_F. The prediction for the ratio of the charged to neutral K pairs may serve as a cleaner tests of the symmetry than the predicted $KK/\pi\pi$ ratio, as the K-π mass difference may induce large kinematical corrections for this last ratio. It is predicted that the number of K^+K^- pairs will be larger by a factor of <u>four</u> than the number of $K_1^0K_2^0$ pairs produced in the annihilation process <u>at rest</u>.

It is interesting to note that if we consider the $\bar{p}p$ system to be really at rest (and not in a "protunium" state with finite relative momentum) we find that the $\bar{p}pV$ vertex is forbidden in this approximation. This follows from the fact that the $\bar{p}p$ system <u>at rest</u> has $\gamma_0 = 6$,[16] while a <u>moving</u> vector meson cannot have $\gamma_0 > 2$. The simplest allowed diagram for the annihilation process is, in this case, a one-baryon exchange diagram. Similarly the $\bar{p}p$ annihilation <u>at rest</u> into an electron-positron pair cannot go through a one-photon intermediate state, if we assume that the static $U(6) \otimes U(6)$ invariance can be applied to the $\bar{p}p$ system.

8. $\varphi \to \rho\pi$. The φ is usually defined as the $S = 1$ state of the $\lambda'\bar{\lambda}'$ system whereas the ω is built out of $(p'\bar{p}' + n'\bar{n}')$. These definitions are motivated mainly by the approximately known octet-singlet mixing angle, predicted by the Gell-Mann-Okubo SU(3) mass formula. If

the φ is really identical with the $\lambda'\bar{\lambda}'$ vector meson, we can show that the decay $\varphi \to \rho\pi$ is forbidden by $SU(6)_W$. The simplest way of seeing this is to consider the separate conservation[23] of $W_{\lambda'}$, the total W-spin of all λ' and $\bar{\lambda}'$ "particles." The $\varphi \to \rho\pi$ decay may go only through the non-zero helicity state of the φ as the $V_0V_0P_0$ vertex is forbidden (prediction No. 1). However, this helicity state of the φ has $W = W_{\lambda'} = 1$ while the $\rho_1\pi$ system contains no λ' or $\bar{\lambda}'$ quarks and has $W_{\lambda'} = 0$ (but $W = 1$ or 2). $W_{\lambda'}$ conservation, then, forbids the process. This prediction is found to be in very good agreement with the data as the $\varphi \to \rho\pi$ partial width is experimentally very small ($18 \pm 8\%$ of the total φ width)[24] and phase-space considerations lead us to a still much smaller matrix element for the decay.

9. The famous $-\frac{2}{3}$ ratio between the magnetic form factors of the neutron and proton is also a result[6] of $SU(6)_W$.

10. Various interesting predictions concerning a large number of photon-baryon, meson-baryon, baryon-baryon and antibaryon-baryon forward scattering amplitudes have been derived[25] from $SU(6)_W$. These are discussed in detail by S. Meshkov in these proceedings. We mention here only two points concerning these predictions. First, there are various cases in which $SU(6)_W$ gives a definite ratio between two otherwise independent <u>isospin</u> channels (i.e., $\pi + N \to \pi + N^*$). In these cases there are no problems of mass differences and the difficult problem of "how to compare the predictions of the higher symmetries to the experimental scattering data"[22] is avoided. However, we encounter here another difficult problem: What is exactly meant by "forward scattering amplitude?" It is, at present, impossible in most cases to measure differential cross sections in such small angles that will give us clear information about

the expected 0° angle amplitude. Comparing such predictions with experiment is, hence, very difficult.

The other point that we would like to make is the following: consider the set of reactions $P + B \rightarrow P + B^{*}$ where P, B and B^{*} are respectively the pseudoscalar meson octet, the baryon octet and the decuplet of baryon resonances. If we consider the forward scattering amplitude in the t-channel we find that the two mesons can be coupled to $W = 2$ or $W = 0$ while the BB^{*} system can be in a $W = 2$ or a $W = 1$ state. The process can proceed only via the exchange of a $W = 2$ (but not necessarily $S = 2$) object. This is not surprising as we know that a pseudoscalar meson cannot be coupled to the two mesons and if we exchange a vector meson we get only an M1 contribution in the VNN^{*} vertex and the M1 amplitude is known to vanish in the forward direction. This means that W-spin invariance + one vector meson exchange predict zero forward scattering for all $P + B \rightarrow P + B^{*}$ reactions. It also means that the ratios predicted by W-spin for the cases of a dominant vector meson exchange refer to the "background" contribution of other terms which are induced by the exchange of a more complicated system or, possibly, by simple poles in the S-channel. In such cases we cannot expect both the predictions and their comparison with experiment to be too significant.

11. Using a broken $SU(6)_W$, one can derive predictions for the non-leptonic decays of baryons.[26] However, these predictions strongly depend on the assumed transformation properties of the non-leptonic weak Hamiltonian. We will not discuss them in detail in these notes.

12. It is well known that the SU(3) symmetry is badly broken for various scattering processes.[22] Consequently, we have to include these

symmetry-breaking terms in all our calculations. On the other hand, it is apriori clear that other symmetry-breaking effects are comparable to, if not greater than, the SU(3) breaking terms. These are the terms which break the $SU(2)_W$ symmetry (e.g., the π-ρ mass difference) or the ones which appear in the transition stage between an SU(3) $\otimes$ $SU(2)_W$ invariant theory and a complete $SU(6)_W$ invariance (e.g., the N-N^* mass difference). A detailed analysis of the $SU(6)_W$ transformation properties of all the various symmetry-breaking terms is clearly needed. Such an analysis has not yet been done. It is, however, known that the simplest possible assumption, namely that of a $\underline{35}$ dominance of the symmetry-breaking terms is, in general, not correct.[27]

We conclude that the assumption of an approximate W-spin and $SU(6)_W$ invariance may lead us to a reasonable description of various physical situations, including certain vertex functions and forward-scattering amplitudes. The agreement with experiment is in some cases remarkably good; in fact, it is by far better than what we should expect from a theory in which the ρ and π masses are degenerate. It is perhaps worth mentioning that all the major successes of $SU(6)_W$ occur in cases where mass differences are irrelevant (μ_n/μ_p, $\varphi \to \rho\pi$, M1 dominance, Johnson-Treiman relation, etc.). Is this an accident or a meaningful result? We do not know.

Are we allowed (and if yes, why?) to exclude non-colinear intermediate states?

How should one include all symmetry-breaking effects in the calculations?

What can be predicted by the coplanar U(3) $\otimes$ U(3) group?

Is there any simple $U(6) \otimes U(6)$ mass formula?

How do the 0°-angle predictions of W-spin change when we go to small angles?

These are characteristic problems which we still have to answer if we want to apply all these ideas consistently to strong interaction physics.

LIST OF REFERENCES

1. B. Sakita, Phys. Rev. 136, B1756 (1964). F. Gursey and L. A. Radicati, Phys. Rev. Letters 13 173 (1964).
2. See e.g., L. O'Raifeartaigh, Phys. Rev. Letters 14, 575 (1965).
3. R. F. Dashen and M. Gell-Mann, Phys. Letters 17, 142 (1965).
4. H. Harari and H. J. Lipkin, to be published in Phys. Rev.
5. H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 670 (1965). See also: S. Meshkov, these proceedings.
6. K. J. Barnes, Phys. Rev. Letters 14, 798 (1965); K. J. Barnes, P. Carruthers and F. von Hippel, Phys. Rev. Letters 14, 82 (1965).
7. A. Salam, R. Delbourgo and J. Strathdee, Proc. Roy, Soc. A284, 146 (1965).
8. V. Bargmann and E. P. Wigner, Proc. Nat. Acad. Sc. Wash. 34, 211 (1948).
9. B. Sakita and K. C. Wali, Phys. Rev. Letters 14, 404 (1965); Phys. Rev. 139, B1355 (1965). M.A.B. Beg and A. Pais, Phys. Rev. Letters 14,269 (1965).
10. This approach was used by Sakita and Wali, ref. 9.
11. M.A.B. Beg and A. Pais, Phys. Rev. Letters 14, 509 (1965); R. Blankenbecler, M. L. Goldberger, K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 518 (1965).
12. H. Harari, Phys. Rev. Letters 14, 1100 (1965).
13. E. P. Wigner, Phys. Rev. 51, 106 (1937).
14. A detailed explanation of this point can be found in references 5 and in H. J. Lipkin, to be published in the proceedings of the Trieste Seminar on High-Energy Physics, May-June 1965.
15. See e.g. R. Adair, these proceedings.

16. H. Harari, H. J. Lipkin and S. Meshkov, Phys. Rev. Letters 14, 845 (1965).

17. M. Gourdin and Ph. Salin, Nuovo Cimento 27, 193 and 309 (1963).

18. C. W. Akerlof *et al.*, Phys. Rev. Letters 14, 1036 (1965).

19. L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters 11, 90 (1963); B. Kehoe, Phys. Rev. Letters 11, 93 (1963); J. Leitner, these proceedings.

20. K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965).

21. See e.g. the results presented by S. J. Lindenbaum, these proceedings.

22. This point is discussed in detail in: H. Harari, to be published in the proceedings of the Trieste Seminar on High-Energy Physics, May-June 1965.

23. H. J. Lipkin, Phys. Rev. Letters 13, 590 (1964); 14, 513 (1965).

24. J. S. Lindsey and G. A. Smith, Phys. Rev. Letters 15, 221 (1965).

25. J. C. Carter, J. J. Coyne, S. Meshkov, D. Horn, M. Kugler and H. J. Lipkin, Phys. Rev. Letters 15, 373 (1965).

26. D. Horn, M. Kugler, H. J. Lipkin, J. C. Carter, J. J. Coyne and S. Meshkov, Phys. Rev. Letters 14, 717 (1965).

27. H. Harari, D. Horn, M. Kugler, H. J. Lipkin and S. Meshkov, Phys. Rev., in print; H. Harari and M. A. Rashid, preprint.