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AXIAL MESON EXCHANGE AND THE RELATION OF HYDROGEN HYPERFINE SPLITTING TO ELECTRON SCATTERING*

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In a recent note to this Journal, Fenster, Köberle, and Nambu[1] suggested that the exchange of an axial vector meson ($J^{PC}= 1^{++}$) might remove much of the apparent discrepancy between theory and experiment for the ground state hyperfine splitting (hfs) of hydrogen. The existence of such an electromagnetic Gamow-Teller type interaction was postulated by Nambu and collaborators in earlier publications[2] in order to maintain divergence free axial currents. Its strength was determined on the basis of a theoretical model and the observed $\pi^0 \to 2\gamma$ decay rate; its phase was undetermined by their arguments. We wish to show in this letter that such a suggestion is difficult to reconcile with existing data on the ratio of positron-proton to electron-proton elastic scattering.[3]

The diagram considered by Fenster, et.al. is shown in Fig. 1, where the axial vector meson of mass m_A is indicated by A. We denote its phenomenological couplings to protons and electrons by B and b respectively. The true coupling of A to electrons presumably occurs through a two photon intermediate state; however, it is not necessary to concern ourselves with the detailed mechanism here. The diagram of Fig. 1 gives for the S-matrix element between e^-p states[4]

$$S_{e^-p;e^-p} = (2\pi)^4\,\delta^4(p'+\ell'-p-\ell)\sqrt{\frac{m_e^2 m_p^2}{p_0' p_0 \ell_0' \ell_0}}\;iBb\,\bar{u}(p')\,\gamma_\mu\gamma_5\,u(p) \times \bar{u}(\ell')\,\gamma_\nu\gamma_5\,u(\ell)\;\frac{\left(g^{\mu\nu}-\dfrac{q^\mu q^\nu}{m_A^2}\right)}{q^2-m_A^2} \tag{1}$$

where $q = p' - p$.

From Eq. (1) in the non-relativistic limit one infers an effective hyperfine Hamiltonian

$$\delta H = -\frac{Bb}{m_A^2}\, \underline{\sigma}_e \cdot \underline{\sigma}_p\, \delta(\underline{r}_e - \underline{r}_p) \tag{2}$$

in addition to the usual Fermi-Segre term

$$H = \frac{e^2}{12 m_e m_p}\, g_p\, \underline{\sigma}_e \cdot \underline{\sigma}_p\, \delta(\underline{r}_e - \underline{r}_p) \tag{3}$$

where $g_p = 5.58$ is the proton g factor. Hence, the exchange of an axial vector meson generates a fractional change in the triplet-singlet ground state hyperfine splitting $\nu = \nu_T - \nu_S$ of

$$\frac{\delta\nu}{\nu} = -\left(\frac{Bb}{m_A^2 e^2}\right)\left(\frac{12 m_e m_p}{g_p}\right) . \tag{4}$$

If the apparent discrepancy between theory and experiment[5]

$$\frac{\nu_{expt} - \nu_{theory}}{\nu_{expt}} = (45 \pm 17) \times 10^{-6} \approx 4(1 \pm \tfrac{1}{2}) \times 10^{-5} \tag{5}$$

is attributed entirely to the axial exchange mechanism the resulting coupling is determined to be

$$\frac{Bb}{4\pi} = -\, 5.5\, (1 \pm \tfrac{1}{2})\, \alpha^2 \left(\frac{m_A}{m_p}\right)^2 ; \quad \alpha = \frac{1}{137} . \tag{6}$$

We should perhaps point out the relation of the axial vector exchange of the type considered here to the exhaustive discussion of the hydrogen hyperfine splitting presented in the latest analysis of Iddings (Ref. 5). In that work Iddings expressed the proton structure contributions to the hfs in terms of a dispersion relation for the forward Compton scattering $\gamma + p \rightarrow \gamma + p$ of off-mass shell photons. The weight functions in this dispersion relation are measurable in $e^- p$ scattering. There remains, however, the possible presence of subtraction terms proportional to the photon mass. (On-the-mass-shell Compton scattering has a subtraction term uniquely fixed by the Thomson limit.) Axial vector exchange (with coupling to the electron line via two photons) is precisely an off-the-mass-shell subtraction term of this type; it has no absorptive part in the $e^- p$ channel, and it vanishes as the photons go on the mass shell ($1^+ \rightarrow \gamma + \gamma$ is forbidden by angular momentum conservation and statistics for real photons).

The S-matrix element Eq. (1) also occurs in electron and positron-proton scattering in addition to the usual one photon exchange contribution and leads to a first order correction to the Rosenbluth cross section that is readily calculated to be[6]

$$\left(\frac{d\sigma}{d\Omega}\right)_{e^\pm p} = \left(\frac{d\sigma}{d\Omega}\right)_M \left\{ \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2 \theta/2 \pm \frac{bBG_M}{\pi\alpha} \frac{\tau\left[\frac{E}{m} - \tau\right]}{\left[\tau + \frac{1}{4}\left(\frac{m_A}{m_p}\right)^2\right]} \tan^2 \theta/2 \right\} \tag{7}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_M = \frac{\alpha^2 \cos^2 \theta/2}{4E^2 \sin^4 \theta/2} \frac{1}{\left[1 + \frac{2E}{m} \sin^2 \theta/2\right]}$$

$$\tau \equiv -\frac{q^2}{4m_p^2} > 0$$

E is the incident electron energy, and G_E and G_M are familiar electric and magnetic form factors normalized to $G_E(0) = 1$, $G_M(0) = \frac{1}{2} g_p = 2.79$. Two features of Eq. (7) are of particular experimental interest. The last term changes sign from + for positron-proton to − for electron-proton scattering since it is an interference term between the odd charge conjugation amplitude for photon exchange and the even amplitude one for axial exchange (equivalent to two photons). Also the last term has a factor (E/m) leading to a deviation from the Rosenbluth straight line.[6]

If (Bb) has the minus sign required to fit the hyperfine splitting we see immediately from Eq. (7) that $\left(\frac{d\sigma}{d\Omega}\right)_{e^+p} / \left(\frac{d\sigma}{d\Omega}\right)_{e^-p} \leq 1$. Using Eqs. (6) and (7) and allowing Bb to have a form factor varying like $G_E(q^2)$ we have calculated the ratios shown in Fig. 2 at energies and angles corresponding to the experimental points of Browman, et.al.[3] It is clear from that figure that axial vector exchange sufficient to explain all or most of the hyperfine discrepancy is difficult if not impossible to reconcile with the observed ratio. The theoretical points are insensitive to the choice of the axial meson mass for all $m_A^2 > -q^2$ as assumed here and their error bars reflect the uncertainty in Eq. (6). Only with an ad hoc assumption that the coupling Bb has a form factor that falls for large $q^2 < 0$ much more rapidly than $G_E(q^2)$ and $G_M(q^2) \approx 2.79 \times G_E(q^2)$ is it possible to avoid this contradiction. Alternatively one must turn to additional and compensating 2γ exchange contributions to $e^{\pm}p$ scattering which have not been indicated by earlier studies[7] or one must look for an interpretation of the apparent hfs discrepancy in Eq. (5) in terms of other inadequacies of the theoretical calculations of the 2γ exchange contribution. Pseudoscalar exchange cannot provide such a compensation since its coupling via two photons to the electron line vanishes in the high energy limit $m_e \to 0$. (See the footnote on page 36 of Ref. 6.)

It is also possible in principle to test for this axial vector exchange by means of electron-proton scattering alone.[6] From Eq. (7) we see that in the plot of $\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_M$ vs. $\tan^2 \theta/2$ at fixed q^2 the explicit presence of $\frac{E}{m}$ in the interference term causes a departure from straight line behavior. However, with the coupling parameter of Eq. (6) it is found that experiments to better than a 1% accuracy would be required in order to detect the predicted deviation from linearity with present accelerator (including SLAC) parameters. This does not seem to be feasible.

REFERENCES

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FIGURE CAPTIONS

Fig. 1. Axial vector exchange contribution to the electron-proton interaction.

Fig. 2. The positron-proton to electron-proton differential cross section ratio vs. momentum transfer. Shown are the experimental points of Browman, et.al. and the corresponding points that would be expected on the basis of axial vector exchange using the coupling of Eq. (6) and a form factor varying as $G_E(q^2)$. The theoretical points are calculated for $m_A = 1.5\, m_p$ and are insensitive to this choice.

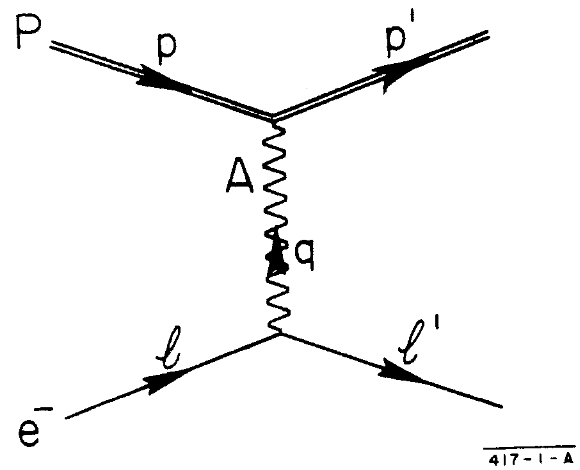
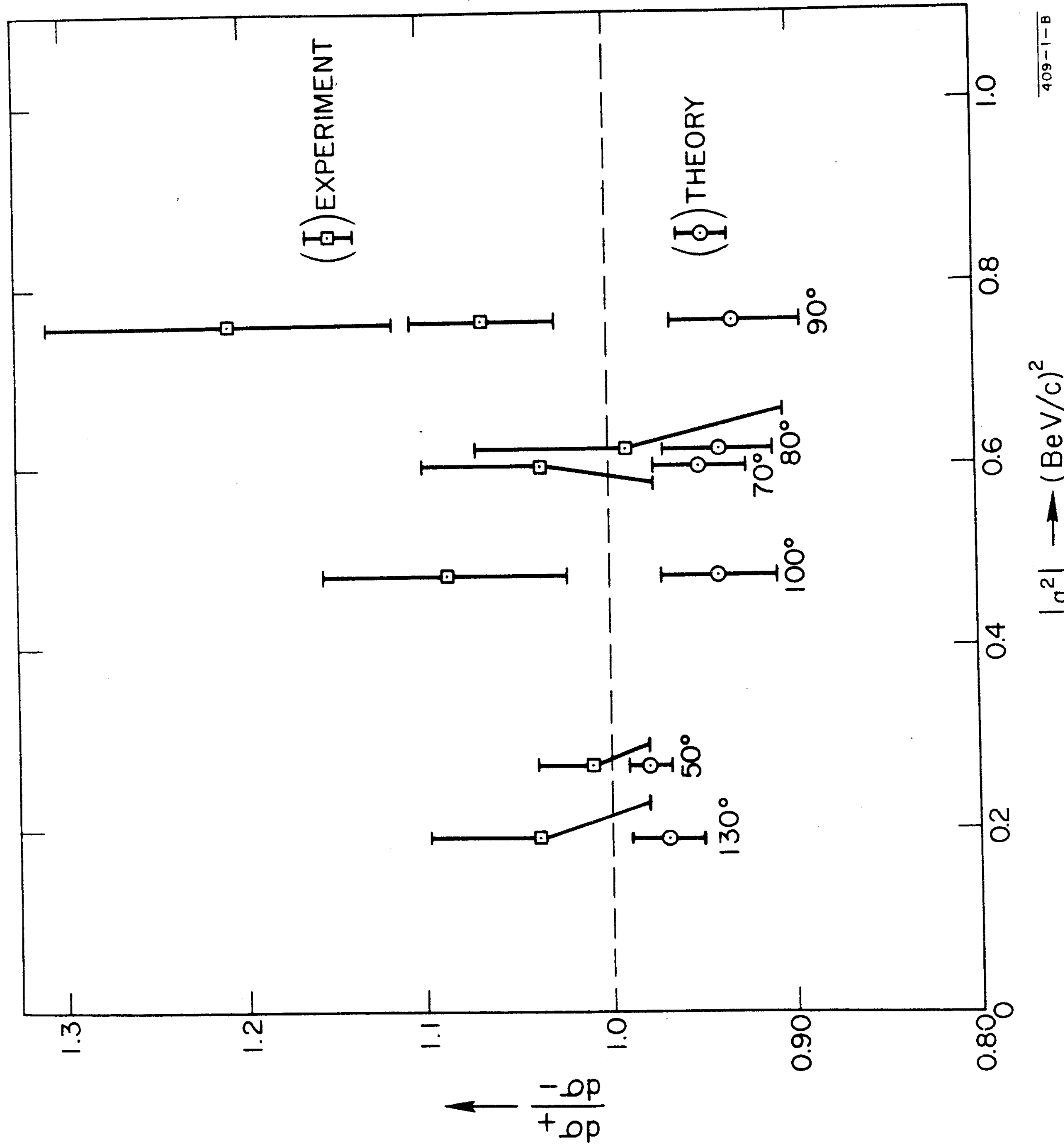


FIG. 1



EXPERIMENT
THEORY
90°
80°
70°
100°
50°
130°
1.3
1.2
1.1
1.0
0.90
0.80
0
0.2
0.4
0.6
0.8
1.0
$|q^2| \longrightarrow (\mathrm{BeV}/c)^2$
$\frac{d\sigma^+}{d\sigma^-} \longrightarrow$
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FIG. 2