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RADIATIVE CORRECTIONS TO COLLIDING BEAM EXPERIMENTS*

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The general procedure of calculating the radiative corrections is well known. It is also well realized that the real photon emission part of the radiative corrections has to be calculated separately for different types of experimental set up. Most experiments in the colliding beam program will probably be carried out using spark chambers. In this paper we give a general procedure of treating the radiative corrections to the two-body final state problems

$$e_1 + e_2 \rightarrow A + B\ , \tag{1}$$

in the e^-e^- or e^+e^- colliding beam experiment using spark chamber as a detector. Experimentally, two-body final states are characterized by the fact that in the absence of radiative corrections A and B must come out with opposite momenta $\vec{P}_A = -\vec{P}_B$ whose magnitude is given by

$$P^2_{max} = \left[E^2 - \tfrac{1}{2}\,(m^2_A - m^2_B) + (m^2_A - m^2_B)^2(16E^2)^{-1}\right]\ . \tag{2}$$

However, since photons are always emitted in the process, A and B will not, in general, be colinear and their momenta will be less than given by Eq. (2). Thus experimenters have to give so-called "criteria of coincidence." The most sensible criteria are as follows: An event of the type (1) shall be called coincident if for every particle A going in the direction θ_{1A} (see Fig. 1a), the particle B comes out within the cone opposite A having a half-angle $\Delta\theta$, i.e.,

$$0 < \pi - \theta_{AB} < \Delta\theta \tag{3}$$

and further, the magnitude of the momenta of the particles A and B shall be within the ranges

$$P_A^{min} < P_A < P_{max} \quad , \tag{4}$$

and

$$P_B^{min} < P_B < P_{max} \tag{5}$$

respectively.

Given these restrictions on the phase space of particles A and B, one can proceed to calculate the radiative corrections. It is most convenient to transform the restrictions on the phase space on A and B given by Eqs. (3), (4), and (5) into those on the photon phase space by the energy momentum conservation. In general the resultant photon phase space will look like a bomb as shown by the dotted lines in Fig. 1.

The shape of this phase space can be obtained as follows: From momentum conservation P_A, P_B, and k must form a triangle. From energy conservation the sum of the three sides of this triangle must be fixed (see Fig. 2). We are interested in obtaining the maximum value of k as a function of θ_{KA}.

1. To obtain a to b in Fig. 1a we let $P_A = P_A^{min}$.
2. To obtain b to c in Fig. 1a we let $\theta_{AB} = \pi - \Delta\theta$.
3. To obtain c to d in Fig. 1a we let $P_B = P_B^{min}$. Analytical expressions for k_{max} as a function of θ_{KA} can thus be obtained from elementary manipulations.

In order to obtain a reasonably compact formula for the radiative corrections, we may approximate this bomb-shaped photon phase space by three regions indicated by solid lines in Fig. 1b. The construction of this approximate phase space is based on the following considerations:

1. Most of the photons are emitted in the directions along either e_1, e_2, A or $B_{e\ell}$ due to the δ function-like behavior of the matrix element in these four regions. Hence, only in the vicinity of these four regions one needs to be very careful and can safely deform other parts of the phase space to simplify the calculation.

2. The phase space of one photon emission can be written as

$$Y = \int \frac{d^3k}{2\omega} \frac{d^3P_A}{2E_A} \frac{d^3P_B}{2E_B} \delta^4(P_1 + P_2 - P_A - P_B - k)$$

$$= \frac{d\Omega_A}{8} \int_{\omega_{min}}^{\omega_{max}} k\,d\omega \int_{\cos(\theta_{Ak}^{max})}^{1} d(\cos\theta_{Ak}) \frac{2\pi P_A}{2E - \omega(1 - \cos\theta_{Ak})} \tag{6}$$

The deformed photon phase space is to be chosen such that the ω dependence of $\cos\theta_{Ak}^{max}$ shall be so simple that the subsequent integration can be done analytically.

3. Let $Y = Y_{soft} + Y_A + Y_B$ as shown in Fig. 1b. The spherically symmetric part Y_{soft} corresponds to the phase space for the soft photon emission. For this part $\cos(\theta_{Ak}^{max}) = -1$, $\omega_{min} = \lambda$ (fictitious photon mass) and[1] $\omega_{max} = (\overline{Of} \times \overline{Og})^{\frac{1}{2}}$ (see Fig. 1b). The contribution of this part contains infrared divergence and its treatment is well known. The top and bottom parts of the dotted lines are replaced by spherical surfaces with radii $\overline{Oa}$ and $\overline{Od}$ respectively.

Rather than proceeding with general discussions, we give the result of our calculation for the process $e^- + e^- \rightarrow e^- + e^-$. For this case we let $P_A^{min} = P_B^{min} \equiv E_{min}$, we have then $\overline{Oa} = \overline{Od} = E - E_{min}$. The equation satisfied by line bc of Fig. 1a is

$$\omega_{max}(\theta_{Ak}) = \frac{2E\Delta\theta}{\Delta\theta + \sin\theta_{Ak} + \sin(\theta_{Ak} - \Delta\theta)} \tag{7}$$

From this we can obtain the maximum energies of photons emitted along e_1 and e_2 and by taking their geometrical average we obtain the radius (defined as. ΔE) for Y_{soft}.

$$\begin{aligned} \Delta E &\equiv [\omega_{max}(\theta_{Ak} = \theta_{1A}) \times \omega_{max}(\theta_{Ak} = \pi - \theta_{1A})]^{\frac{1}{2}} \\ &\approx \frac{2E\Delta\theta}{2\sin\theta_{1A} + \Delta\theta} \end{aligned} \tag{8}$$

For Y_A we replace two sides by straight lines parallel to $\overline{Oa}$ as shown in Fig. 1b. We have then for Y_A, $\cos\theta_{Ak}^{max} = (1 - E^2\Delta\theta^2\omega^{-2})^{\frac{1}{2}}$, $\cos\theta_{Ak}^{min} = 1$, $\omega_{min} = \Delta E$ and $\omega_{max} = E - E_{min}$. For Y_B we have $\cos\theta_{Ak}^{max} = -1$, $\omega_{min} = \Delta E$, $\omega_{max} = E - E_{min}$, and $\cos\theta_{Ak}^{min} = \frac{\Delta\theta^2}{2}\left(\frac{E}{\omega} - 1\right)^2 - 1$.

The radiative corrections to the e^-e^- scattering were considered by the author[2] in 1960. The virtual radiative correction plus the soft emission part [Eq. (T.23)] can be used without change except that the new definition of ΔE, given by Eq. (8), must be used. An approximate expression for the matrix element squared for the hard photon emission is given by Eq. (T.33), which can be used to evaluate the hard photon emitted into Y_A and, by appropriate change of particle indices, it can be used to evaluate those emitted into Y_B. The result can be written as

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{d\sigma}{d\Omega}_{Møller}(1 + \delta) \\ \delta &= \delta_{soft} + \delta_A + \delta_B \end{aligned} \tag{9}$$

where δ_{soft} is given by Eq. (T.24), with ΔE defined by Eq. (8),

$$\delta_A = \frac{\alpha}{\pi}\left[\left(\ln\frac{E - E_{min}}{\Delta E} - \frac{E - E_{min} - \Delta E}{E}\right)\ln\frac{2E^2}{m^2} - \ln\frac{E - E_{min}}{\Delta E}\right.$$
$$\left. - \ln^2\frac{\Delta\theta E}{\sqrt{2}(E - E_{min})} + 2\,\frac{E - E_{min}}{E}\ln\frac{\sqrt{2}(E - E_{min})}{\Delta\theta E}\right], \quad (10)$$

and

$$\delta_B = \frac{\alpha}{\pi}\left[\left(\ln\frac{E - E_{min}}{\Delta E} - \frac{E - E_{min} - \Delta E}{E}\right)\ln\left(\frac{E}{m}\Delta\theta\right)^2 - \ln\frac{E - E_{min}}{\Delta E} + \ln^2\frac{E}{\Delta E}\right] \quad (11)$$

The numerical examples are given in Fig. 3. It should be noted that δ is more sensitively dependent on the choice of $\Delta\theta$ than E_{min} in the range of values considered. The relatively insignificant dependence of δ on E_{min} implies that the hard photon emission is rare, and thus one is allowed to make very crude approximations on both the phase space (Y_A and Y_B) and the matrix element in calculating δ_A and δ_B (see Ref. 2 for details).

We can treat similarly all the e^+e^- colliding experiments with two-body final states

$$e^+ + e^- \rightarrow A + B .$$

We have not made detailed analyses of all the processes of this type. However, the following comments may be of general interest.

1. The "virtual" radiative corrections to many of the e^+e^- colliding beam experiments such as $e^+ + e^- \rightarrow e^+ + e^-$, $e^+ + e^- \rightarrow \pi^+ + \pi^-$, $e^+ + e^- \rightarrow 2\gamma$, $e^+ + e^- \rightarrow \mu^+ + \mu^-$, and $e^+ + e^- \rightarrow p + \bar{p}$ can be obtained from the existing results[1,2,3,4,5] of $e^- + e^- \rightarrow e^- + e^-$, $e^- + \pi^- \rightarrow e^- + \pi^-$, $e^- + \gamma \rightarrow e^- + \gamma$, $\bar{e} + \mu^- \rightarrow e^- + \mu^-$, and $e^+ + p \rightarrow e^+ + p$ by the well-known substitution rule.

2. The cross section for one soft photon emission from the process $e^+ + e^- \to A + B$ can be written as

$$d\sigma_{soft} = -d\sigma_o \frac{\alpha}{4\pi^2} \int_\lambda^{\Delta E} k d\omega \int d\Omega_k \left[\frac{p_+}{p_+ \cdot k} - \frac{p_-}{p_- \cdot k} + Z\left(\frac{p_A}{p_A \cdot k} - \frac{p_B}{p_B \cdot k} \right) \right]^2 \quad (11)$$

where $d\sigma_o$ is the lowest-order cross section, and $Z = 1, -1, 0$, depending on whether the charge of particle B is $+1, -1$, or 0, respectively. The integration of the type (11) is well known.[7] For simplicity we treat the case

$$p_A^2 = p_B^2 = M^2 \text{ and } p_+^2 = p_-^2 = m^2$$

The result can be written as

$$d\sigma_{soft} = d\sigma_o \frac{\alpha}{\pi} \ln \frac{E}{\Delta E} \left[1 + Z^2 - 2\ln \frac{4E^2}{m^2} - Z^2 \frac{2E^2 - M^2}{EP_A} \ln \frac{(E+P_A)^2}{M^2} + 4Z\ln \frac{(p_+ \cdot p_A)}{(p_+ \cdot p_B)} \right]$$

$$-d\sigma_o \left[\frac{\alpha}{\pi} \; k(+,+) - K(+,-) + Z^2K(A,A) - Z^2K(A,B) + 2ZK(+,A) - 2ZK(+,B) \right] \quad (12)$$

where[1] K's are the infrared terms and always cancel out completely against similar terms in the virtual radiative corrections.

3. If we ignore the radiative corrections, all the processes of the type $e^+ + e^- \to A + B$ must be symmetric with respect to 90^o in the c.m. ($e^+ + e^- \to e^+ + e^-$ is the only exception). From the term linear in Z in Eq. (12) we notice that if the final particles are charged, then there will be more positively charged final particles going in the direction of P_+ than the negatively charged ones. This phenomenon is very similar to the

difference between the e^+p and e^-p scatterings where e^+p in general has a larger cross section at a finite fixed angle than e^-p if higher-order terms are included.[1]

4. In reaching the above conclusion we have assumed that the two-photon exchange graphs do not contribute anything significant except for supplying infrared terms

$$d\sigma_o \frac{\alpha}{\pi} 2Z[K(+,A) - K(+,B)] \tag{13}$$

which cancel out with the similar terms in Eq. (12). This assumption has been verified in all the calculations[2,4,5] done by perturbation theory for ee, eμ, and eπ scatterings. When the final particles A and B are strongly interacting, there may be some additional non-negligible contributions beside the infrared terms in the two-photon exchange diagrams.

LIST OF REFERENCES

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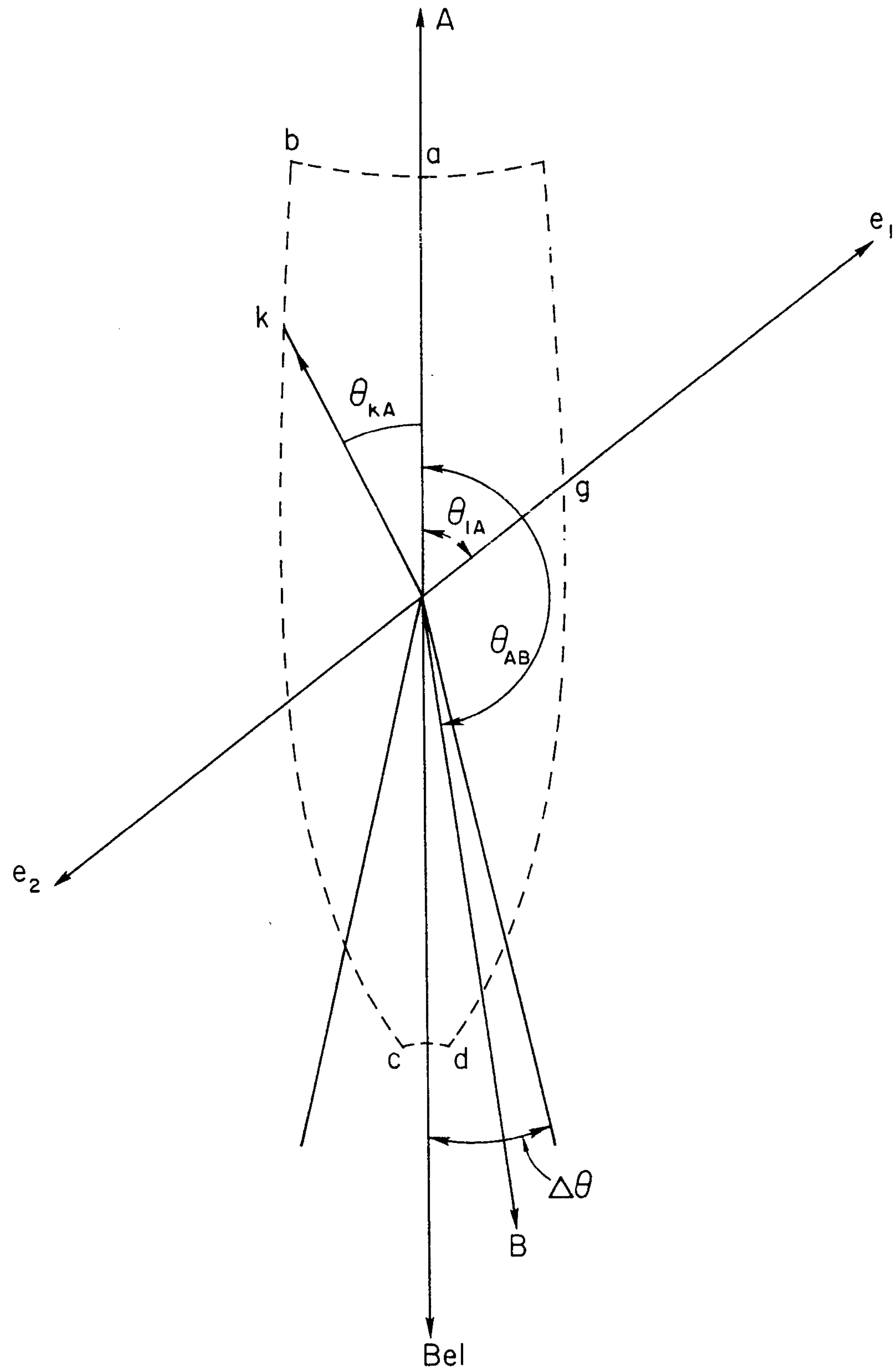

Fig. 1a

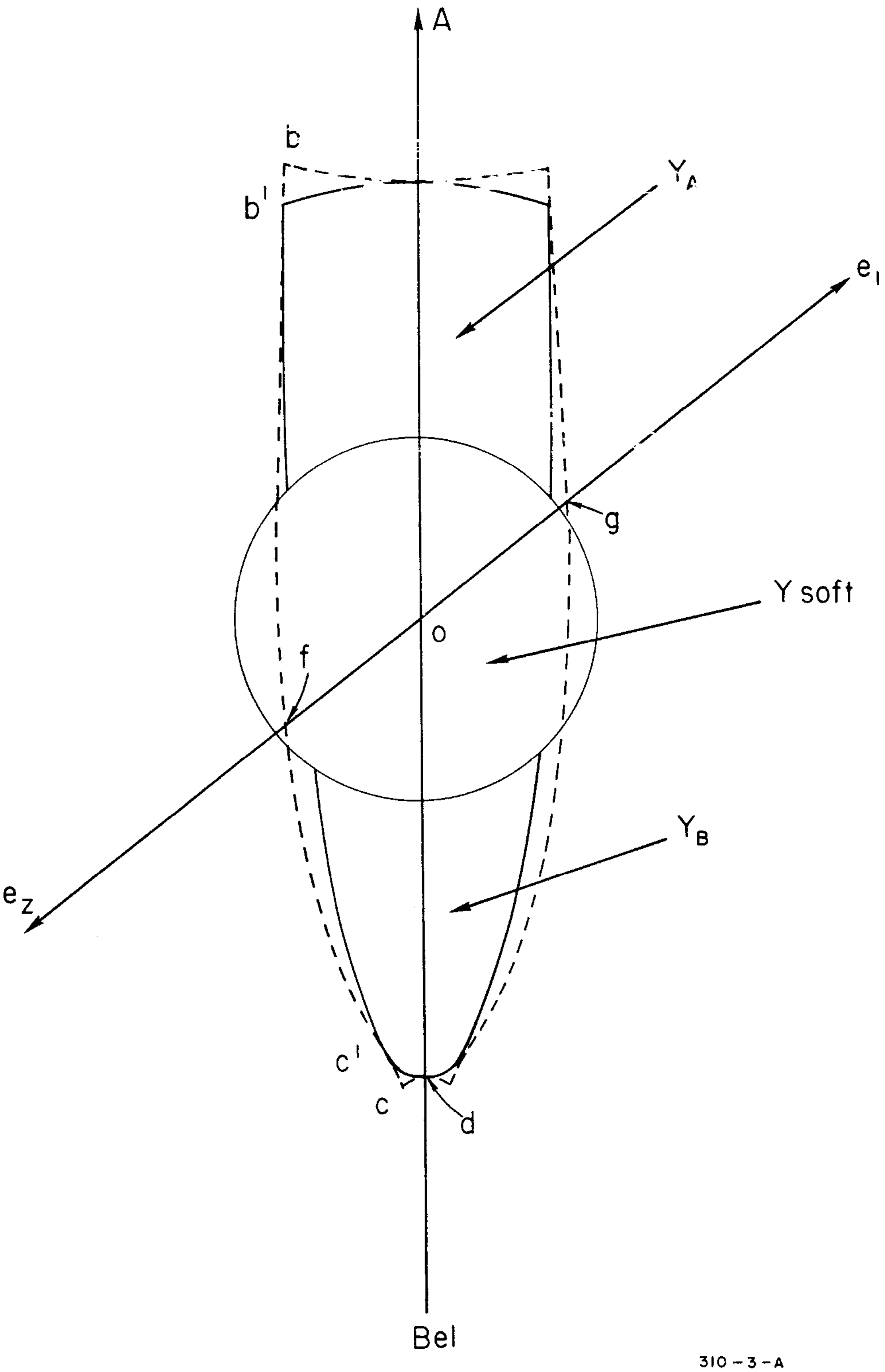

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Fig. 1b

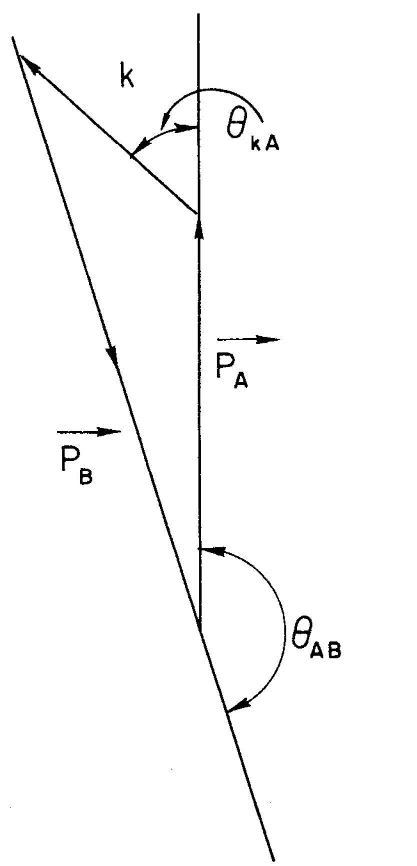
k
θ_{kA}
$\overrightarrow{P_A}$
$\overrightarrow{P_B}$
θ_{AB}
310-2-A

Fig. 2

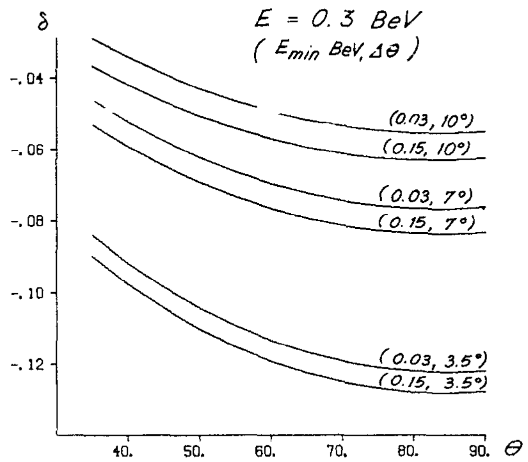

Fig 3 a

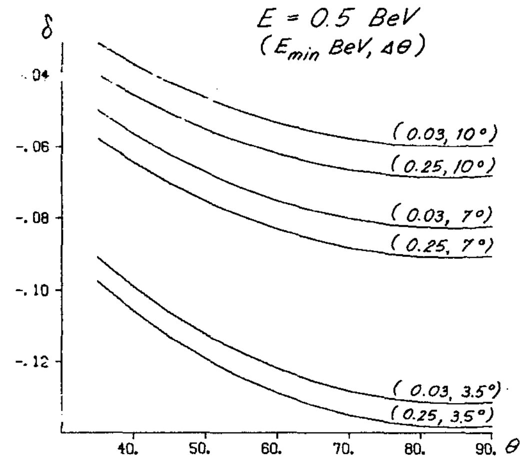

Fig 3 b

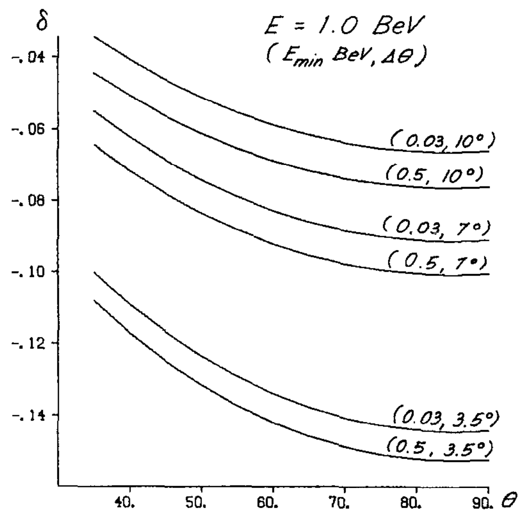

Fig 3 c

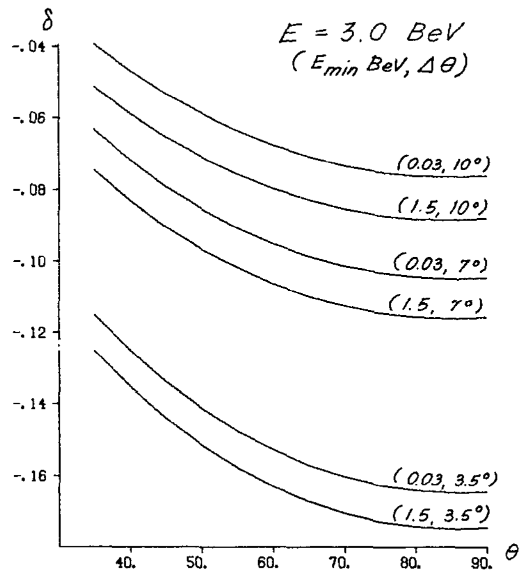

Fig 3 d