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ABSORPTIVE CORRECTIONS AND FORM FACTORS IN THE PERIPHERAL MODEL

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ABSTRACT

Absorptive corrections applied to the peripheral model have provided a relatively successful interpretation of a variety of high energy production processes. There exist, however, a number of difficulties associated with these calculations. We consider the reaction $\pi N \rightarrow \rho N$ which is dominated by π exchange in order to study the following three ambiguities: i) the actual dependence of the absorptive corrections on the initial and final state elastic scattering phase shifts, ii) the role of a form factor, iii) the numerical values of the final state elastic scattering phase shifts. The comparison of our calculation with the experimental data, in particular the density matrix of the ρ, lead to the following results. The $j = \frac{1}{2}$ partial waves must be totally suppressed by the absorptive corrections and the form factor must play a very minor role in order to fit the observed deviation of the ρ's density matrix from that predicted by the exchange of a π in the peripheral model. (A form factor cuts down the low partial waves in a manner which leaves the density matrix unmodified from the simple peripheral model and thus reduces the effect of absorption corrections on the density matrix.) We expect any form factor associated with π exchange to have a weak t dependence since there exists no resonance with the appropriate

quantum numbers (to couple to the π) with energy $\lesssim$ 1.3 BeV. It is plausible that form factors (in addition to the absorptive corrections) while unimportant for π exchange may play a significant role in vector exchange (since there seems to be an abundance of high spin resonances).

I. INTRODUCTION

The analysis of many high energy production processes in terms of a peripheral or single particle exchange model (PM) has proved to be very useful. The failure of the simple PM to yield the striking forward peaking observed experimentally as a function of production angles (or the momentum transfer variable t) as well as the violation by the PM of the unitarity limit for the low partial waves was well known. Motivated by the fact that the exchanged particle is off the mass shell the PM was modified (PMF) by the introduction of empirical form factors to take care of these descrepancies with experiment (and theory).[1] However the form factor

$$F(t) = \left(\frac{m^2 - \Lambda^2}{t - \Lambda^2}\right) \tag{1}$$

where m is the mass of the exchanged particle, introduced phenomenologically into the scattering amplitude required a quite unphysically small Λ to fit the data.[2,3] A second difficulty with the PMF has recently developed due to accurate experimental data now being available on the polarization of the produced particles. The new data[2,4] show significant deviations from the predictions of the PM, and the PMF gives, of course, the same results for the density matrix as the PM. Moreover, in the energy range under consideration, any one particular reaction is a small part of the total inelastic cross section, i.e., there are many open competing channels. These considerations recently have led to a number of calculations[5,9] UPM (and UPMF) which modify the PM (and PMF) to take into account the competing absorption processes (for the initial and final state) in a manner analogous to distorted-wave Born approximation calculations of low-energy nuclear physics. These calculations, in a number of cases, yield good fits both to the production cross section as a function of t and the density matrix for the produced particles.

There are several basic difficulties or uncertainties connected with the UPM (and UPMF). In particular we will be concerned with the following three points: a) numerous approximations are made in getting the UPM into a simple useful form: Several different forms have been suggested. It is not our purpose to discuss the theoretical foundations[5,14] of these schemes, although a critique of some of the derivations is presented in the Appendix. b) Although the form factors used in the PMF were clearly unphysical, there must be some modification of the cross section due to form factors in addition to the absorptive corrections. c) The absorptive corrections require knowledge of elastic scattering of the particles in the final state as well as the initial state. In general the final state scattering parameters are not known.

It is the purpose of this article to study the above three ambiguities from a purely pragmatic approach for the production process

$$\pi^- + p \rightarrow \rho^- + p \qquad (2)$$

at an energy corresponding to an laboratory pion momentum P_L of 4.0 BeV/c. Note that (2) is dominated by π exchange.[15]

We shall consider in detail a) the schemes employed in the absorption corrections, b) the effect of form factors and c) the nature of the ρN elastic scattering. In fitting the experimental data[2,4] for the reaction (2) we wish to see how unique are the effects due to these three factors. The question of uniqueness is not only important from an aesthetic view point but also in the question extending these calculations from just a fit to existing data to that of a predictive nature, e.g., at different energies.

The relevant formalism is given in Section II. The results of the numerical calculations are presented and discussed in Section III. The absorptive corrections we consider have the following structure: Let $f_{\{\lambda\}}$ represent the helicity amplitudes for the PM or PMF. Analyze $f_{\{\lambda\}}$ into partial waves

$$f_{\{\lambda\}} = \frac{1}{2} \sum_j (2j+1) d^j_{\{\lambda\}} \, f^j_{\{\lambda\}} \quad . \qquad (3)$$

Then the UPM or UMPF we use has the form

$$f^u_{\{\lambda\}} = \frac{1}{2} \sum_j (2j+1) d^j_{\{\lambda\}} \, g({}^I\delta^j_{\{\lambda\}}) \, f^j_{\{\lambda\}} \, g({}^F\delta^j_{\{\lambda\}}) \quad . \qquad (4)$$

where ${}^{I,F}\delta$ is the phase shift for elastic scattering in the initial, final state. We make the usual simplifying assumptions i) that δ does not connect different helicity states and ii) that δ is pure imaginary.[16] Then the forms for g which we will investigate become[8,9]

$$g({}^{I,F}\delta^j) = ({}^{I,F}\eta^j)^{\frac{1}{2}} \tag{5}$$

and[6,10]

$$g({}^{I,F}\delta^j) = \left(\frac{1 + {}^{I,F}\eta^j}{2}\right) \tag{6}$$

where $\eta^j \equiv |S^j| \equiv |e^{2i\delta^j}|$. We note that (5) and (6) are equal for $\eta^j \approx 1$, i.e., when the absorption corrections are small.

In order to investigate the effect of a form factor we use for convenience the phenomenological form (1) which lumps together any t dependence of both vertices and the propagator of the exchanged particle.

To obtain the η's needed for the absorptive corrections we use the following parametrization of the elastic scattering cross sections:

$$\frac{d\sigma_{e\ell}^{I,F}}{d\Omega} = |f^{I,F}|^2 = \left|i\frac{{}^{I,F}k\,{}^{I,F}\sigma_{total}}{4\pi}\exp\left[-\frac{1}{8}\,{}^{I,F}R^2\,{}^{I,F}k^2(1-\cos\theta)\right]\right| \tag{7}$$

Then associating η^j with $\eta^{\ell=j-\frac{1}{2}}$ where

$$\eta^{\ell} = \frac{k}{2}\int_{-1}^{1} d(\cos\theta)\, P_{\ell}(\cos\theta) f \tag{8}$$

we have, e.g.,

$$\eta^{j=\frac{1}{2}} = 1 - \frac{\sigma_{total}}{\pi R^2}\left(1 - e^{-4k^2/R^2}\right) \tag{9}$$

Now from πp elastic scattering we have[17] ${}^{I}\sigma_{total} \approx 2.5\ f^2$ and $R \approx 1.07$ f. This yields, for the production process (2) ${}^{I}\eta^{j=\frac{1}{2}} \approx .28$.

We, of course, have no knowledge of ρN elastic scattering. We assume that ${}^{F}R = {}^{I}R$ and take ${}^{F}\sigma_{total}$ as an adjustable parameter ($\leq \pi R^2$).

Thus for both absorption forms (5) and (6) we adjust the form factor parameter Λ^2 (see Eq. (1)) and the ρN total cross section ${}^{F}\sigma_{total}$ to fit the experimental data on process (2). The results of our calculations of process (2) may be summarized as follows:

The UPMF calculations using form (6) for the absorption correction factor g do not fit the data for reaction (2). Whereas we are able to fit the angular distribution for the production cross section with physically reasonable values for Λ^2, we cannot fit the density matrix using (6). Good fits to all the data are obtained using (5) with the ρN total cross section $\approx \pi R^2$ (see (9)) so that ${}^{F}\eta^{j=\frac{1}{2}} \approx 0$ and the form factor parameter $\Lambda^2 \gtrsim 75\ m_\pi^2$.[18,19] This means that the $j = \frac{1}{2}$ partial waves must be suppressed by the absorption corrections by a large factor and that only a small correction can come from a form factor. Whereas a form factor cuts down the low partial waves it does so in a manner which leaves the density matrix unmodified from the simple peripheral model. Thus a form factor reduces the effect of the absorption corrections on the density matrix.

The large value of Λ^2 is gratifying since we expect it to be determined by some $I = 0$, $j = 0^-$ 3 pion resonance which can couple to the pion. However we know that no resonance exists with these quantum numbers and mass less than 1.3 BeV.[19] On the other hand the very large suppression of the $j = \frac{1}{2}$ partial waves required is somewhat disturbing since the theoretical form for g seems to be unambiguous only for relatively small correction factors. Since the ρN cross section is an adjustable parameter in our calculation and there exists no estimate of it from other sources, we cannot separate the effect of the form (5) from the parameter ${}^{F}\sigma_{total}$. Using the parameters which fit the data at $P_L = 4$ BeV/c, calculations are performed at higher energies. Good experimental data at these higher energies should provide a test of the present model.

We note that practically all the absorptive corrections to the scattering amplitude at $P_L = 4$ BeV/c came from the low partial waves (see Fig.2 and the accompanying discussion in Section III). Thus we feel that it is worthwhile to make the corrections on each partial wave as was done in this paper rather than make the further approximation[8,9] of large j and small scattering angles and use the asymptotic expression for the d functions which simplifies the calculations. At very high energies ($P_L \gtrsim 15$ BeV/c) it is probably a good approximation to use this simplifying assumption.

Finally we make an observation concerning other production processes. Our calculations were confined to process (2) in which π exchange dominates. We have shown that any form factor must play a small role. On the other hand, UPM calculations of processes where vector particle exchanges dominate have not had the same degree of success[20] as UPM calculations where $j = 0$ particles are exchanged. We suggest that the form factors may play a much more important role for vector exchange. Experimentally there seems to be an abundance of high spin resonances at moderate energies; these might provide some structure in the vector interaction.[21] We are presently performing a number of UPFM calculations for vector particle exchanges.

II. PROCEDURE OF INCLUDING ABSORPTIVE CORRECTIONS

The first step in the inclusion of absorptive effects is to perform a complete helicity and partial wave decomposition[22] of the unmodified amplitude. Let the peripheral amplitude for process (2) be

$$\langle\lambda_1|T|\lambda_2,\mu\rangle \tag{10}$$

where, as shown in Fig. 1, λ_1,λ_2 are the helicities of the initial and final protons and μ the helicity of the ρ meson. Eq. (10) may represent the PM or PMF. The partial wave projection of (10) is

$$\langle\lambda_1|T|\lambda_2,\mu\rangle = \frac{1}{2} \sum_j (2j+1) \quad \langle\lambda_1|T^j|\lambda_2,\mu\rangle d^j_{\mu-\lambda_2,-\lambda_1}(\theta) \tag{11}$$

The absorptive modifications consist of replacing the partial wave amplitude $\langle\lambda_1|T^j|\lambda_2,\mu\rangle$ by a "unitarized" one:

$$\langle\lambda_1|T^j_U|\lambda_2,\mu\rangle = \sum_{\lambda_1',\lambda_2',\mu'} g({}^{\pi N}\delta^j_{\lambda_1\lambda_1'}) \ \langle\lambda_1'|T^j|\lambda_2',\mu'\rangle \qquad g({}^{\rho N}\delta^j_{\lambda_2\lambda_2'\mu\mu'}) \tag{12}$$

where the absorptive correction factors g are functions of the initial and final state elastic scattering phase shifts. The forms (5) and (6) in Section I in terms of S matrix elements generalize to

$$g({}^{\pi N}\delta^{j}_{\lambda_1\lambda_1'}) = ({}^{\pi N}S^{j}_{\lambda_1\lambda_1'})^{\frac{1}{2}} \quad , \tag{13}$$

and

$$g({}^{\pi N}\delta^{j}_{\lambda_1\lambda_1'}) = ({}^{\pi N}S^{j}_{\lambda_1\lambda_1'} + \delta_{\lambda_1\lambda_1'})/2 \tag{14}$$

respectively. In all our discussions we shall assume that the πN and ρN phase shifts are diagonal in and independent of helicity so that (12) becomes

$$\langle\lambda_1|T^j_U|\lambda_2,\mu\rangle = g({}^{\pi N}\delta^j)\; g({}^{\rho N}\delta^j) < \lambda_1|T^j|\lambda_2,\mu\rangle \quad . \tag{15}$$

In addition we take the δ's to be pure imaginary so that the g's we study are given by (5) and (6).

The partial decomposition evaluated in the center of mass system for the process in Fig. 1 yields in the absence of a form factor:[23]

$$\langle \tfrac{1}{2}|T^j|\tfrac{1}{2},1\rangle = -\frac{a}{\sqrt{2}}\ \xi_-\ (\tfrac{\beta+1}{2})(\tfrac{\beta-1}{2})^{\frac{1}{2}}\ e^j_{\frac{1}{2},-\frac{1}{2}}(\beta) - \frac{a}{4}\ \xi_-\ \delta_{j,\frac{1}{2}}\ ,$$

$$\langle \tfrac{1}{2}|T^j|-\tfrac{1}{2},1\rangle = -\frac{a}{\sqrt{2}}\ \xi_+\ (\tfrac{\beta-1}{2})(\tfrac{\beta+1}{2})^{\frac{1}{2}}\ e^j_{\frac{3}{2},-\frac{1}{2}}(\beta)\ ,$$

$$\langle -\tfrac{1}{2}|T^j|-\tfrac{1}{2},0\rangle = -\frac{a}{2}\ \xi_-\ h(\tfrac{\beta+1}{2})^{\frac{1}{2}}\ e^j_{\frac{1}{2},\frac{1}{2}}(\beta) - \frac{a}{4}\frac{E_\rho}{m_\rho}\ \xi_-\ \delta_{j,\frac{1}{2}}\ ,$$

$$\langle -\tfrac{1}{2}|T^j|\tfrac{1}{2},1\rangle = \frac{a}{\sqrt{2}}\ \xi_+\ (\tfrac{\beta-1}{2})(\tfrac{\beta+1}{2})^{\frac{1}{2}}\ e^j_{\frac{1}{2},\frac{1}{2}}(\beta) - \frac{a}{4\sqrt{2}}\ \xi_+\ \delta_{j,\frac{1}{2}}\ , \qquad (16)$$

$$\langle -\tfrac{1}{2}|T^j|-\tfrac{1}{2},1\rangle = \frac{a}{\sqrt{2}}\ \xi_-\ (\tfrac{\beta+1}{2})(\tfrac{\beta-1}{2})^{\frac{1}{2}}\ e^j_{\frac{3}{2},\frac{1}{2}}(\beta)\ ,$$

$$\langle \tfrac{1}{2}|T^j|-\tfrac{1}{2},0\rangle = \frac{a}{2}\ \xi_+\ h(\tfrac{\beta-1}{2})^{\frac{1}{2}}\ e^j_{\frac{1}{2},-\frac{1}{2}}(\beta) - \frac{a}{4}\ \xi_+\ \frac{E_\rho}{m_\rho}\ \delta_{j,\frac{1}{2}}$$

where

$$\beta = (m_\pi^2 + q^2 + q'^2 + (m_\rho^2 - m_\pi^2)/4s)/(2qq')\ ,$$

$$\xi_\pm = q\left(\frac{E_2+M}{E_1+M}\right)^{\frac{1}{2}} \pm q'\left(\frac{E_1+M}{E_2+M}\right)^{\frac{1}{2}}\ ,$$

$$h = (q'E_\pi - qE_\rho\beta)/(qm_\rho)\ , \qquad (17)$$

$$a = \frac{ig_{\rho\pi\pi}g_{N\bar{N}\pi}}{4\pi} \approx i5.5\ .$$

Also terms of the incident pion laboratory momentum

$$s = m_\pi^2 + M^2 + 2\ M\sqrt{P_L^2 + m_\pi^2}$$

and

$$t = -q^2 - q'^2 + 2\ qq'\cos\theta + (E_\pi - E_p)^2$$

where θ is the production angle of the ρ .

The $e^j_{\mu,\nu}(\beta)$ are rotation matrices of the second kind related to the $d^j_{\mu,\nu}(z)$ by

$$\left(\frac{\beta-1}{2}\right)^{\frac{\mu-\nu}{2}} \left(\frac{\beta+1}{2}\right)^{\frac{\mu+\nu}{2}} e^j_{\mu,\nu}(\beta) = \frac{1}{2}\int_{-1}^{1} \frac{d^j_{\mu,\nu}(z)dz}{\beta-z} \left(\frac{1-z}{2}\right)^{\frac{\mu-\nu}{2}} \left(\frac{1+z}{2}\right)^{\frac{\mu+\nu}{2}} . \qquad (18)$$

The other six amplitudes may be obtained from (16) by the symmetry

$$-\langle -\lambda_1 | T^j | -\lambda_2, -\mu\rangle = \langle \lambda_1 | T^j | \lambda_2, \mu\rangle \ . \qquad (19)$$

Now the form factor (2) times the propagator for Fig. (1) can be written as $\left(\frac{1}{t-m_\pi^2} - \frac{1}{t-\Lambda^2}\right)$. Thus the properties for the amplitude modified by the form factor (2) are obtained simply by subtracting from each term in (16) an analogous term with β replaced by $\beta + (\Lambda^2 - m_\pi^2)/(2\ qq')$. We note that all the "exceptional" terms proportional to $\delta_{j,\frac{1}{2}}$ in (16) drop out. Then the unitarized partial wave amplitude with a form factor (1) becomes

$$^U T^j_{\{\lambda\}}(\Lambda^2) = g(^{\pi N}\delta^j)\ g(^{\rho N}\delta^j) \left[T^j_{\{\lambda\}}(\beta) - T^j_{\{\lambda\}}\left(\beta + \frac{\Lambda^2 - m_\pi^2}{2\ qq'}\right)\right] \qquad (20)$$

with $T^j_{\{\lambda\}}(\beta)$ given by (16). The unitarized amplitude $^U T_{\{\lambda\}}$ is obtained by summing the partial wave series. In terms of this amplitude the cross section is given by[24]

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\ qq'} \sum_{\lambda_1 \lambda_2, \mu} |\langle \lambda_1 | U_T | \lambda_2, \mu \rangle|^2 \tag{21}$$

whereas the density matrix of the ρ meson is given by

$$\rho_{\mu\mu'}(\theta) = (2\ qq' d\sigma/d\Omega)^{-1} \sum_{\lambda_1 \lambda_2, \alpha\beta} d^1_{\mu\alpha}(\psi)\ \langle \lambda_1 | U_T | \lambda_2 \alpha \rangle$$

$$\langle \lambda_1 | U_T | \lambda_2 \beta \rangle^* \ d^1_{\beta\mu'}(-\psi) \tag{22}$$

with

$$\sin\psi = \frac{4 m_\rho\ q \sin\theta}{[t - (m_\rho + M)]^2 [t - (M - m_\rho)^2]} \quad . \tag{23}$$

III. CALCULATIONS AND CONCLUSIONS

The production cross section for process (2) and the density matrix of the ρ are calculated from the UPMF amplitude ${}^{U}T_{\{\lambda\}}$ using (21) - (24). If $T_{\{\lambda\}}$ is the PM amplitude and $T^{j}_{\{\lambda\}}$ the partial wave projection given by (16) then for the form factor (1) we have

$$ {}^{U}T_{\{\lambda\}} = \frac{m_\pi^2-\Lambda^2}{t-\Lambda^2} T_{\{\lambda\}} + \frac{1}{2} \sum_{j-\frac{1}{2}=0}^{\ell_{max}} (2j+1)\, d^{j}_{\{\lambda\}} [{}^{U}T^{j}_{\{\lambda\}} - {}^{U}T^{j}_{\{\lambda\}}] \qquad (25) $$

where ${}^{U}T^{j}_{\{\lambda\}}$ is given by (20). We found that at the energies we are interested in, only a moderate number of partial waves need be corrected for the absorption effects, e.g., at $P_L \approx 4$ BeV/c, $\ell_{max} \sim 10$ is sufficient. (See discussion below concerning Fig. 2.) The d functions and e functions, Eq. (18), were obtained from the low j values by using recursion relations.[22]

We investigate the form (5) and (6) for the absorptive corrections. The effect of a form factor was determined by considering Λ^2 as a free parameter. The elastic absorptive phase shifts ${}^{\pi N}\eta_j$, ${}^{\rho N}\eta_j$ are obtained using (7) and (8) with ${}^{\pi N}\sigma_{total} = 2.5\ f^2$ and ${}^{\pi N}R \approx 1.07$ f.[17] We then assume ${}^{\rho N}R = {}^{\pi N}R$ and vary ${}^{\rho N}\sigma_{total}$ ($\leq \pi R^2$). Thus our second free parameter is ${}^{\rho N}\sigma_{total}$ which we equivalently state by giving ${}^{\rho N}\eta_{j=\frac{1}{2}}$.

Fig. 2 shows the results of a calculation using form (5) with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and no form factor, i.e., $\Lambda^2 = \infty$ in which we vary ℓ_{max} (see Eq. (25)). We thus see the effect of successive absorption of more partial waves at $P_L = 4$ BeV/c. The main effect comes from the first 2 partial waves. We also observe that it is sufficient to make the absorptive corrections for the lowest 10 partial waves. Hence we feel that it is worthwhile at these energies to deal directly with the partial wave projections instead of making the large j approximation in order to use the simplified expressions[8,9] involving the asymptotic expressions for the d's.

We know that there is not a unique prescription to fit the ρ production cross section $\frac{d\sigma}{d\Omega}$ alone. The PMF (using the form factor (1)) can fit this data with $\Lambda^2 \sim 6\ m_\pi^2$.[2] However this value of Λ^2 is clearly unphysical.[19] On the other hand a calculation including absorption corrections of type (5) with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and no form factor $(\Lambda^2 = \infty)$ gives a good fit to the data on $\frac{d\sigma}{d\Omega}$. This is shown in Fig. 3. We observe from Fig. 4 that changing Λ^2 to $50\ m_\pi^2$ does not qualitatively change the fit to the data obtained in Fig. 3. Fig. 5 shows a comparison between the UPMF calculations using (5) with $\Lambda^2 = 50\ m_\pi^2$ and ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ with one using (6) with $\Lambda^2 = 30\ m_\pi^2$ and ${}^{\rho N}\eta_{\frac{1}{2}} = 0$. Since the latter value of Λ^2 is not unreasonably small, the form (6) for the absorptive corrections is quite acceptable.

Now we turn to the data on the density matrix $\rho_{\mu,\mu'}$ for the ρ meson. Experimentally we have[25]

$$\rho_{0,0} = .53\,^{+.12}_{-.11}\ , \qquad \rho_{1,-1} = .16\,^{+.09}_{-.10}\ , \tag{25}$$

$$\mathrm{Re}\ \rho_{1,0} = -.06 \pm .05 \quad ,$$

where these represent an average over the production angle $\cos\theta > .9$ $(-t \leq 15\ m_\pi^2)$. Now both the PM and PMF give $\rho_{0,0} = 1$ and all others zero, independent of $\cos\theta$. Thus the data (25) rule both of these models out. The results of our calculations of the UPMF show that form (6) cannot fit (25): We find e.g., that for ${}^{\rho N}\eta_{\frac{1}{2}}=0$ and $\Lambda^2 = 30$ (the same parameters as in Fig. 5), $\rho_{0,0}$ stays above .95 and the others are ≈ 0. The data (25) can be fit using form (5) but only with ${}^{\rho N}\eta_{\frac{1}{2}} \approx 0$. The $j = \frac{1}{2}$ partial waves must be almost completely suppressed by the absorptive corrections and only a small correction can come from a form factor. A form factor cuts down the low partial waves in a manner which leaves the density matrix unmodified from the PM or PMF. Thus the effect of a form factor in addition to the absorption corrections is to bring the $\rho_{\mu,\mu'}$ back to the PM predictions. This is demonstrated in Fig. 6. Here we compare UPFM calculations using form (5) with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = 50m_\pi^2$ to those with $\Lambda^2 = \infty$. (If in the former case ${}^{\rho N}\eta_{\frac{1}{2}}$ had been chosen > 0 to compensate the effect of the form factor in a fit to $\frac{d\sigma}{d\Omega}$, the descrepancy between the two calculations in Fig. 6 would of course be larger.)

Thus we find that only the UPMF calculations using (5) with ${}^{\rho N}\eta_{\frac{1}{2}} \approx 0$ and Λ^2 large $(\gtrsim 75\ m_\pi^2)$ can fit the data on process (2) at $P_L = 4$ BeV/c. Since the practical prescription for the UPMF calculations seems reasonably unique, it would be of interest to perform experiments at high energy to test these conclusions. We give in Fig. 7 and 8 the theoretical prediction for ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = \infty$ at $P_L = 10$ BeV/c.

We note that our calculations have been based on the usual assumption that the elastic scattering did not involve spin flip. The presence of appreciable spin flip elastic amplitudes would invalidate our conclusions.

Finally we speculate that although form factors play a small role in the exchange of a π (or probably any pseudoscalar particle), they have a significant effect in the exchange of vector particles: Experimentally there seems to be an abundance of high spin resonances at moderate energies.[19]

APPENDIX

Absorptive Corrections from a K Matrix Model in a Random Phase Approximation

In order to get the absorptive corrections into a simple useful form, a number of different approaches have been considered.[5-14] Different approaches and approximations lead to different forms, i.e., from a theoretical point of view we have no unique simple prescription for the absorption corrections. Just to illustrate this dilemma we consider the following simple model to include the effects of unitarity due a number of competing channels. We are interested in the high energy region in which there are a large number of open channels and the elastic scattering is mainly absorptive.[16]

Consider n (open) coupled 2 body channels in a given partial wave. The $n \times n$ scattering matrix T, defined in terms of the S matrix by

$$T \equiv (S-I)/2i \tag{A1}$$

where I is the $n \times n$ identity matrix, can also be written in terms of a K matrix as

$$T = K(I-iK)^{-1} = (K+iK^2)(I+K^2)^{-1} \quad . \tag{A2}$$

where K is real and symmetric. Note that any approximation on K still leads to a unitary S matrix. Now we make the random phase approximation:

$$(K^2)_{ij} = \sum_{\ell} K_{i\ell}K_{\ell j} = 0 \quad \text{for} \quad i \neq j \tag{A3}$$

Now using (A3) we have

$$(K^3)_{ij} = (K^2)_{ii}\, K_{ij} = K_{ij}(K^2)_{jj} \quad .$$

Thus K^2 = positive constant times I:

$$K^2 = d^2 I \quad . \tag{A4}$$

Hence we obtain for (A2)

$$T_{ij} = (K_{ij} + id^2\delta_{ij})/(1+d^2) \tag{A5}$$

Taking the elastic scattering amplitude to be pure imaginary in our high energy region, (i.e., neglecting K_{ii} with respect to d^2)

$$T_{ii} = id^2/(1+ d^2) = \frac{\eta - 1}{2i} \quad . \tag{A6}$$

Our first result is that all the elastic scattering amplitudes are equal. Now using (A6) we obtain

$$T_{ij} = K_{ij}\left(\frac{1+\eta}{2}\right) \tag{A7}$$

In addition if we take K_{ij} to be given by the peripheral diagram, (A7) gives a simple form for the absorptive corrections.[26]

FOOTNOTES

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16. Recently a sizeable real part of the $\pi\rho$ elastic amplitude has been found in the forward direction. (S. Lindenbaum, High Energy Physics Conference, Dubna, USSR (1964), preprint) In principle our calculations should include this fact. However it may be that the real part gets its contribution from the higher angular momentum waves and that the low partial waves are primarily absorptive. To test this hypothesis, one must observe the real part of the elastic scattering amplitude at non-forward angles. As corrections to low partial waves are the most important ones, the assumption of a purely absorptive elastic amplitude may be sufficient. Possibly more important may be the contributions to the absorptive corrections from spin flip amplitudes, especially in the ρp state. At present there is no way of estimating these effects. Spin flip elastic amplitudes if sufficiently large would have a sizeable effect on the density matrix.

17. S. Lindenbaum, p. 105, Nucleon Structure, Edited by R. Hofstadter and L. Schiff, Stanford Press (1964).

18. We use units $\hbar = c = 1$.

19. It is ambiguous how one relates the cutoff mass Λ in the phenomenological form factor (1) to a physical mass. However the form of Eq. (20) suggests that the form factor takes into account the exchange of a higher mass state Λ with the same quantum numbers as that of the particle originally exchanged. Thus we expect the mass Λ to be of the order of magnitude of the masses of states which can couple to the original particle. For pseudoscalar particles, we do not know of any such state below the BeV region.

20. A form factor will not change the energy dependence of the cross section. However it will change the density matrices (given by the absorptive correction calculations) of the produced particles back toward the values of the PM.

21. Also see the last two papers of Ref. 9.

22. M. Jacob and G. Wick, Annals of Physics **7**, 404 (1959).

23. See Ref. 8 and 9 for full PM helicity amplitudes.

24. K. Gottfried and J. Jackson, Nuovo Cimento **33**, 309 (1964).

25. I. Derado, V. Kenney and N. Shephard, Phys. Rev. Letters **13**, 505 (1964).

26. We do not use the form (A6) in our calculations since it gives smaller corrections than form (6) which we found did not fit the data for process (2).

FIGURE CAPTIONS

1. One pion exchange diagram for $\pi^- + p \rightarrow \rho^- + p$ with kinematics and helicities indicated.

2. Plots of the cross section for process (2) versus the production angle showing effect of successive absorption of more partial waves. We use Eq. (5) for the corrections with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and no form factor, i.e., $\Lambda^2 = \infty$.

3. Fit of calculation using (5) with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = \infty$ to the data of Ref. 2. The first bin is dotted to note that the experimental value is uncertain and is an estimate of the correct value by Z. Guiragossian.

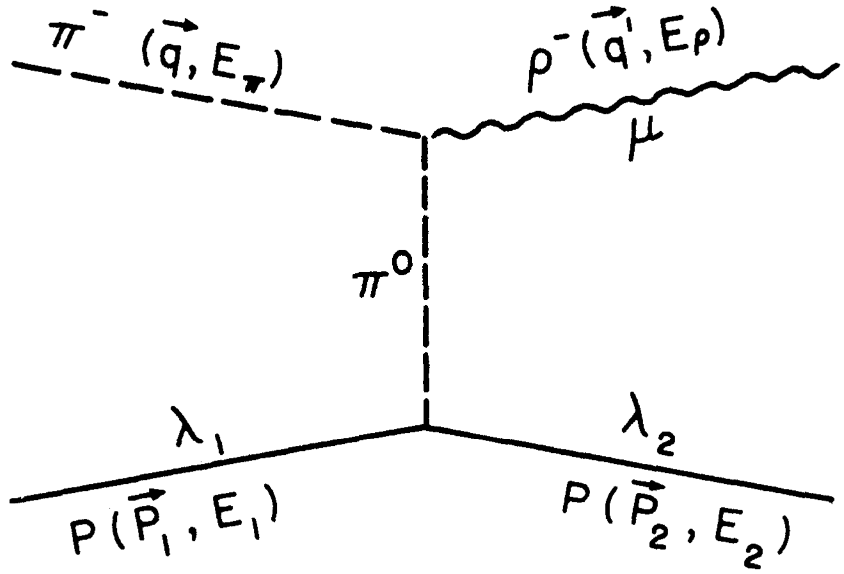
4. Effect of varying the form factor on the cross section. Other parameters are the same as in Fig. 3.

5. Upper curve uses Eq. (5) for the absorptive corrections with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = 50\ m_\pi^2$ whereas the lower one uses (6) with ${}^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = 30\ m_\pi^2$.

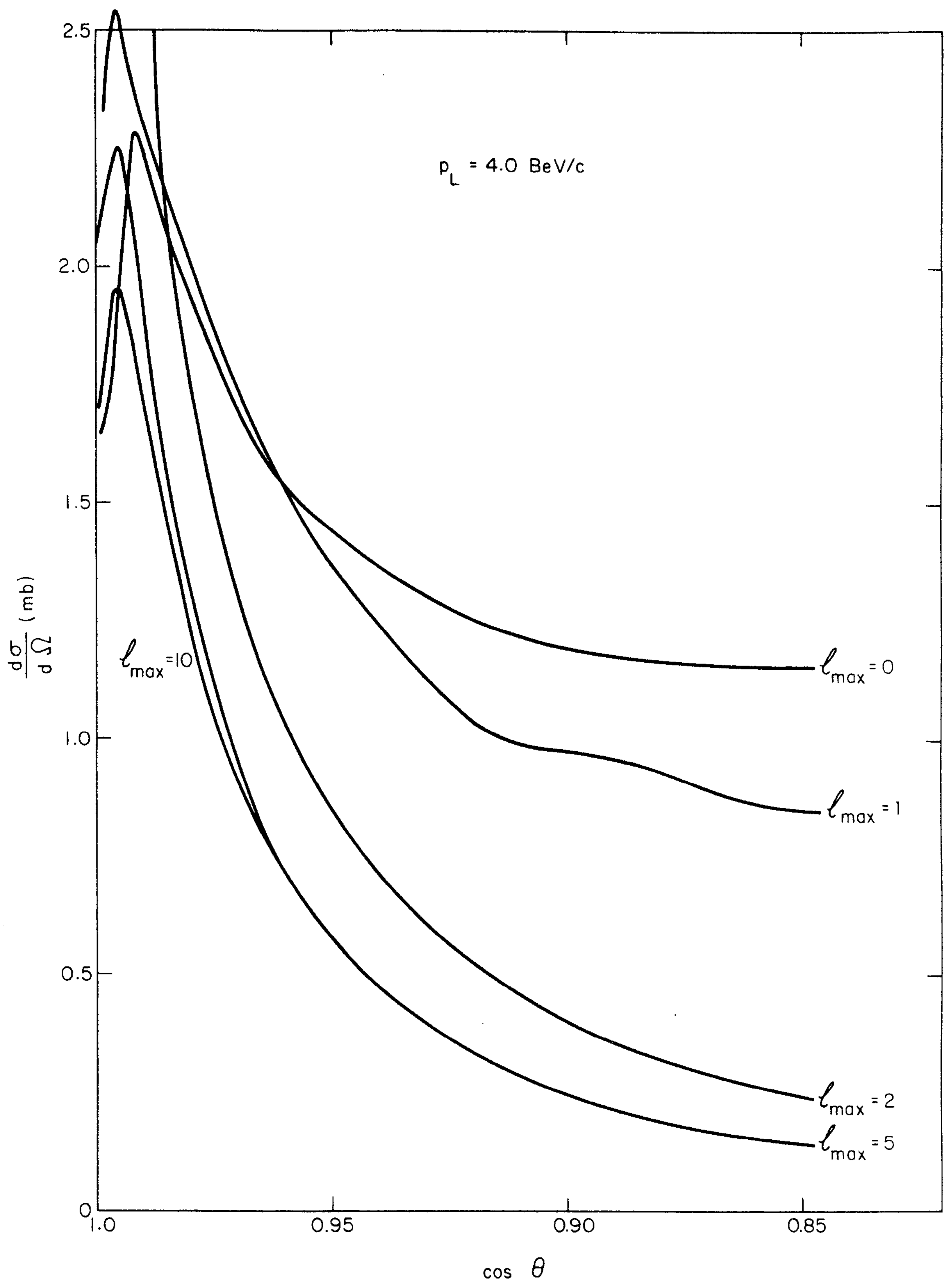
6. Effect of including a form factor on the density matrix elements $\rho_{\mu,\mu'}$ of the produced ρ. These should be compared with the experimental values (25). Other parameters in the calculation are the same as in Fig. 3.

7. The cross section at $P_L = 10$ BeV/c using corrections according to Eq. (5) with $^{\rho N}\eta_{\frac{1}{2}} = 0$ and $\Lambda^2 = \infty$.

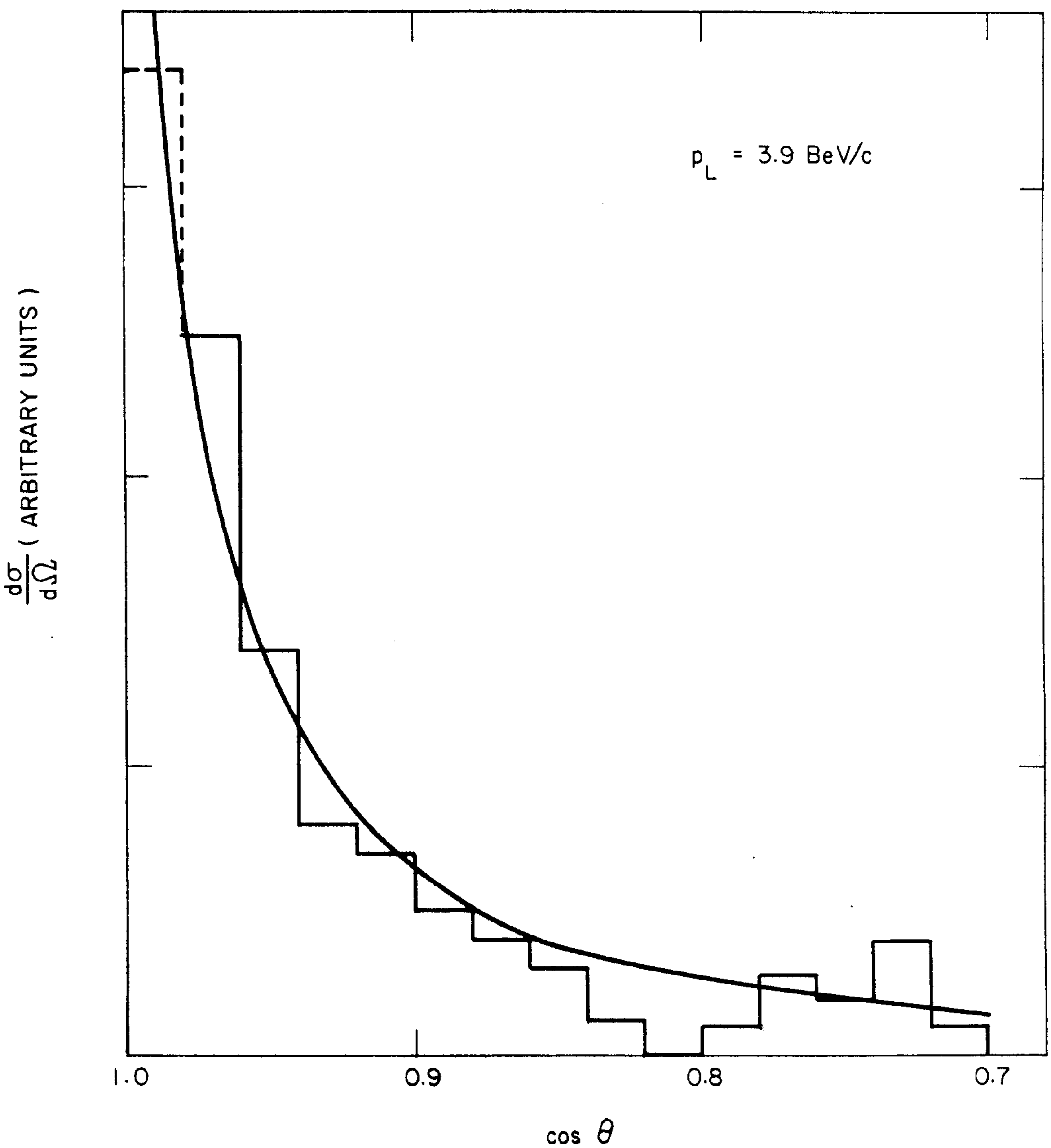
8. Density matrix at $P_L = 10$ BeV/c using the parameters of Fig. 7.



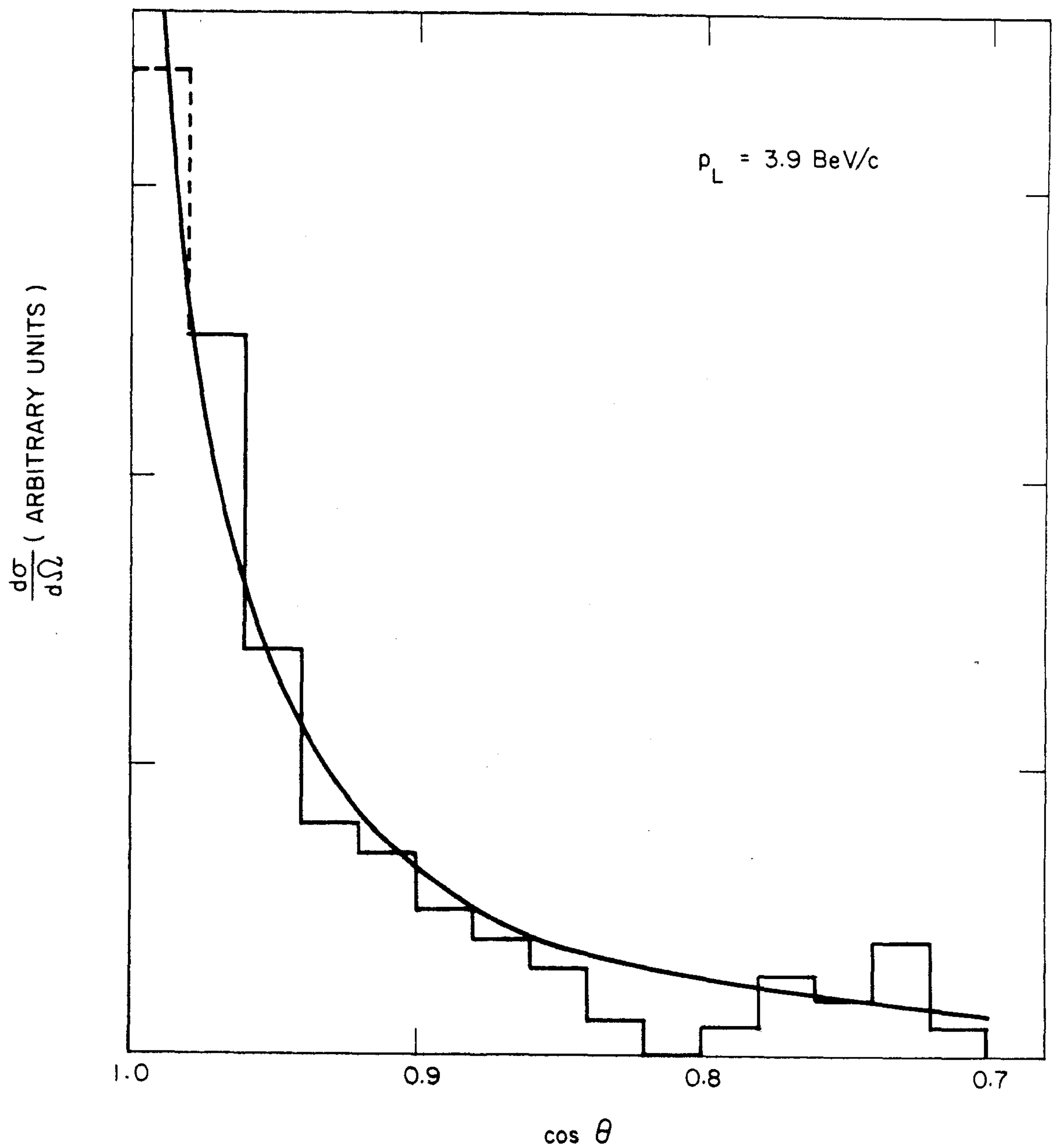
π^- $(\vec{q}, E_\pi)$
$\rho^-(\vec{q}', E_\rho)$
μ
π^0
λ_1
λ_2
$P(\vec{P}_1, E_1)$
$P(\vec{P}_2, E_2)$



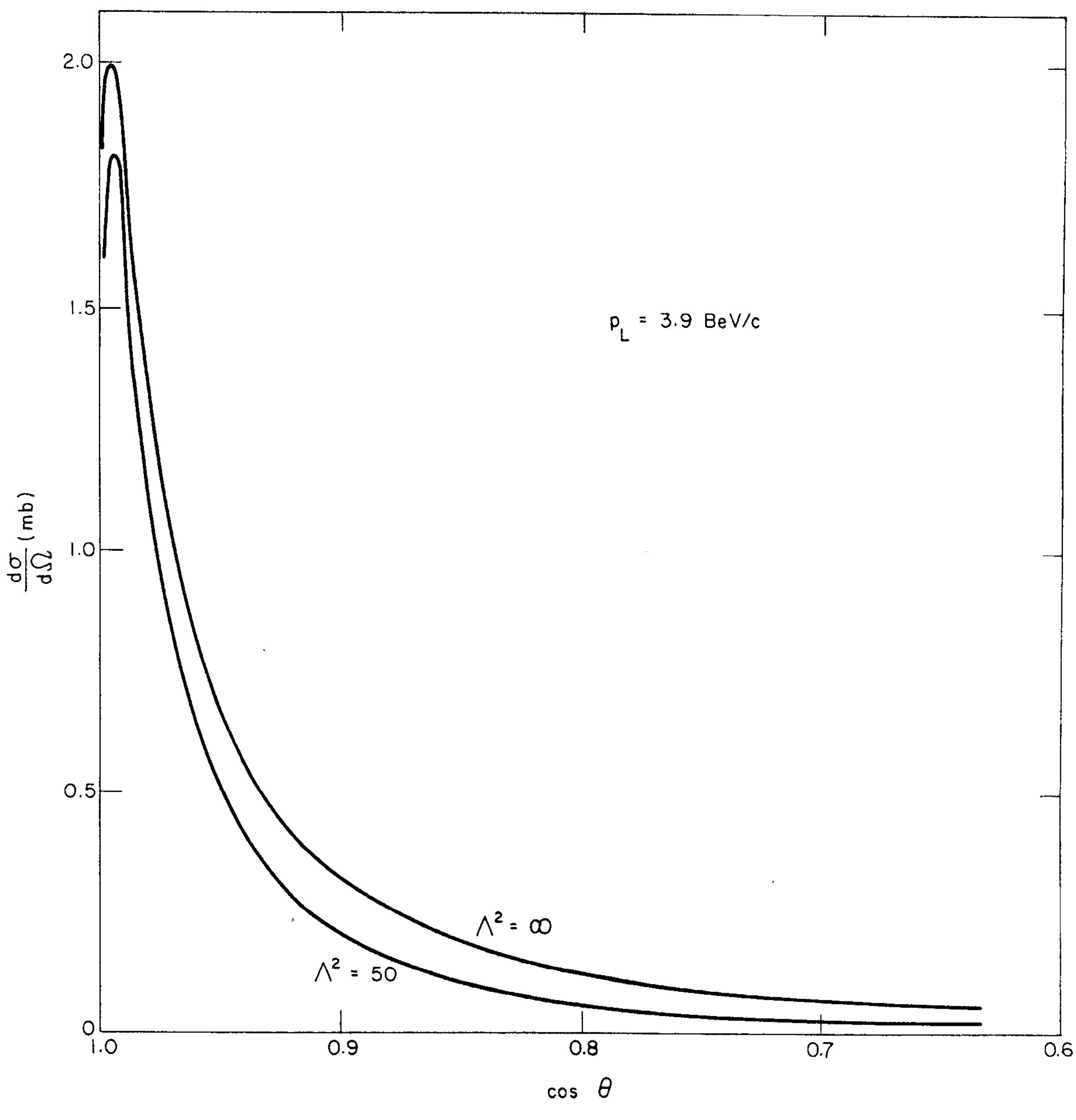
2.5
2.0
1.5
1.0
0.5
0
p_L = 4.0 BeV/c
$\frac{d\sigma}{d\Omega}$ (mb)
ℓ_{max}=10
ℓ_{max} = 0
ℓ_{max} = 1
ℓ_{max} = 2
ℓ_{max} = 5
1.0
0.95
0.90
0.85
cos θ



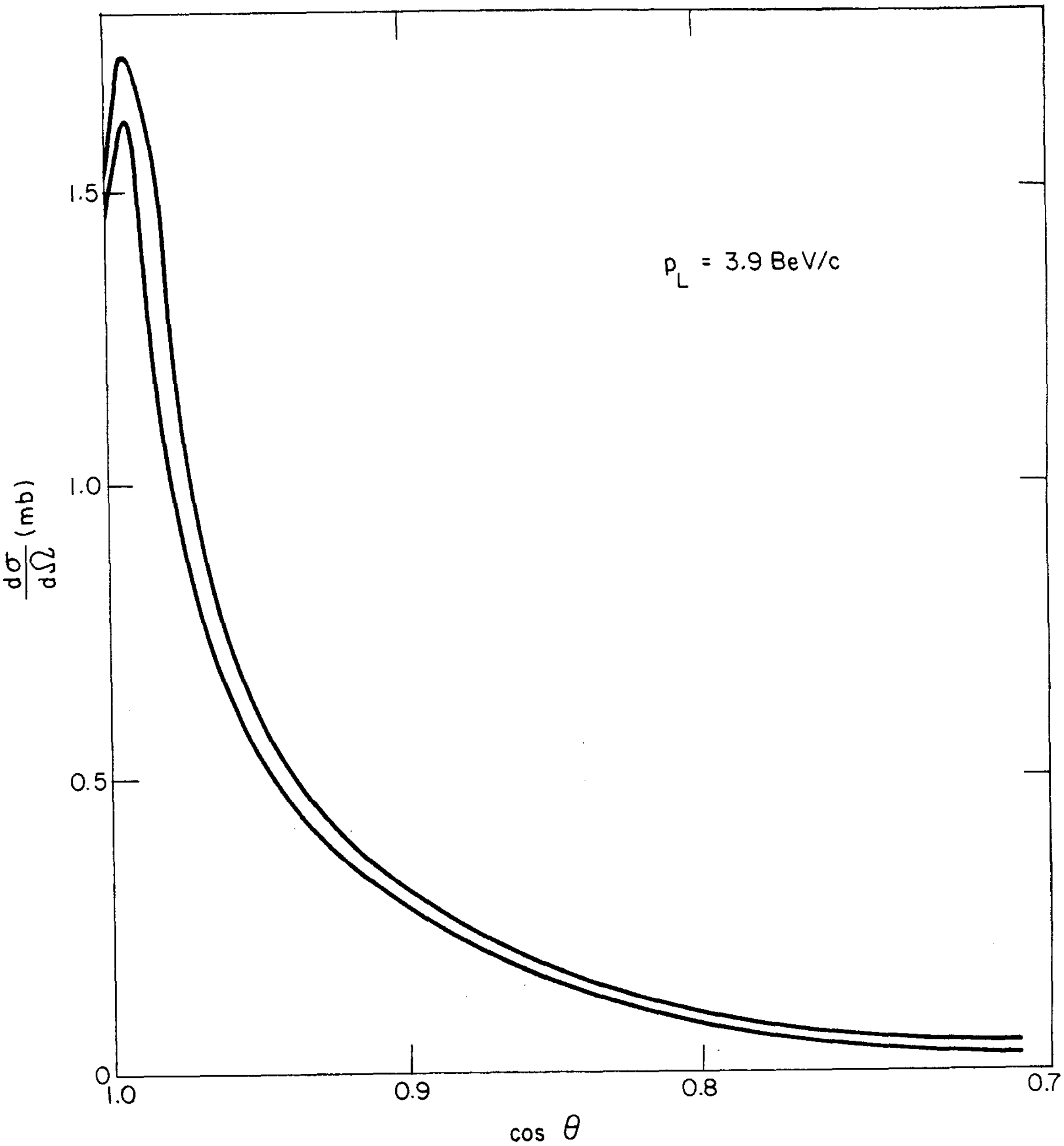
p_L = 3.9 BeV/c
$\frac{d\sigma}{d\Omega}$ (ARBITRARY UNITS)
1.0
0.9
0.8
0.7
cos θ



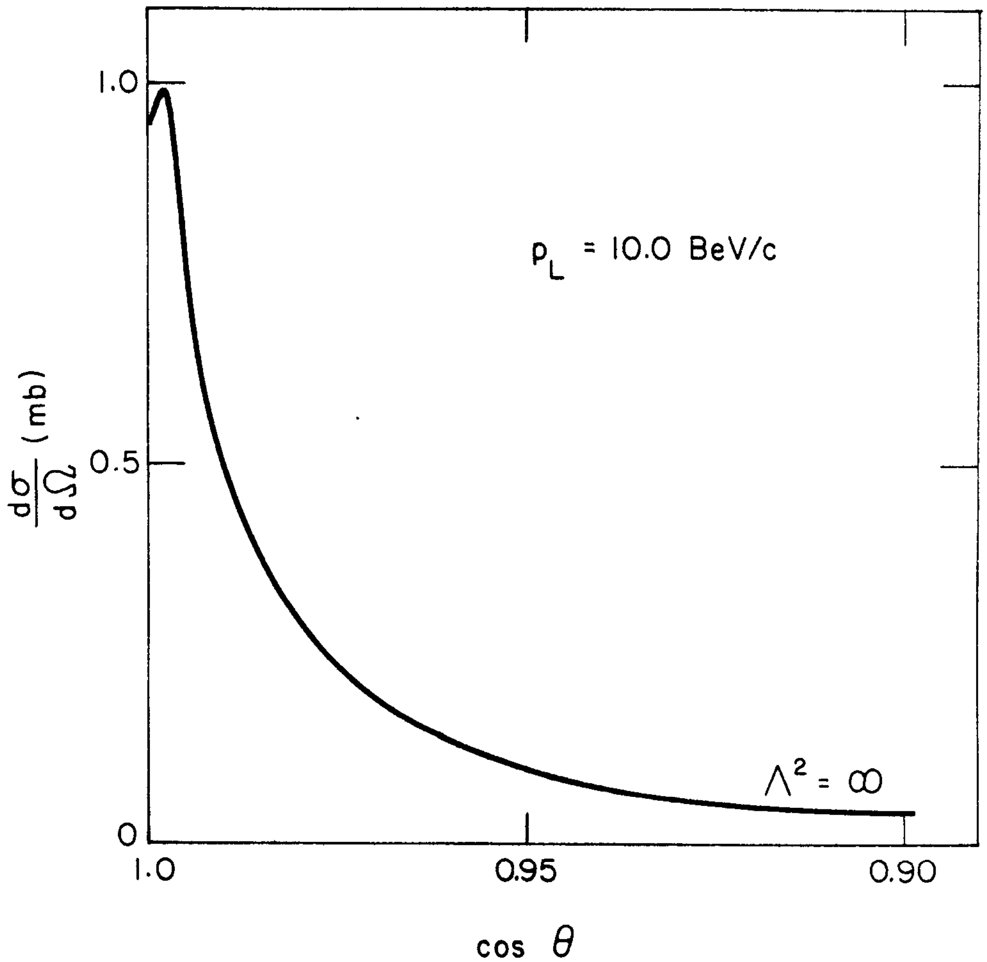
p_L = 3.9 BeV/c
$\frac{d\sigma}{d\Omega}$ (ARBITRARY UNITS)
1.0
0.9
0.8
0.7
cos θ

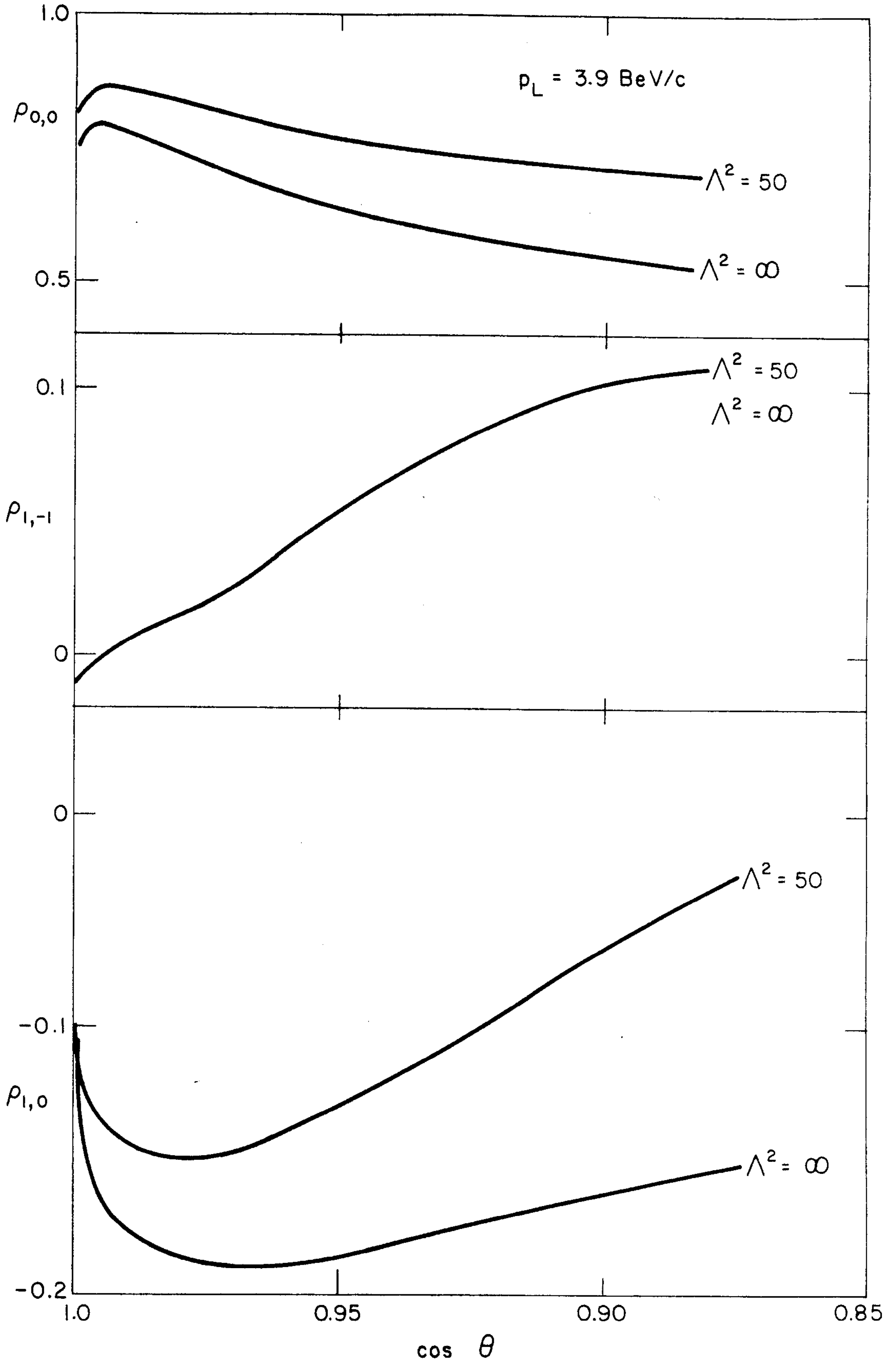


$\frac{d\sigma}{d\Omega}$ (mb)
2.0
1.5
1.0
0.5
0
1.0
0.9
0.8
0.7
0.6
cos θ
p_L = 3.9 BeV/c
$\Lambda^2 = \infty$
Λ^2 = 50

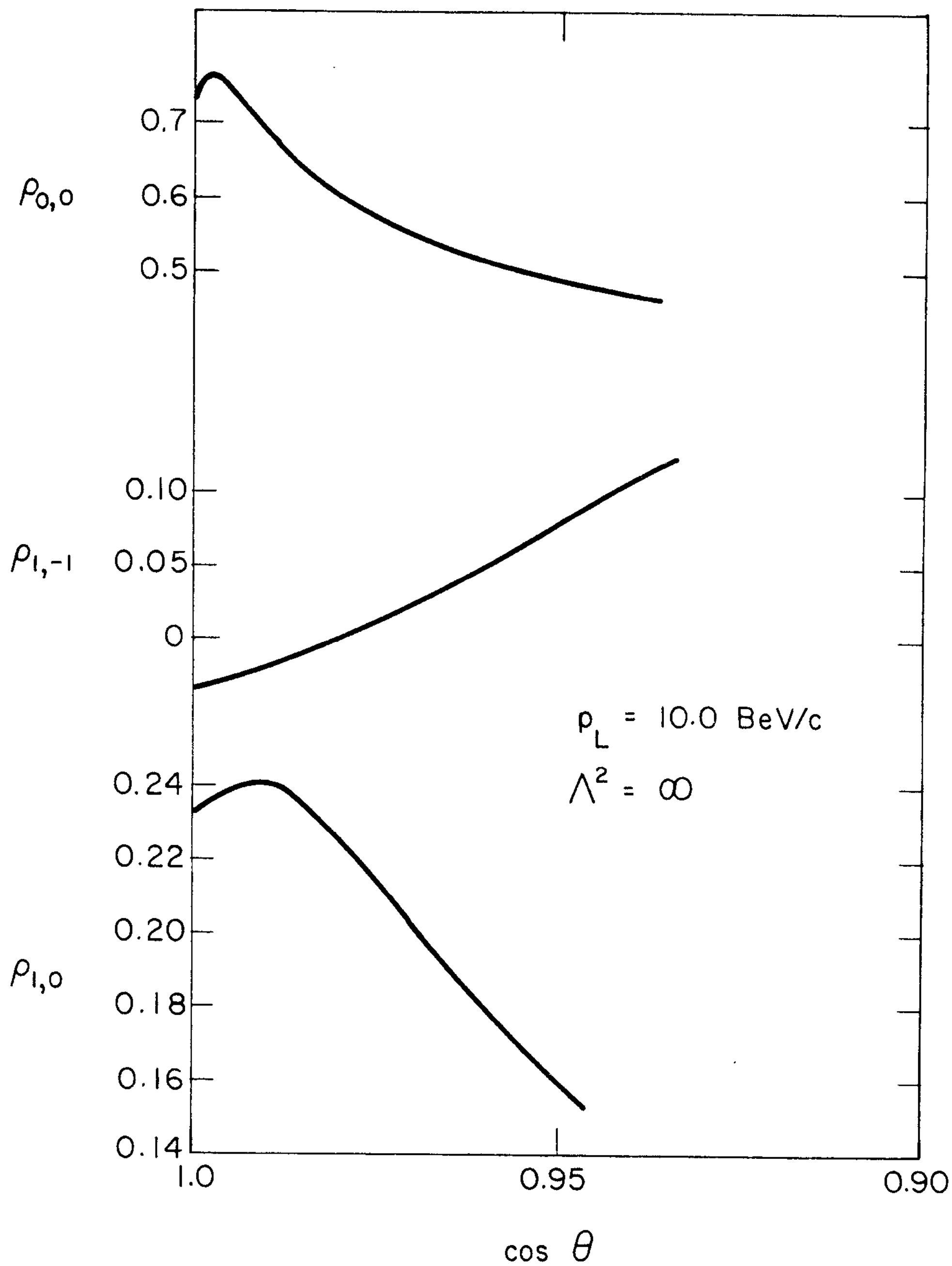


1.5
1.0
0.5
0
1.0
0.9
0.8
0.7
p_L = 3.9 BeV/c
$\frac{d\sigma}{d\Omega}$ (mb)
cos θ


1.0
0.5
0
$\frac{d\sigma}{d\Omega}$ (mb)
p_L = 10.0 BeV/c
$\Lambda^2 = \infty$
1.0
0.95
0.90
cos θ



p_L = 3.9 BeV/c
$\rho_{0,0}$
1.0
0.5
Λ^2 = 50
Λ^2 = ∞
$\rho_{1,-1}$
0.1
0
Λ^2 = 50
Λ^2 = ∞
$\rho_{1,0}$
0
-0.1
-0.2
Λ^2 = 50
Λ^2 = ∞
1.0
0.95
0.90
0.85
cos θ



$\rho_{0,0}$
0.7
0.6
0.5
$\rho_{1,-1}$
0.10
0.05
0
p_L = 10.0 BeV/c
$\Lambda^2 = \infty$
$\rho_{1,0}$
0.24
0.22
0.20
0.18
0.16
0.14
1.0
0.95
0.90
cos θ