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Three Nucleon Problem with Separable Potentials*

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Due to recent advances in the reformation of the three-body scattering problem, specifically the recasting of the Lippmann-Schwinger equations into a Fredholm form,[1,2] as well as an elegant total angular momentum separation[3] for the three-body problem, it has become feasible to treat this problem numerically. In this note we shall discuss the low energy three-nuclear problem. As a further simplification we shall treat the two-particle interactions by separable spin dependent potentials adjusted to fit the n-n and n-p scattering length and effective ranges.[4,5] No tensor or spin-orbit type coupling was included. The difference in local and separable potentials occurs in the extension off the energy-momentum shell. It was felt that the use of such an interaction would be a good approximation to scattering by local potentials. However, as we shall discuss later, this type of interaction introduces a very unphysical situation into the three-body problem.

The two-body interactions between neutron-proton and neutron-neutron were taken to be

$$< \vec{k} | V_{np} | \vec{q} > = (\lambda_t^{np} P_t + \lambda_s^{np} P_s) / (\sqrt{k^2+\beta^2} \sqrt{q^2+\beta^2})$$

$$< \vec{k} | V_{pp} | \vec{q} > = \lambda_s^{pp} P_s / (\sqrt{k^2+\beta^2} \sqrt{q^2+\beta^2}) \qquad ,^6$$

where P_t and P_s are triplet, and singlet projection operators λ_t^{np} and β were adjusted to give the correct n-p triplet scattering length and deuteron binding energy,[7] i.e., a = 5.397F, E_B = 2.2245 MeV. λ_s^{nn} was chosen to give an n-n scattering length of -17.5F. To obtain considerable simplification in the three-body problem the range of the interaction, β, was taken to be the same in all potentials, λ_s^{np} was adjusted to give a singlet n-p scattering length of -23.679F, and as we had no more freedom of varying the range of the singlet interaction the effective range comes out to be 2.257F compared to the experimental 2.459F.

In the three-body system a search was made for poles of the transition matrix in a state of total spin 1/2 and orbital angular momentum zero. The numerical calculation presented two poles. Upon further examination it was found that one of the poles had a residue at the wrong sign, which would place it in the general category of ghosts. It is felt that this difficulty is due to the nonlocality of the interaction. In fact, for sufficiently attractive potentials one may, in the separable case, obtain the same effect in two-body scattering. The results of Ref. 5a likewise showed two poles for a certain choice of parameters.[8]

The conclusion that may be drawn from the above is that although separable and local potentials may give the same physical two-body scattering, the manner of extension off the energy momentum shell is very critical in the three-body problem. A detailed discussion of this calculation shall be presented shortly.

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3. R.L. Omnès, Phys. Rev. 134, B1358 (1964).

4. Three-body interactions with separable potentials have been studied by: A.N. Mitra, Nuclear Physics 32, 529 (1962); A.N. Mitra and V.S. Bhasin, Phys. Rev. 131, 1265 (1963); J.H. Hetherington and L.H. Schick, University of Minnesota preprint (COO-1371-9). The treatment that comes closest to the one discussed here is that of Ref. 5.

5. (a) R. Aaron, R.D. Amado and Y.Y. Yam, Phys. Rev. 136, B650 (1964).
   (b) R. Aaron, R.D. Amado and Y.Y. Yam, Phys. Rev. Letters 13, 574 (1964).

6. All the separable potentials considered in the previous references were of the form $\lambda/(k^2+\beta^2)(q^2+\beta^2)$. We have used the square root form in order to reproduce the effective range form exactly for all energies.

7. The two-body scattering parameters are those obtained by H.P. Noyes (submitted to Nuclear Physics).

8. In Ref. 5 (a) two poles were likewise obtained. Professor Amado has informed me that in that case no anomalous residues were found. This may be due to a difference in potentials employed and more probably in a difference in the treatment of the neutron-neutron forces in the three-body problem.